9/23/ Calc3

Exam 1

Avg 86%

med 92%

150

13

135

5

120

1

105

1

90

2

1(h) $\bar{u} \perp \bar{v}$

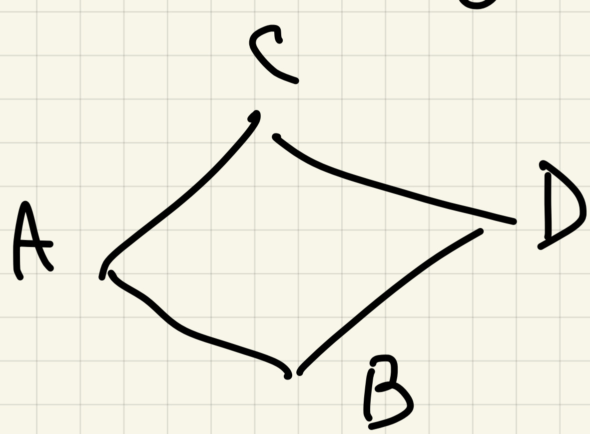
$\langle 0,0,0 \rangle$

2(b) $\overline{AB} \cdot \overline{AC} > 0$ acute

$= 0$ $\perp$

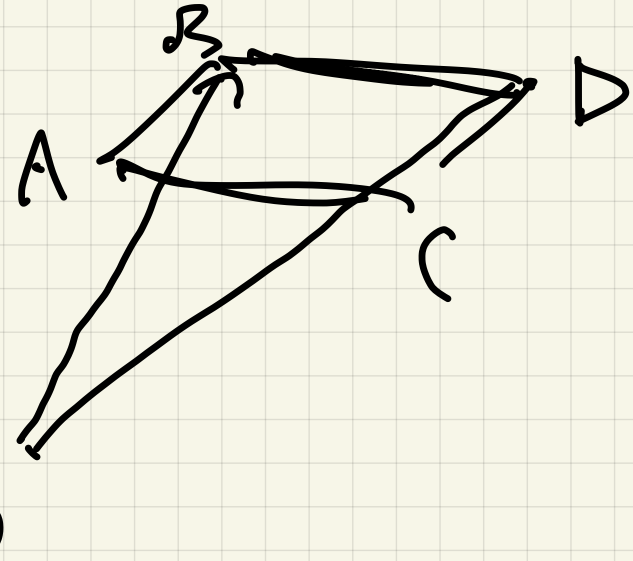
< 0 obtuse

2(f)



Coordinates of $D = \overline{OD}$

$O = (0,0,0)$



$$\overline{OD} = \overline{OB} + \underset{\overline{AC}}{\underset{\parallel}{\overline{BD}}}$$

3(c)

$$\cos\theta = \frac{u\cdot v}{|u||v|} \sim \quad 11.3$$

plane

$$\cos\theta = \frac{|n_1\cdot n_2|}{|n_1||n_2|}$$

3(d)

4(b) complete square

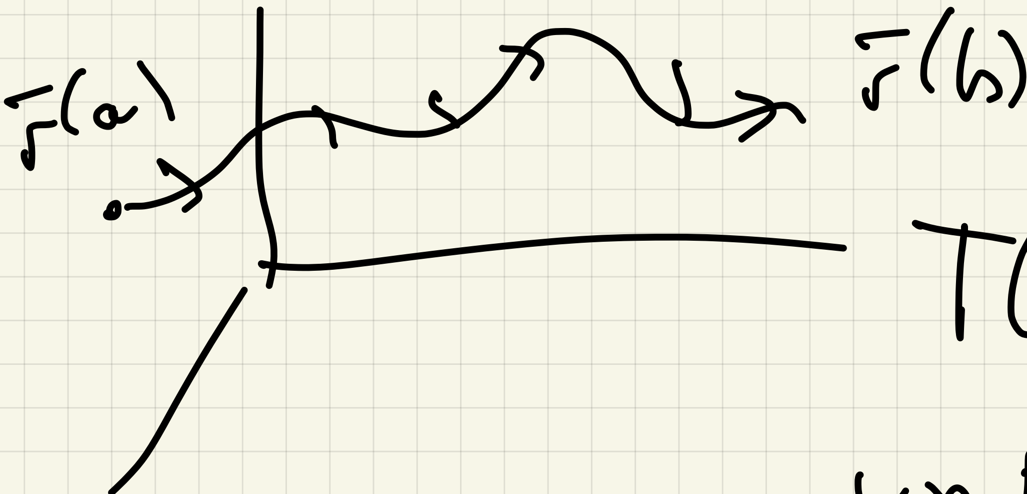
(c) region solid

Goals

Last time

$\vec{r}(t)$ = vector valued

$\vec{r}\,'(t)$ = tangent vector



$$T(t) = \frac{\vec{r}\,'(t)}{|r'(t)|}$$

unit tangent

$$\cancel{\left(N = \frac{T'}{|T'|}\right)}$$

Arclength

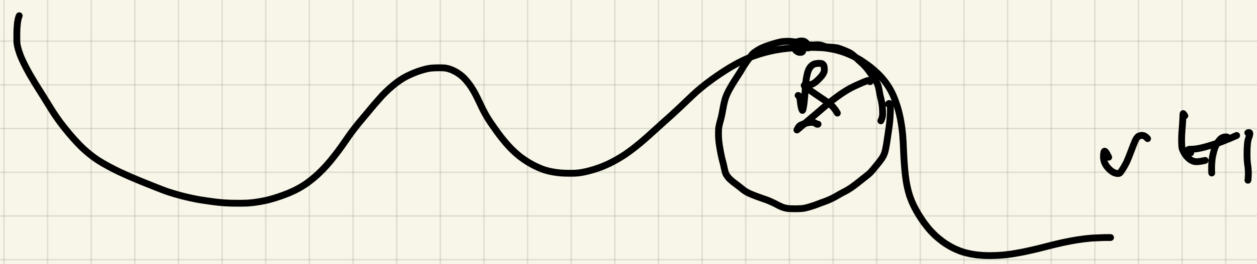
$$L = \int_a^b |r'(t)|\, dx$$

Curvature

$$\kappa = \frac{|T'(t)|}{|r'(t)|} = \frac{|r' \times r''|}{|r'|^3}$$

Book class

Geometric significance!

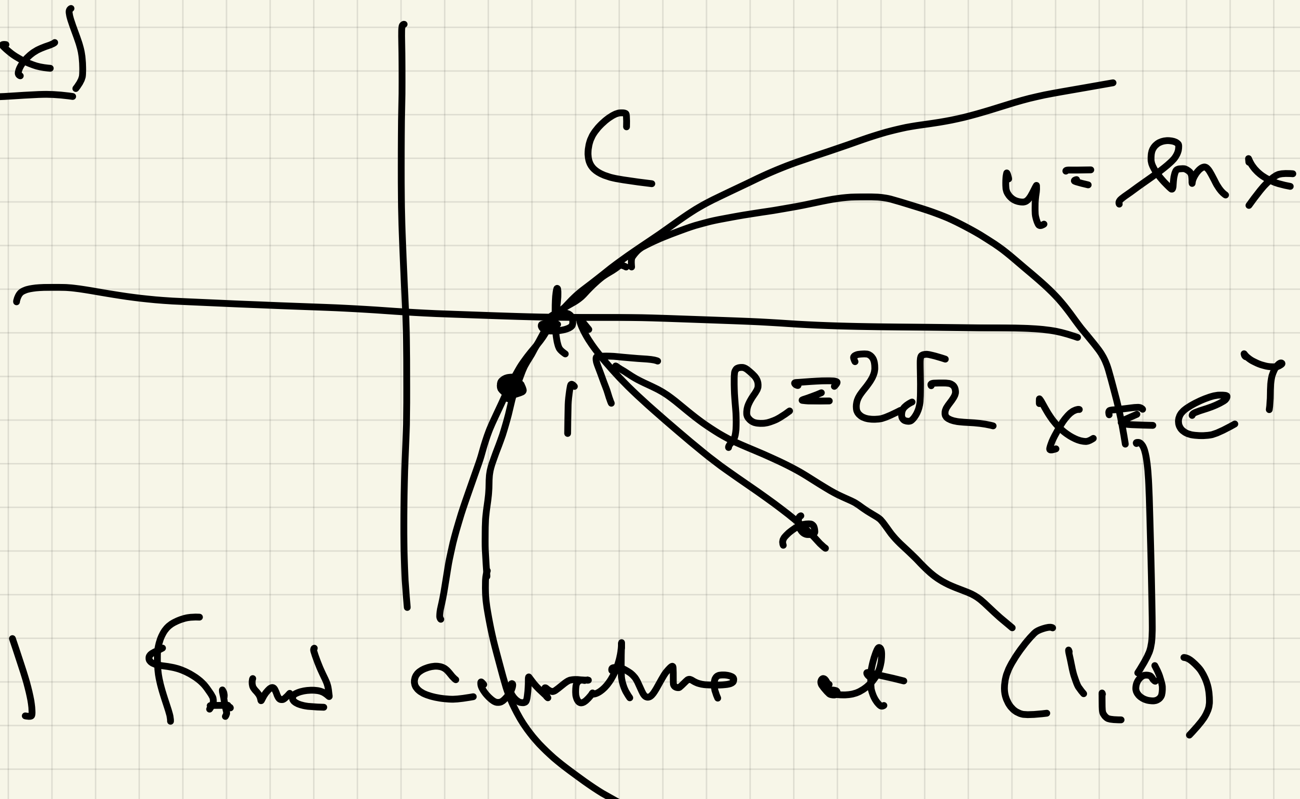


$R = \frac{1}{K}$ = radius of curvature

K big ⇒ circles small

K small ⇒ circles big (flatter)

Ex)



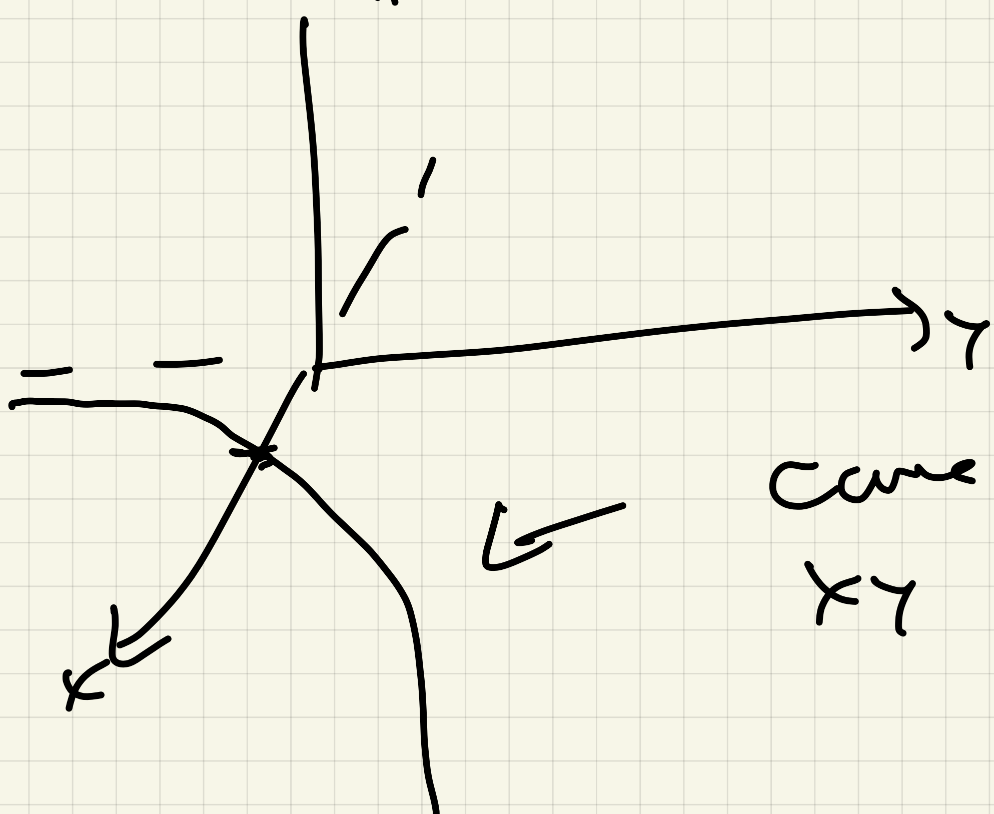
(a) find curvature at (1,0)

(b) Where is curvature largest?

<u>Parametrize</u>: $\bar{r}(t) = \langle t, \ln t \rangle$

or $\quad \overset{x}{} \quad \overset{y}{}$

in $\mathbb{R}^2$ $\bar{r}(t) = \langle e^t, t \rangle$



curve in xy plane

so $\bar{r}(t) = \langle e^t, t, 0 \rangle$

$t = 0$ gives $(1, 0, 0)$

$\bar{r}'(t) = \langle e^t, 1, 0 \rangle$

speed $= |r'(t)| = \sqrt{(e^t)^2 + 1^2 + 0^2}$

$$= \sqrt{e^{2t}+1}$$

$$\bar{r}''(t) = \langle e^t, 0, 0 \rangle$$

$$r' \times r'' = \begin{vmatrix} i & j & k \\ e^t & 1 & 0 \\ e^t & 0 & 0 \end{vmatrix} =$$

$$\langle 0, 0, -e^t \rangle \checkmark$$

$$k = \frac{|r' \times r''|}{|r'|^3} = \frac{|\langle 0,0,-e^t \rangle|}{\left(\sqrt{e^{2t}+1}\right)^3}$$

$$= \frac{e^t}{\left(\sqrt{e^{2t}+1}\right)^3}$$

(a) $\underline{t=0}$: $\dfrac{e^0}{\sqrt{1+1}^{\,3}} = \dfrac{1}{(\sqrt{2})^3} =$

$$\frac{1}{\sqrt[3]{8}} = \frac{1}{2\sqrt{2}}$$

5. $R = 2\sqrt{2}$

(6) $\kappa = \dfrac{e^t}{(e^{2t}+1)^{3/2}}$

maximize:

Calc: $\dfrac{d\kappa}{dt} = \dfrac{d}{dt}\left(\dfrac{e^t}{(e^{2t}+1)^{3/2}}\right) =$

$$\frac{(e^{2t}+1)^{3/2} \cdot e^t - e^t \cdot \frac{3}{2}(e^{2t}+1)^{1/2} \cdot 2 \cdot e^{2t}}{(e^{2t}+1)^3}$$

$$= \frac{e^t\left[(e^{2t}+1)^{3/2} - 3(e^{2t}+1)^{1/2} e^{2t}\right]}{(e^{2t}+1)^3}$$

$$= \frac{e^t\left[(e^{2t}+1) - 3e^{2t}\right]}{(e^{2t}+1)^{5/2}}$$

$$e^{2t}+1-3e^{2t} = 1-2e^{2t} = 0$$

$$e^{2t} = \frac{1}{2} \Rightarrow 2t = \ln\frac{1}{2}$$

$$t = \frac{1}{2}\ln\frac{1}{2}$$

(12

$-.347$

$$K = \frac{e^{\frac{1}{2}\ln\frac{1}{2}}}{\left(e^{\ln\frac{1}{2}}+1\right)^{3/2}} =$$

$$\frac{\left(e^{\ln\frac{1}{2}}\right)^{\frac{1}{2}}}{\left(\frac{1}{2}+1\right)^{3/2}} = \frac{\left(\frac{1}{2}\right)^{1/2}}{\left(3/2\right)^{3/2}} =$$

$$\frac{\frac{1}{\cancel{\sqrt{2}}}}{\frac{3\sqrt{3}}{2\cancel{\sqrt{2}}}} = \frac{2}{3\sqrt{3}} = K$$

$$R = \frac{3\sqrt{3}}{2}$$

~~Ch~~ §13.1 :

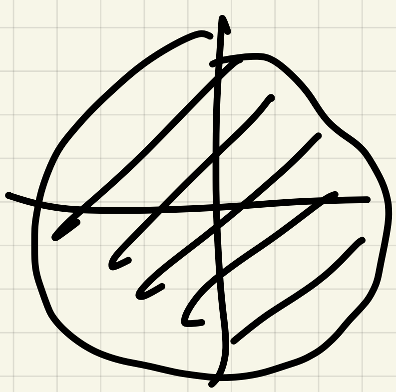
Functions of 2 variables

$z = f(x,y)$

Domain/range/graph

<u>Ex)</u> $z = f(x,y) = \sqrt{9-x^2-y^2}$

<u>Domain</u>: $9 - x^2 - y^2 \geq 0$

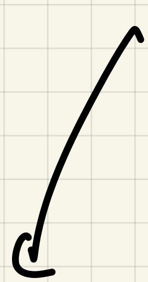


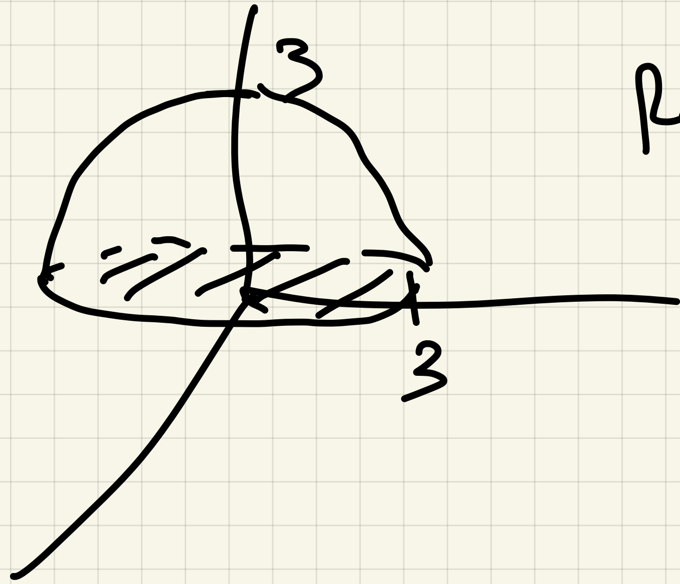
$$\boxed{9 \geq x^2 + y^2}$$

<u>graph</u>: $z = \sqrt{9-x^2-y^2}$

$z^2 = 9 - x^2 - y^2$

$x^2 + y^2 + z^2 = 9$





Range = [0,3]

Ex 1 $\quad z = f(x,y) = \dfrac{1}{\sqrt{x^2+y^2-4}}$

Domain $\quad x^2+y^2-4 > 0$

$x^2+y^2 > 4$