9/26/ Calc3 Quiz 7

avg 83%
med 100%

$\bar{r}(t) = \langle 2\cos 3t, 2\sin 3t, 4t - t^2 \rangle$

1. $\bar{r}'(t) = \langle -6\sin 3t, 6\cos 3t, 4 - 2t \rangle$

$\bar{v}(t)$

$\bar{r}''(t) = \langle -18\cos 3t, -18\sin 3t, -2 \rangle$

$\bar{a}$

2. find $\bar{R}(t)$: $\begin{cases} R'(t) = \\ R(0) = (0,0,0) \end{cases}$

$R(t) = \int \langle 2\cos 3t, 2\sin 3t, 4t - t^2 \rangle dt$

$\langle \frac{2}{3}\sin 3t, -\frac{2}{3}\cos 3t, 2t^2 - \frac{t^3}{3} \rangle + \bar{C}$

$(0,0,0) = \bar{R}(0) = \langle 0, -\frac{2}{3}, 0 \rangle + \bar{C}$

$C = \langle 0, \frac{2}{3}, 0 \rangle$

$\langle \frac{2}{3}\sin 3t, \frac{2}{3} - \frac{2}{3}\cos 3t, 2t^2 - \frac{t^3}{3} \rangle$

Last time: $z = f(x,y)$

function x, y

$w = f(x,y,z) =$ x, y, z

Domain, range, graph

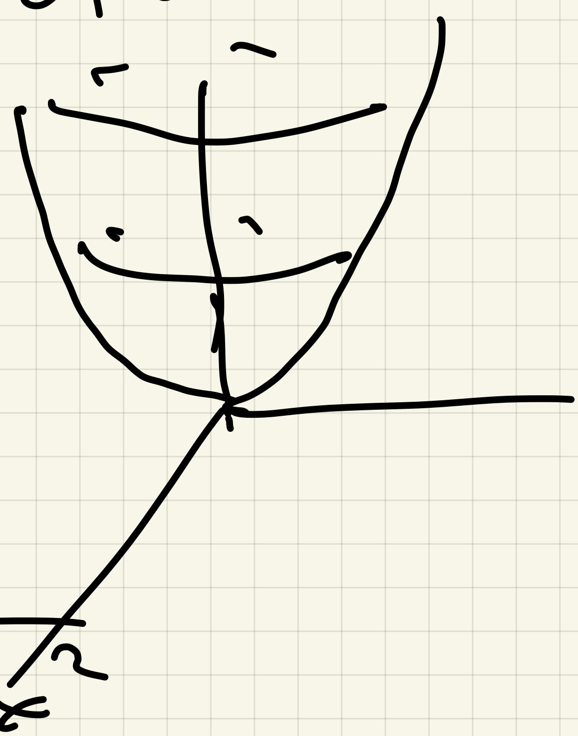
Ex 1 (A few more exs)

(a) $z = f(x,y) = x^2 + y^2$

Dom $= \mathbb{R}^2$

range $= [0, \infty)$

$z = x^2 + y^2$



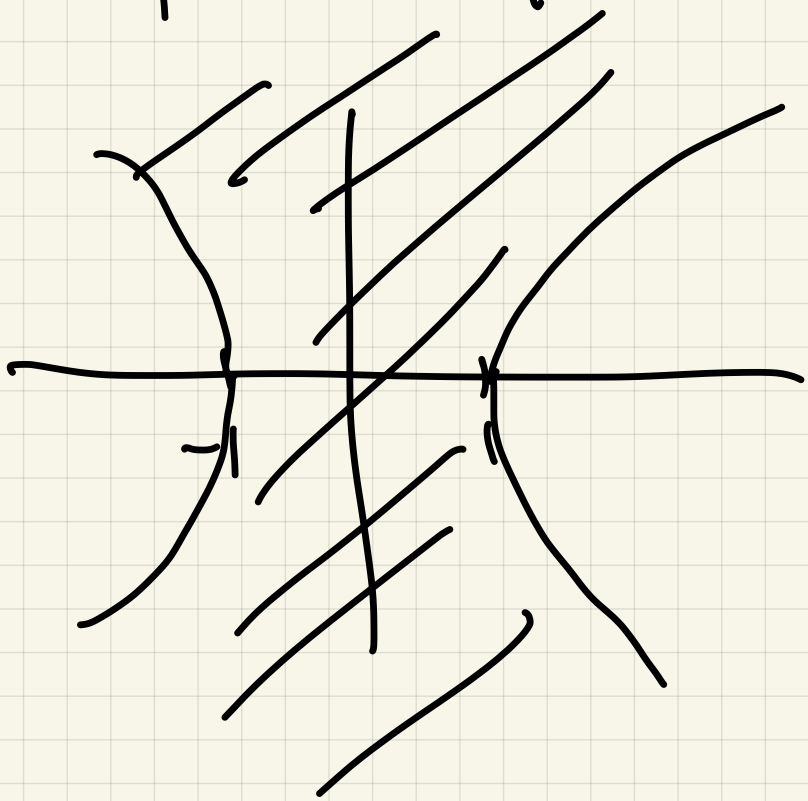
(b) $z = \sqrt{1 + y^2 - x^2}$

Domain $1 + y^2 - x^2 \geq 0$

$1 = x^2 - y^2$

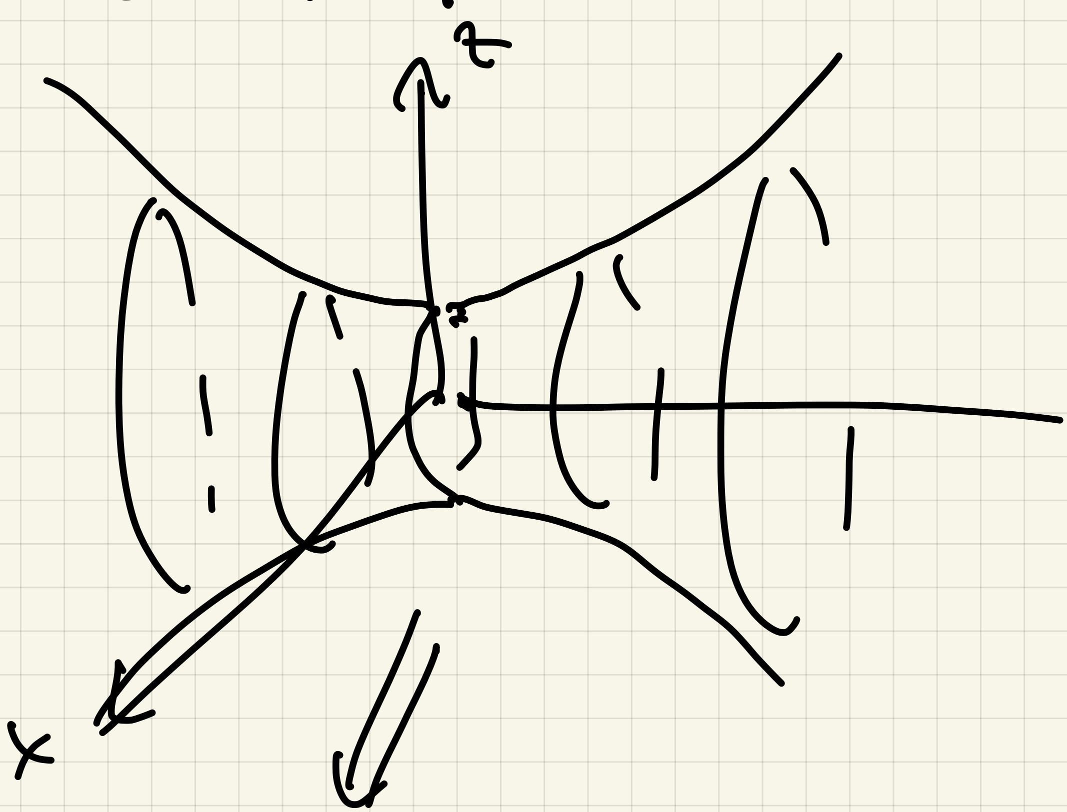
$xy = 1$

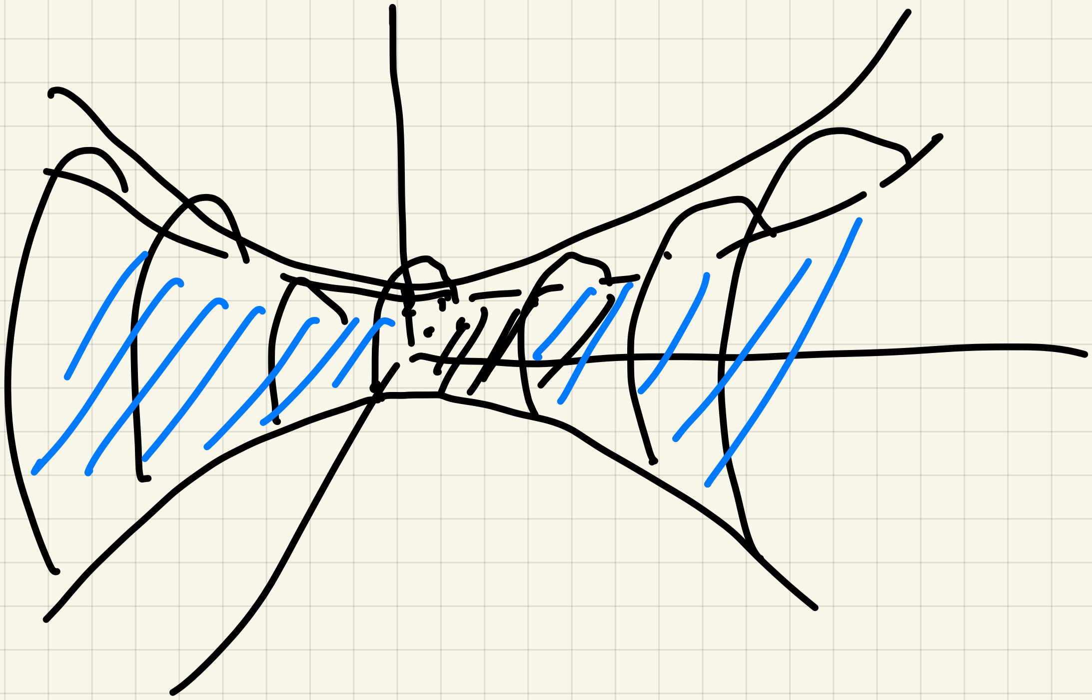
$1 \geq x^2 - y^2$



$z^2 = 1 + y^2 - x^2$

$x^2 + z^2 = 1 + y^2$





§13.2 Limits and continuity

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

① Easy limits

Ⓐ Basic limits:

$$\lim_{(x,y)\to(a,b)} x = a$$

$$\lim_{(x,y)\to(a,b)} y = b$$

$$\lim_{(x,y)\to(a,b)} c = c \qquad c \text{ const}$$

(B) Operations

S Properties of limits:

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

$$\lim f = \lim_{(x,y)\to(a,b)} f(x,y)$$

$$\lim g = \lim_{(x,y)\to(a,b)} g(x,y) = M$$

Then

1. $\lim (f+g) = L+M$
2. $\lim f-g = L-M$
3. $\lim cf = cL$ (c const)
4. $\lim fg = L \cdot M$
5. $\lim \frac{f}{g} = \frac{L}{M}$, $M \neq 0$

Applic: $\lim\limits_{(x,y)\to(1,1)} \dfrac{x^2+xy+7}{x^3+7} = \dfrac{9}{8}$

6. $\lim f^n = L^n$

7. $\lim \sqrt[n]{f} = \sqrt[n]{L}$

$(n>0, \quad L>0 \text{ if } n \text{ even})$

8. If $h: \mathbb{R}\to\mathbb{R}$ continuous at $x=L$, then

$\lim h(f) = h(L)$

Ex) (a) $\lim\limits_{(x,y)\to(3,2)} x+y^3 = 3+2^3 = 11$

(b) $\lim\limits_{(x,y)\to(3,2)} \dfrac{x+y^2}{x^2-y^2} = \dfrac{4}{5}$

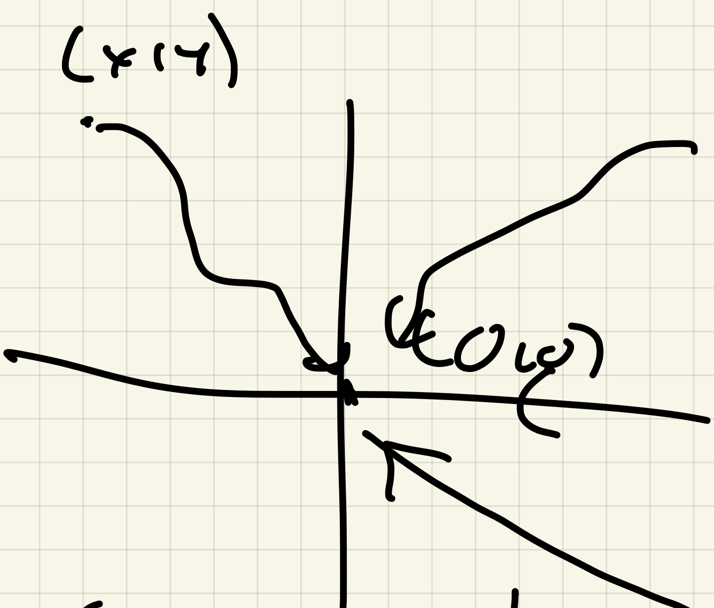
(c) $\lim\limits_{(x,y)\to(3,2)} \ln(x^3-y^3) = \ln 19$

(d) $\lim_{(x,y)\to(3,2)} \dfrac{\ln(\arctan 7/x + x^2)}{\sqrt{\sin y}}$

$= \dfrac{\ln(\arctan 2/3 + 9)}{\sqrt{\sin 2}}$

② Non obvious limits

Ex2 $\lim_{(x,y)\to(0,0)} \dfrac{2x+3y}{x+y} = ??$



(a) find the limit along x-axis

(b) along y-axis

(a) Along x-axis, $y=0$

$$\lim_{x\to 0} \frac{2x+0}{x+0} = \lim_{x\to 0} \frac{2x}{x}$$

$$= \lim_{x\to 0} 2 = 2$$

(b) Along y-axis, $x=0$

$$\lim_{y\to 0} \frac{0+3y}{0+y} = 3$$

so $\lim_{(x,y)\to(0,0)} \frac{2x+3y}{x+y} = \text{DNE}$

Ex 3 $\lim_{(x,y)\to(0,0)} \frac{3x^2+3y^2+x^4-y^4}{x^2+y^2}$

$$x^4-y^4 = (x^2-y^2)(x^2+y^2)$$

$$a^2-b^2 = (a-b)(a+b)$$

$$\frac{3x^2+3y^2+x^4-y^4}{x^2+y^2} = 3 + x^2 - y^2$$

$$\lim_{\to(0,0)} \frac{3x^2+3y^2+x^4-y^4}{x^2+y^2} \overset{\text{sol}}{=} \lim_{(x,y)\to(0,0)} 3 + x^2 - y^2$$

$$\begin{matrix} \text{"} \\ 3 \end{matrix}$$

Ex 4 $$\lim_{(x,y)\to(0,0)} = \frac{\sin(2(x^2+y^2))}{x^2+y^2}$$

$$u = x^2+y^2$$

$$\lim_{u\to 0} \frac{\sin 2u}{u} \overset{\text{LH}}{=} \lim_{u\to 0} \frac{2\cos 2u}{1} = 2$$

Ex 5 $$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

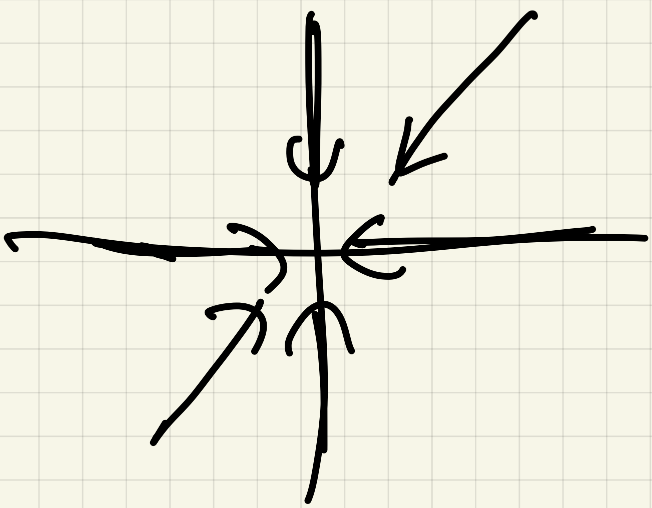
along x-axis: set $y = 0$

$$\lim_{x\to 0} \frac{0}{x^2} = 0$$

along y-axis: set $x=0$

$$\lim_{y\to 0} \frac{0}{y^2} = 0$$

Along line $y=x$



Set $y=x$

$$\lim_{x\to 0} \frac{xy}{x^2+y^2} = \lim_{x\to 0} \frac{x^2}{x^2+x^2}$$

$$= \frac{1}{2} \neq 0$$

So lim DNE

Ex6 $$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}$$

Find limit along lines

x-axis / y-axis

x-axis : $y = 0$

$$\lim_{x \to 0} \frac{0}{x^4} = 0$$

y-axis $x = 0$

$$\lim_{y \to 0} \frac{0}{y^2} = 0$$

x^2

y=x $y = x$

$$\lim_{x \to 0} \frac{x^3}{x^4 + x^2} = \lim_{x \to 0} \frac{x}{x^2 + 1}$$

$$= 0$$

y=mx line slope $m \neq 0$

li Set $y = mx$

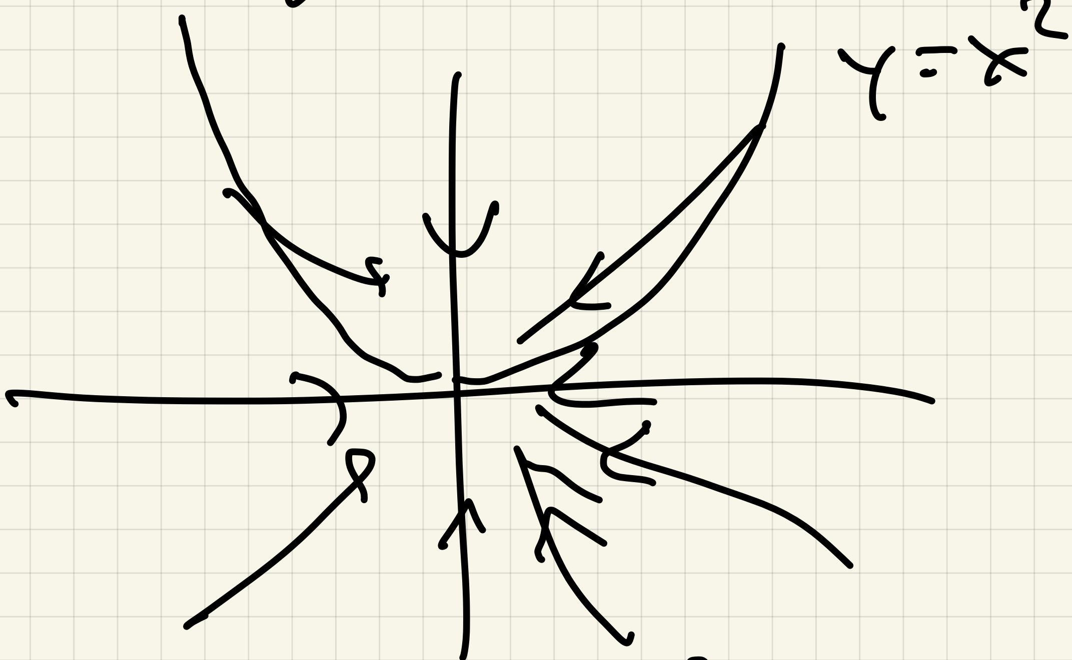
$\div x^2$

$$\lim_{x\to 0} \frac{m x^3}{x^4 + m^2 x^2} =$$

$$\lim_{x\to 0} \frac{m x}{x^2 + m^2} = 0$$

Try path along parabola

$y = x^2$



lim $y = x^2$

$$\lim_{x\to 0} \frac{x^2 y}{x^4 + y^2} = \lim_{x\to 0} \frac{x^4}{x^4 + x^4} = \frac{1}{2}$$

limit DNE

(Note : $y = mx^2$, $m \neq 0$)

§ 13.2

Calc1 : derivative

$y = f(x) \Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Calc3 : $z = f(x,y)$

The first partial derivatives of $z = f(x,y)$ are

$$f_x(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$f_y(x,y) = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

Notation: $f_x = \partial f/\partial x = \partial z/\partial x = z_x$

$$f_y = \partial f/\partial y = \partial z/\partial y = z_y$$

Ex1 $f = \underline{3x} - x^2y^3 + 7y^2$

$$f_x = 3 - 2xy^3$$

$$f_y = 0 - 3x^2y^2 + 14y$$