9/24 | Calc 3

13.1

Last time: Functions of 2 functions

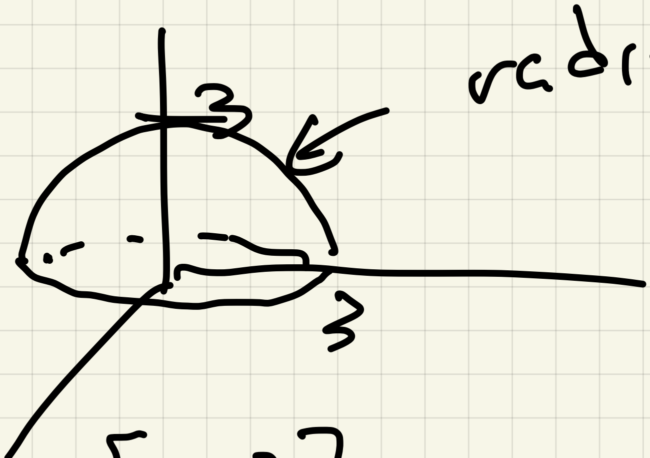
$z = f(x,y)$

Domain / graph / range

Ex 0 $z = f(x,y) = \sqrt{9 - x^2 - y^2}$

Domain: $\{(x,y) : x^2 + y^2 \leq 9\}$

$z^2 = 9 - x^2 - y^2$



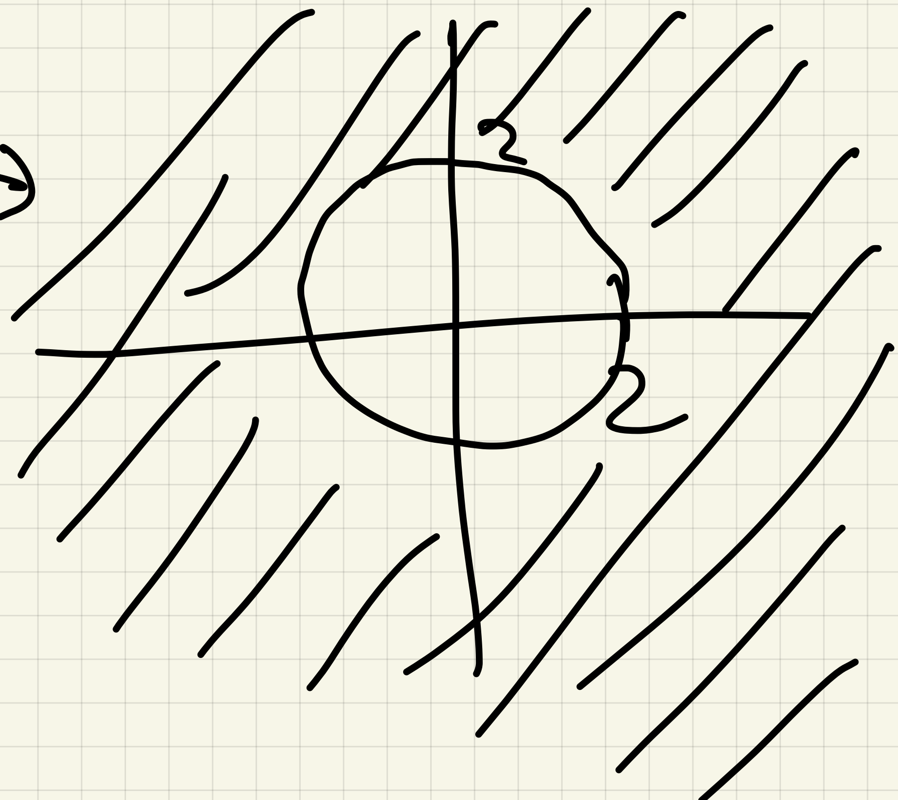
radius 3 sphere

Range = $[0,3]$

Ex 1 $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2-4}}$

Domain: $\{(x,y) : x^2+y^2 > 4\}$

~~Graph~~



Graph:

Can make a "contour map" by sketching

$$\{(x,y) : z = f(x,y) = \text{constant}\}$$

Level sets

$$z = \frac{1}{\sqrt{x^2+y^2-4}}$$

$z = 0$ empty

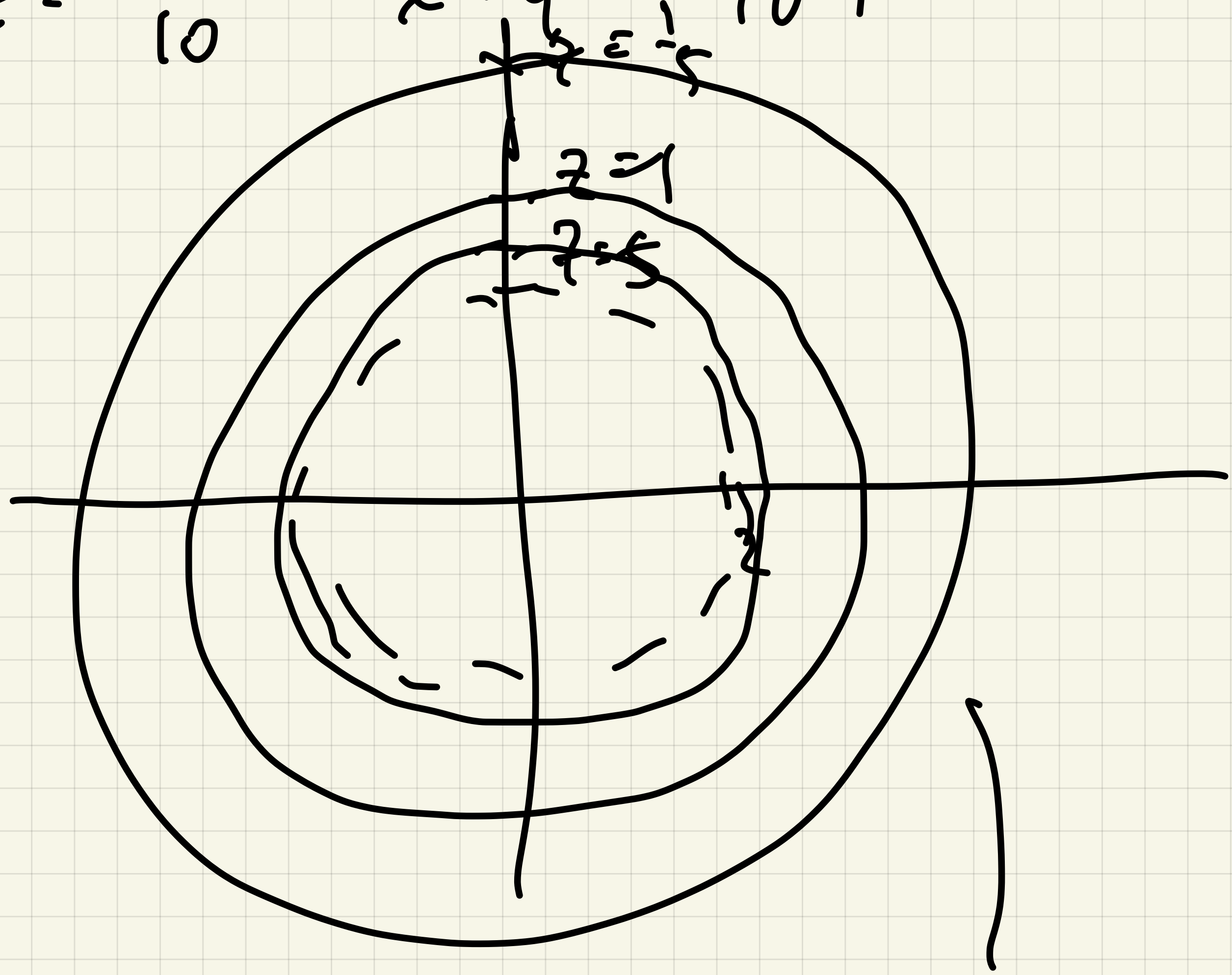
$z = 1 \quad x^2 + y^2 = 5 \qquad 1 = \dfrac{1}{\sqrt{x^2+y^2-4}}$

$z = 5 \quad x^2 + y^2 = 4 + \dfrac{1}{25} \qquad \sqrt{x^2+y^2-4} = 1$

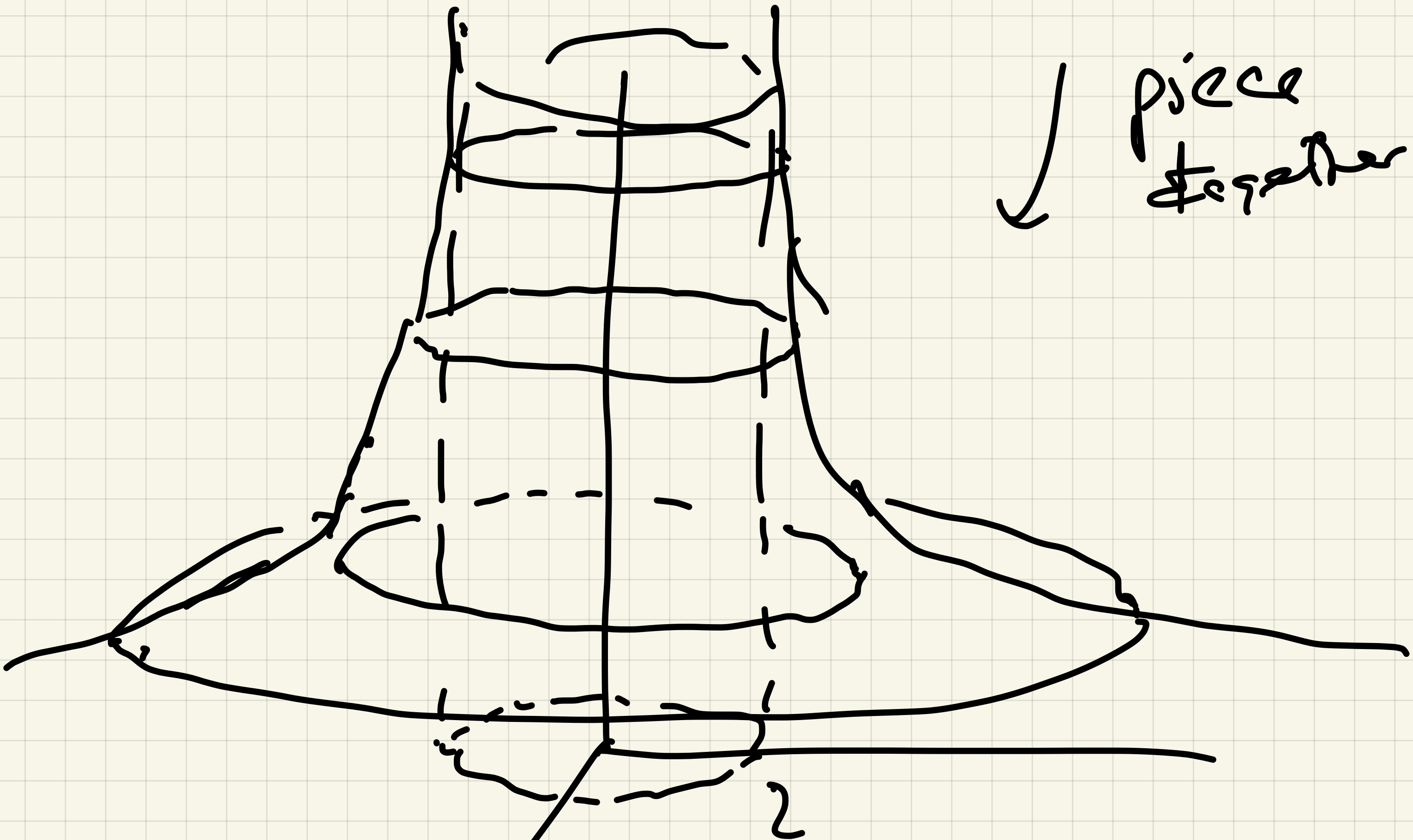
$z = 10 \quad x^2 + y^2 = 4 + \dfrac{1}{100} \qquad x^2 + y^2 = 5$

$z = \dfrac{1}{5} \quad x^2 + y^2 = 29$

$z = \dfrac{1}{10} \quad x^2 + y^2 = 104$



piece together



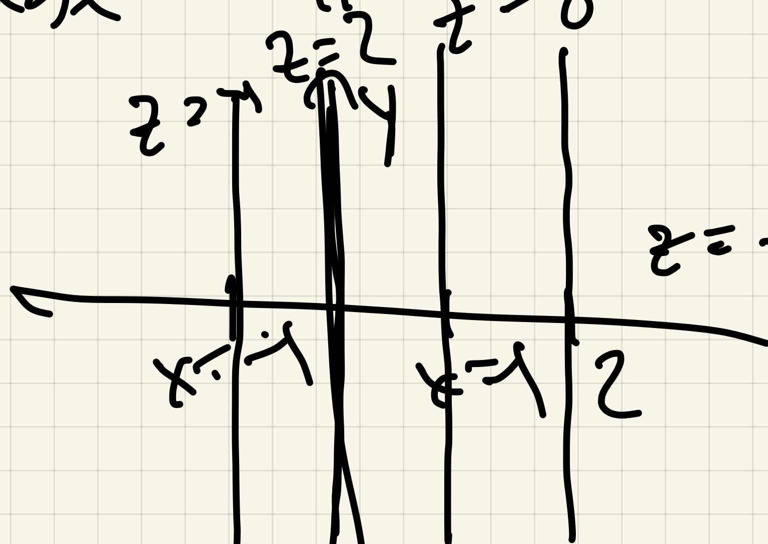
Range $= (0, \infty)$

Ex 2 $z = f(x,y) = -2x + 2$

Domain $= \mathbb{R}^2 =$ all $\{(x,y)\}$

Range $= \mathbb{R}$

sketch level sets:



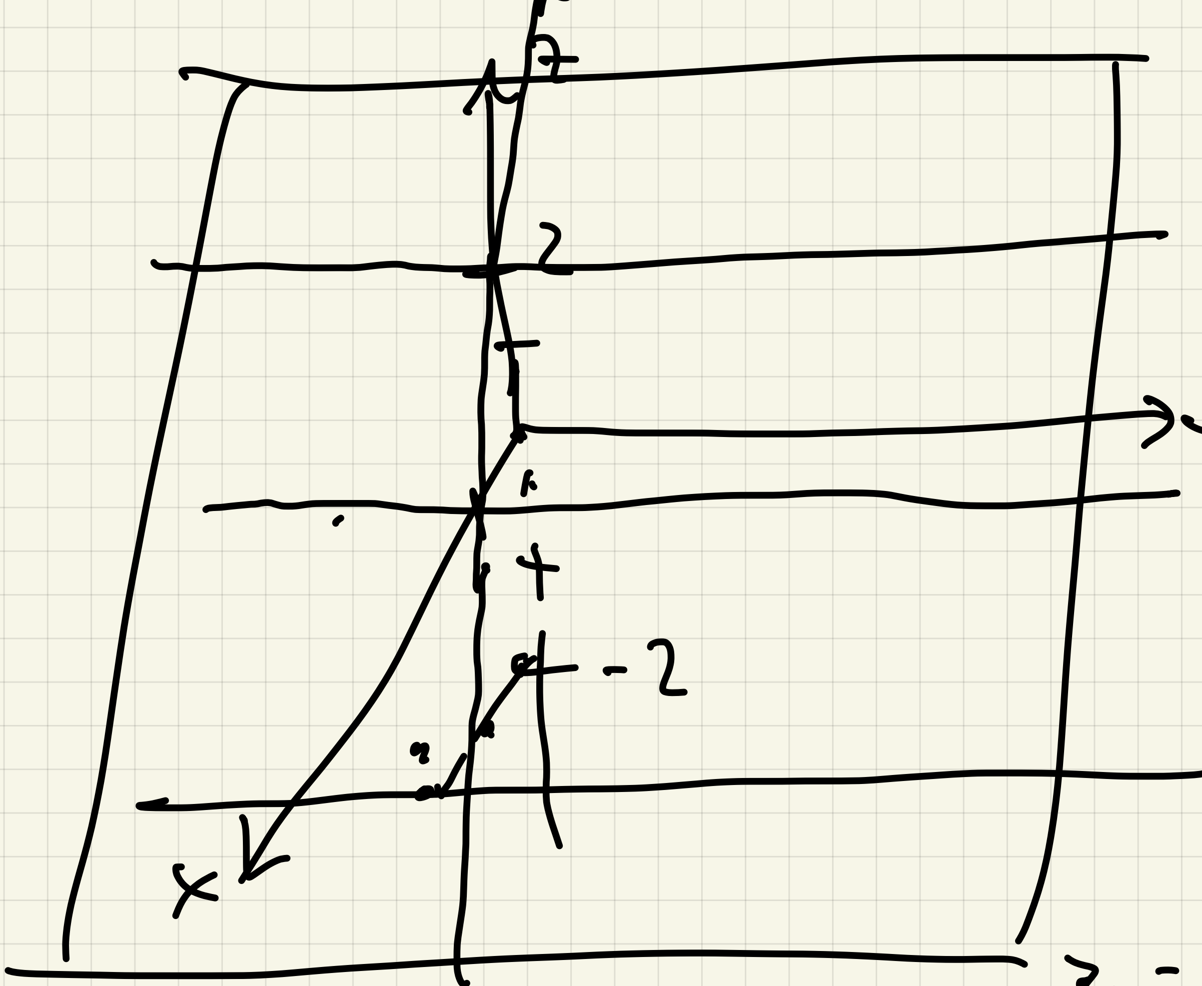
$z=0$: $-2x+2=0$

$z=1$: $1=-2x+2$

$-1=-2x$

$x=\frac{1}{2}$

$z=2$ $\quad x=0$



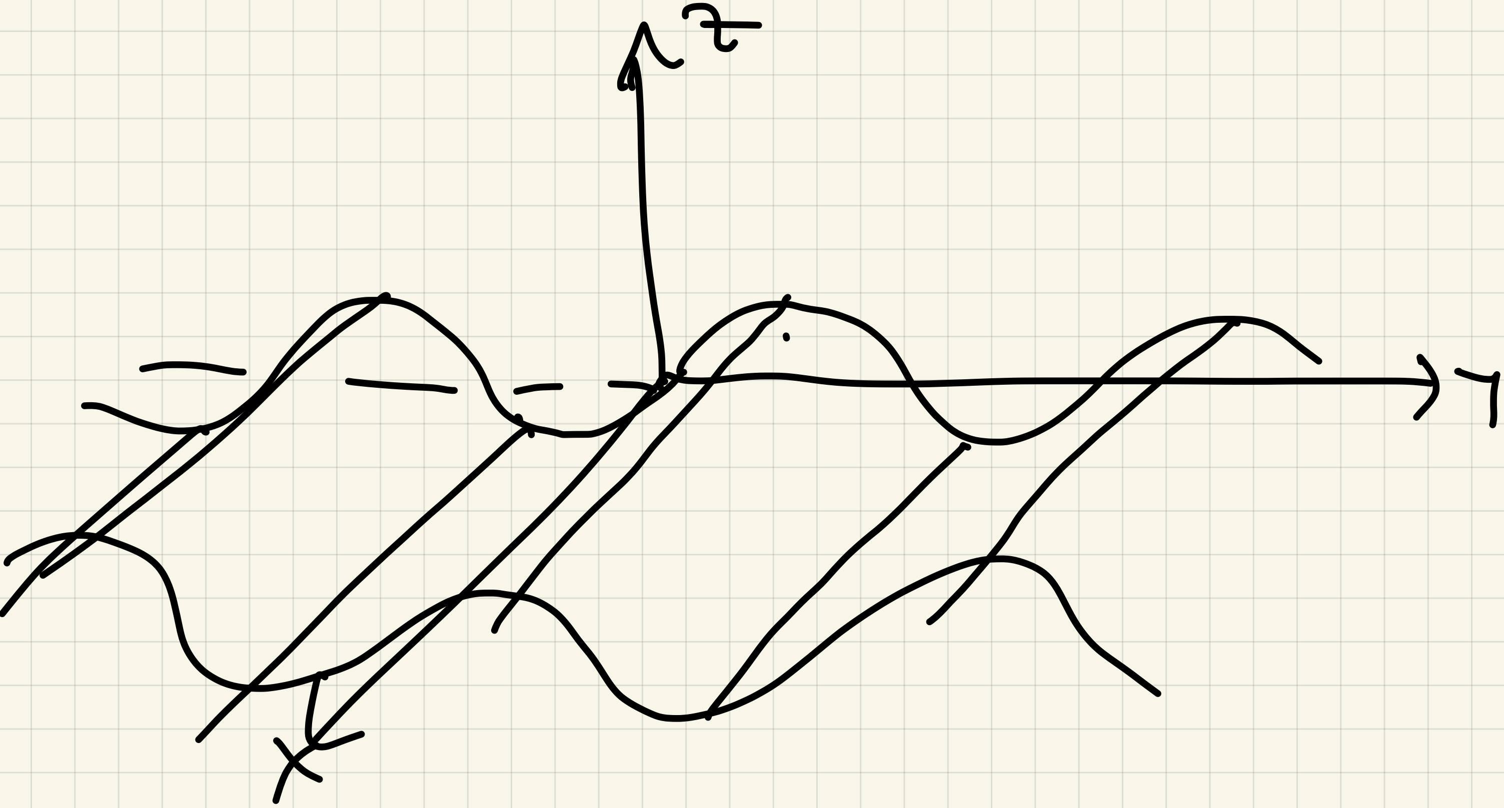
$z=-2x+2$

$2x+z=2$

Ex3 (a) $z=\sin(y)$

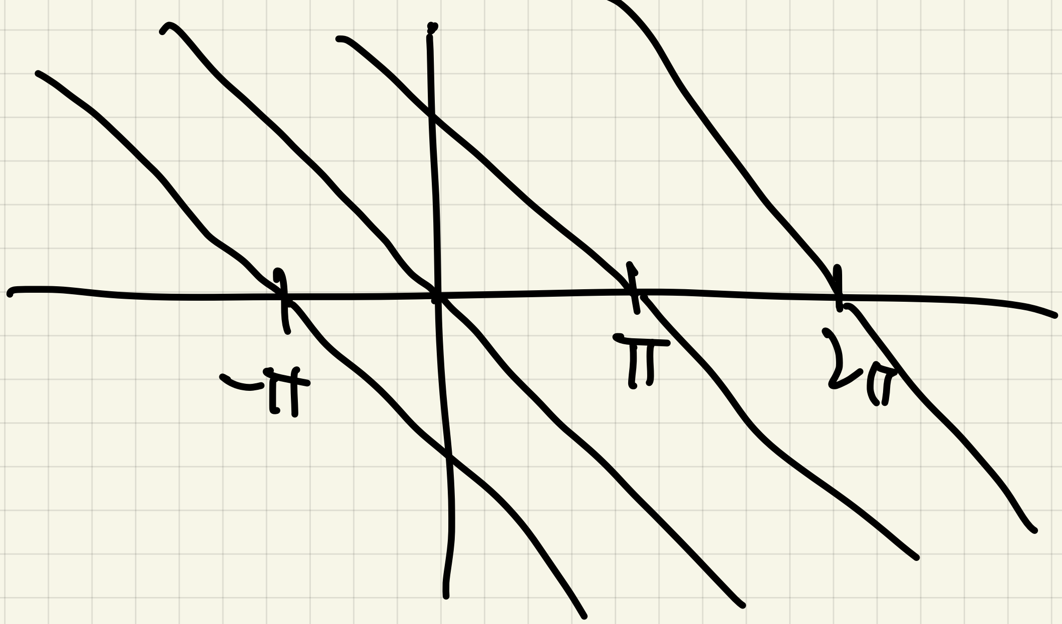
domain $=\mathbb{R}^2$

range $=[-1,1]$

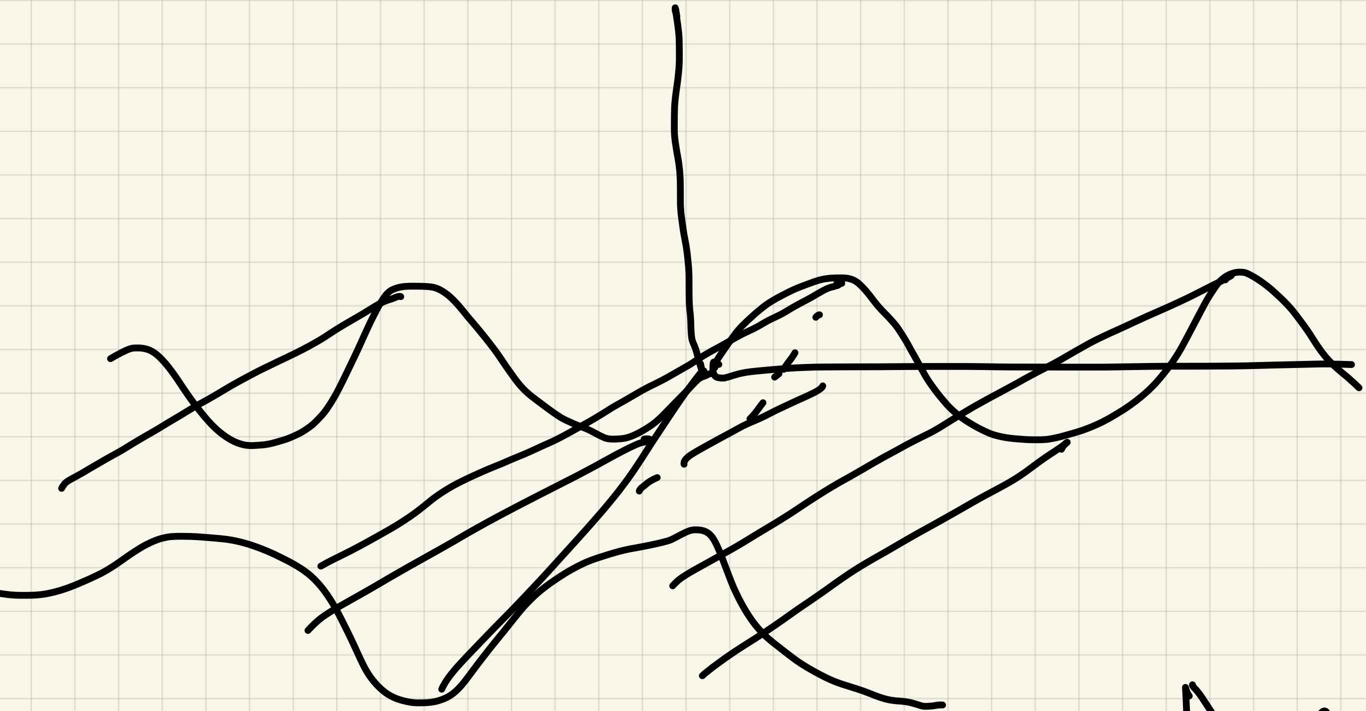


(b) $z = \sin(x+y)$

$z = \text{const} \Rightarrow x+y = \text{constant}$



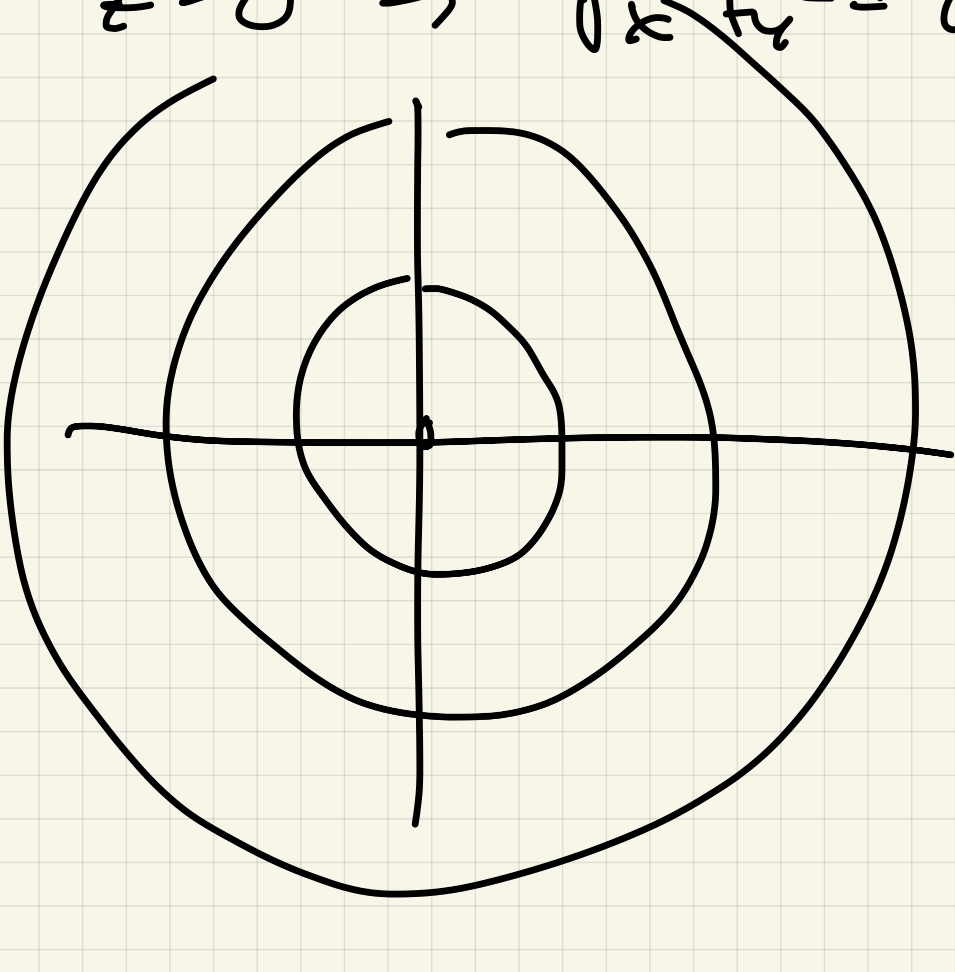
Level sets are lines of slope -1 ($x+y = \text{const}$)

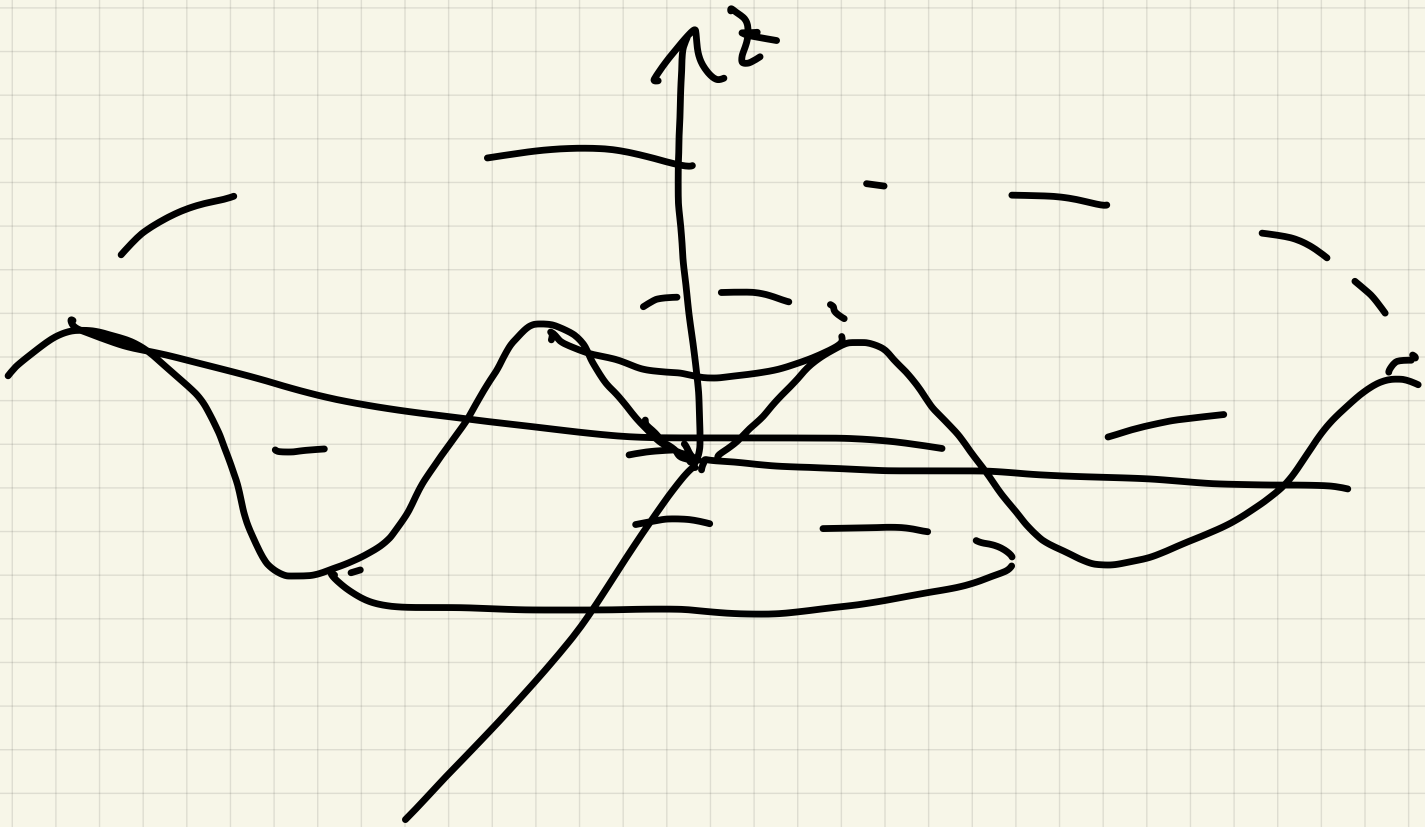


$\text{Dom} = \mathbb{R}^2$
range $[-1, 1]$

(c) $z = \sin\left(\sqrt{x^2+y^2}\right)$

$z = 0 \Rightarrow \sqrt{x^2+y^2} = 0, \pi, 2\pi, 3\pi, \ldots$



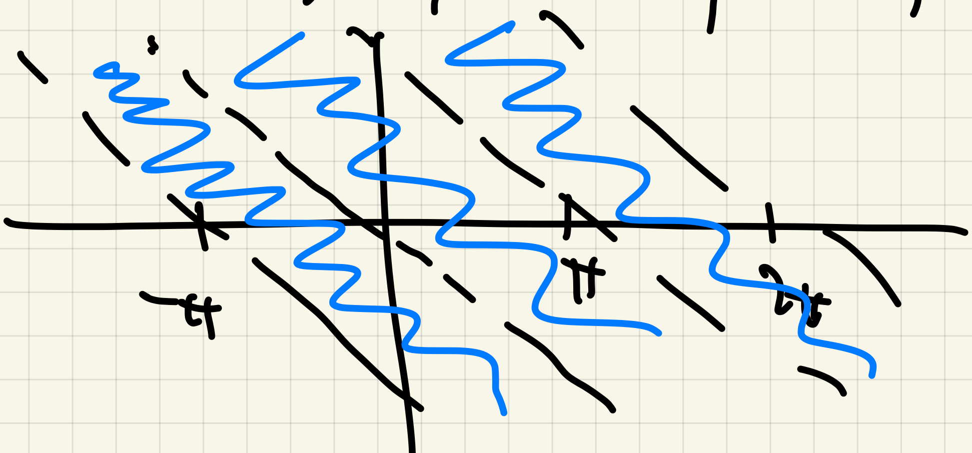


Ex4 Find domain + range

(a) $z = \dfrac{1}{\sin(x+y)} = \csc(x+y)$

Domain $\sin(x+y) \neq 0$

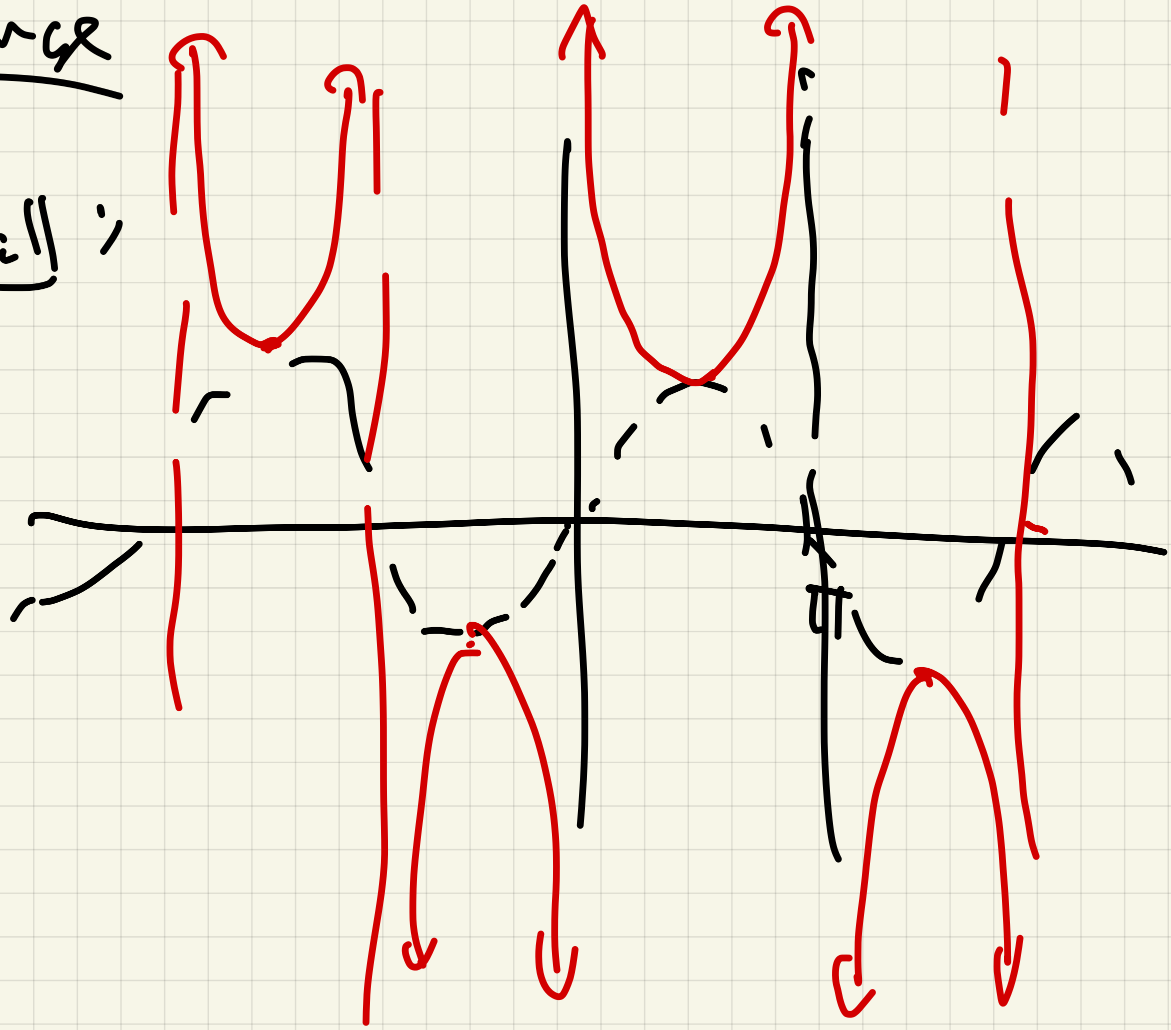
$x+y \neq 0, \pm\pi, \pm 2\pi, \pm 3\pi$



Domain $= \{(x,y) : x+y \neq n\pi,\ n \in \mathbb{Z}\}$

range

Recall ;



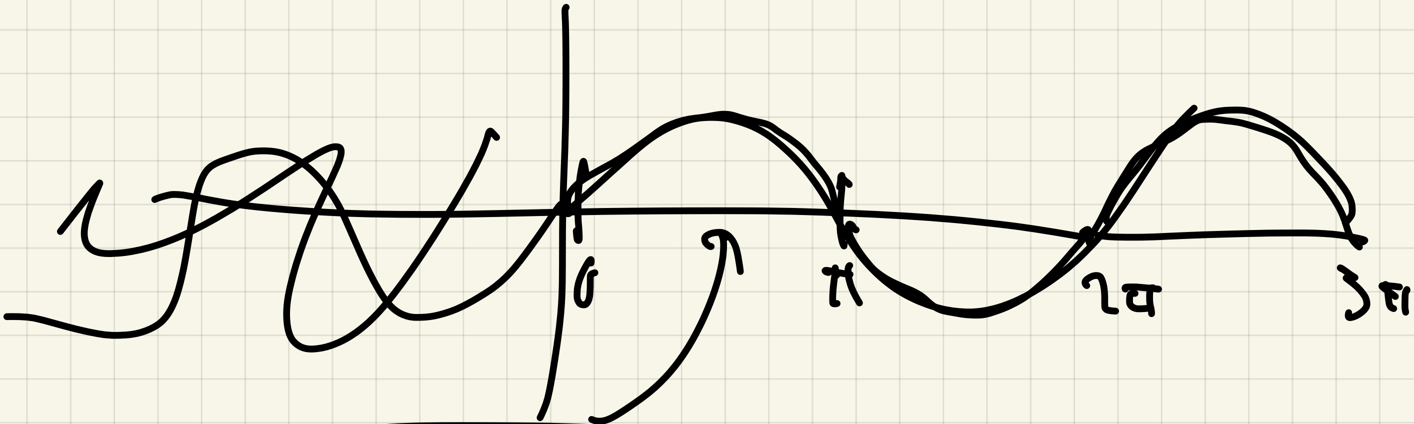
range $(-\infty, -1] \cup [1, \infty)$

(b) , $z = \sqrt{\sin\left(\sqrt{x^2+y^2}\right)}$

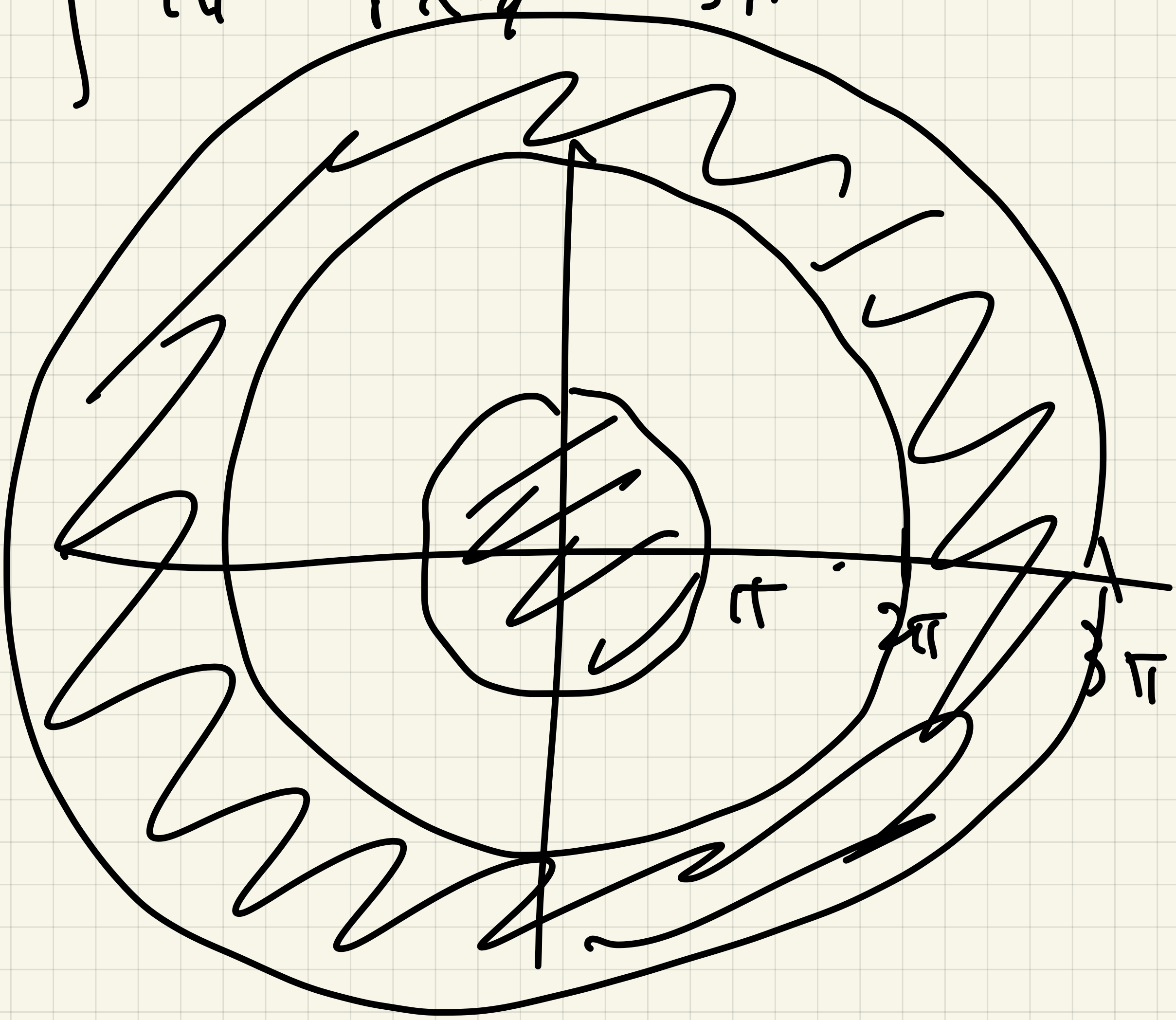
Domain

Need $\sin\sqrt{x^2+y^2} \geq 0$

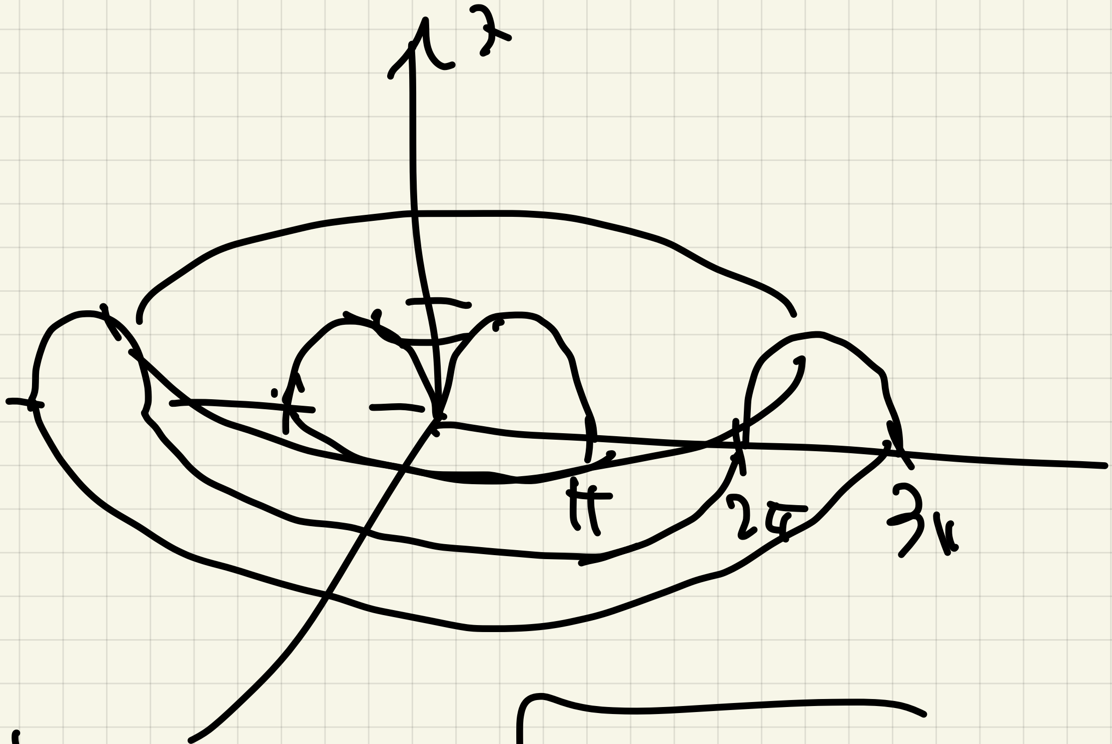
where : $\sin\sqrt{x^2+y^2} \geq 0$



domain
$$\begin{cases} \boxed{0 \le \sqrt{x^2+y^2} \le \pi} \\ 2\pi \le \sqrt{x^2+y^2} \le 3\pi \\ 4\pi \le \sqrt{x^2+y^2} \le 5\pi \end{cases}$$

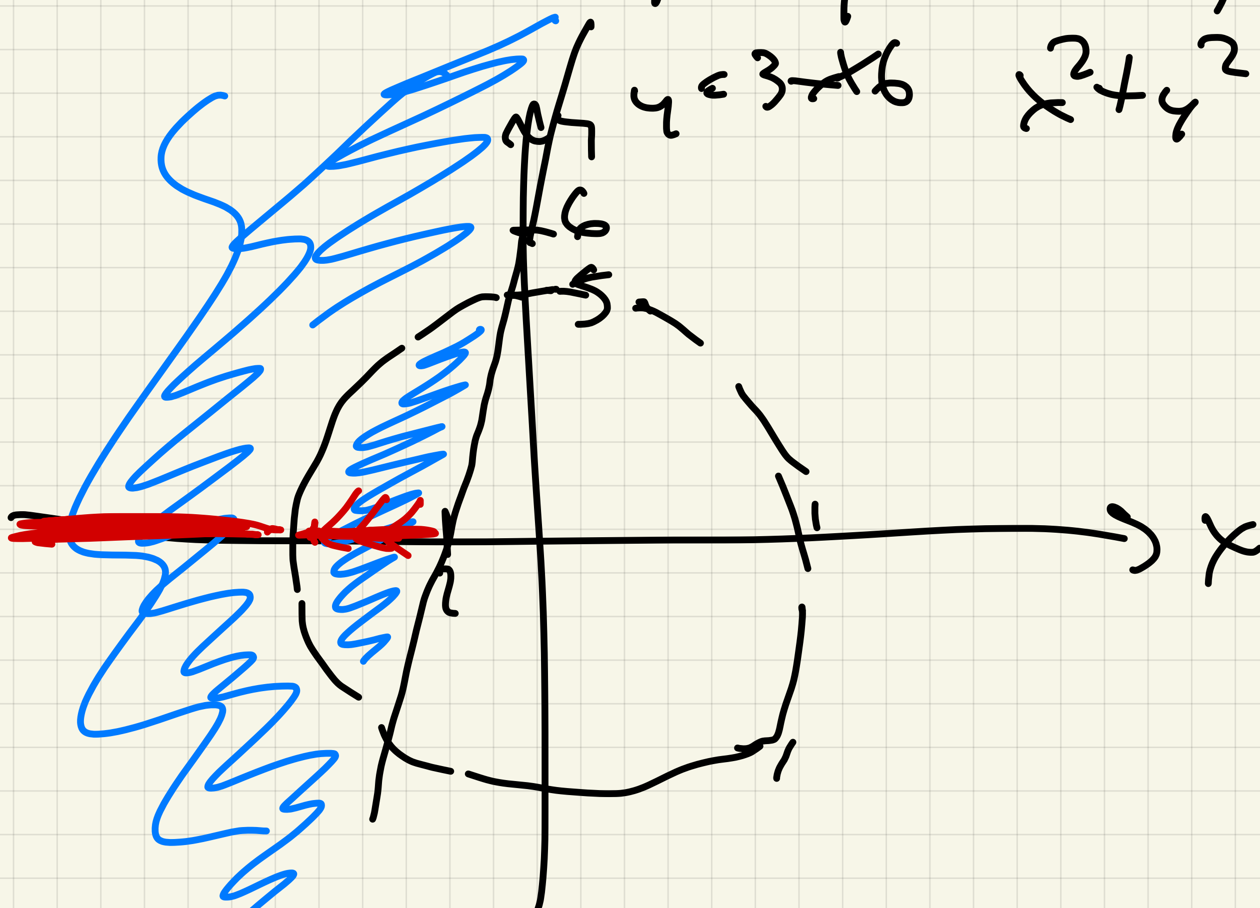


Range = [0,1]



6)
$$z = \frac{\sqrt{y-3x-6}}{25-x^2-y^2}$$

Domain $= \{(x,y) : y \geq 3x+6, \text{ and } x^2+y^2 \neq 25\}$



range: $z = \dfrac{\sqrt{y-3x-6}}{25-x^2-y^2}$

$\boxed{\text{Set } y=0}$ $z = \dfrac{\sqrt{-3x-6}}{25-x^2}$

Inside circle $-5 \le x \le -2$

$x = -2 \Rightarrow z = 0$

What happens as $x \to -5^+$

$$\lim_{x \to -5^+} \frac{\sqrt{-3x-6}}{\boxed{25-x^2}} = \frac{\sqrt{-3x-6}}{(5-x)(5+x)} \to \sqrt{9}$$

0^+ $\Downarrow$ -10

$+\infty$

$[0, \infty)$

Outside circle: $\lim_{x \to -5^-} \dfrac{\sqrt{-3x-6}}{25-x^2} = -\infty$

$$\frac{\int}{0^-} \to$$

$(-\infty, \partial)$

so range $= (-\infty, \infty)$

__More variables:__

$$w = f(x, y, z)$$

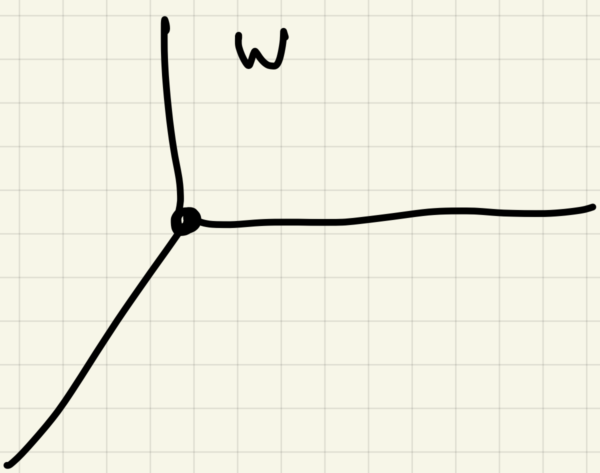
__Sketch level sets__

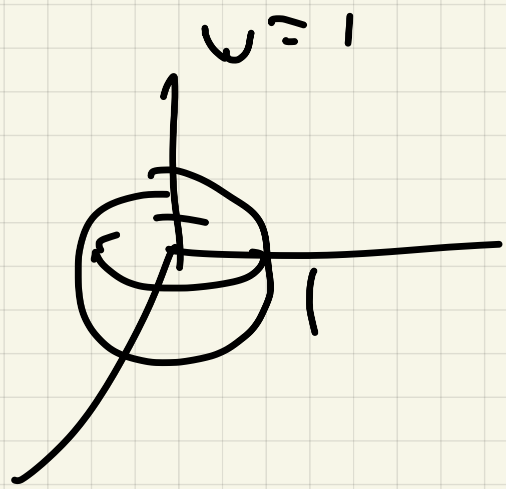
(a) $w = \sqrt{x^2+y^2+z^2}$

$w=0$ $(0,0,0)$

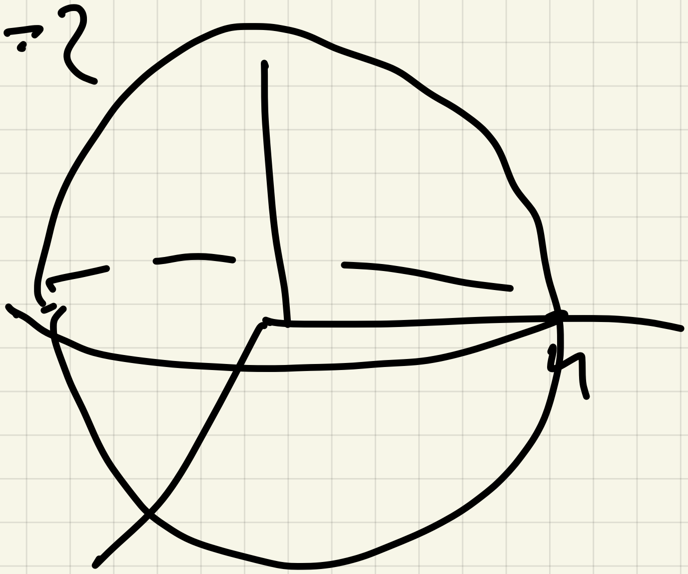
$w=1$ $1 = \sqrt{x^2+y^2+z^2}$

$w=2$ $2 = \sqrt{}$





$w=2$

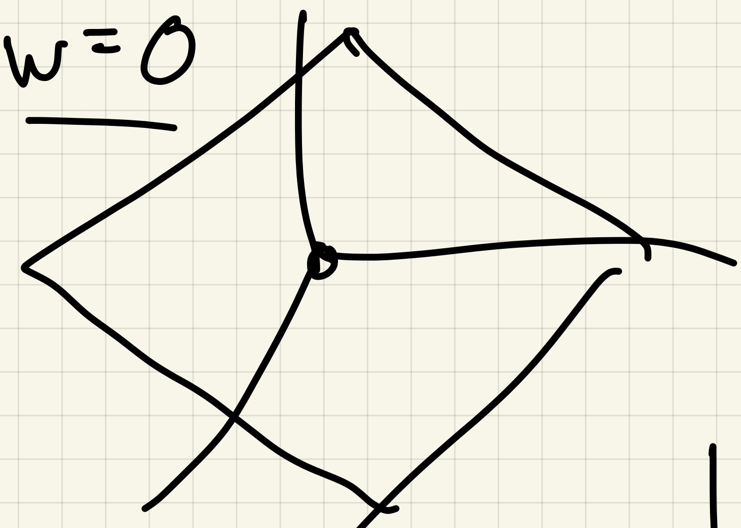


Level sets are spheres

(b) $w = x+2y+3z$

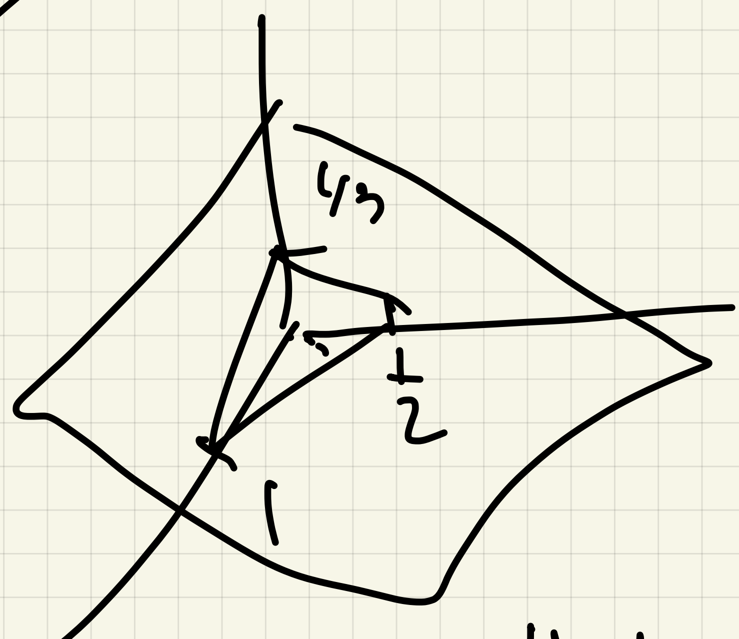
$\underline{w=0}$

$x+2y+3z=0$



$\underline{w=1}$

$1 = x+2y+3z$

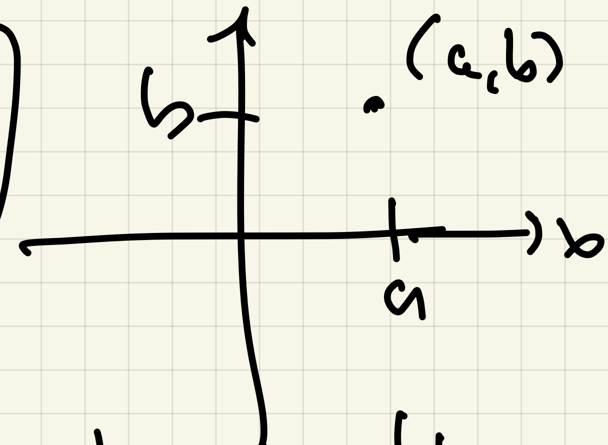


Level sets are parallel planes

§13.2 Limits and Continuity

Defn

$$\boxed{\lim_{(x,y)\to(a,b)} f(x,y) = L}$$



if we can make $|f(x,y) - L|$ arbitrarily small by choosing (x,y) close enough to (a,b)

"Obvious" limits work as usual:

Ex1 $$\lim_{(x,y)\to(2,3)} 5x^2+3xy+17$$

$$5(2)^2+3(2)(3)+17$$

$$20+18+17=55$$

$$\lim_{(x,y)\to(2,5)} \arctan\left(\frac{x+1}{\sqrt{x+y^2}}\right)$$

$$= \arctan\left(\frac{3}{\sqrt{2+25}}\right) =$$

$$\arctan \frac{3}{3\sqrt{3}} =$$

$$\arctan \frac{1}{\sqrt{3}} = \pi/6$$

Ex2 $$\lim_{(x,y)\to(0,0)} \frac{2x+3y}{x+y}$$