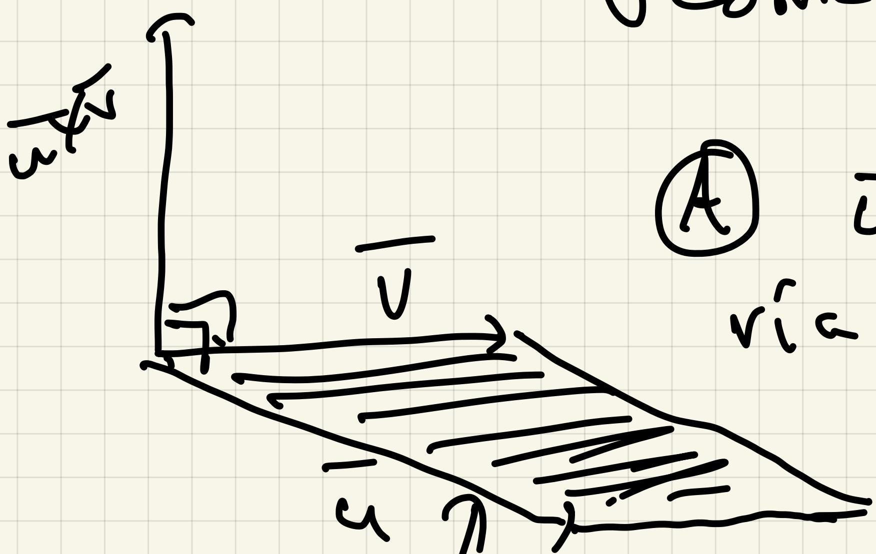
9/21 Calc 3

Last time: cross-product
compute with determinants
Algebraic properties
Geometric properties

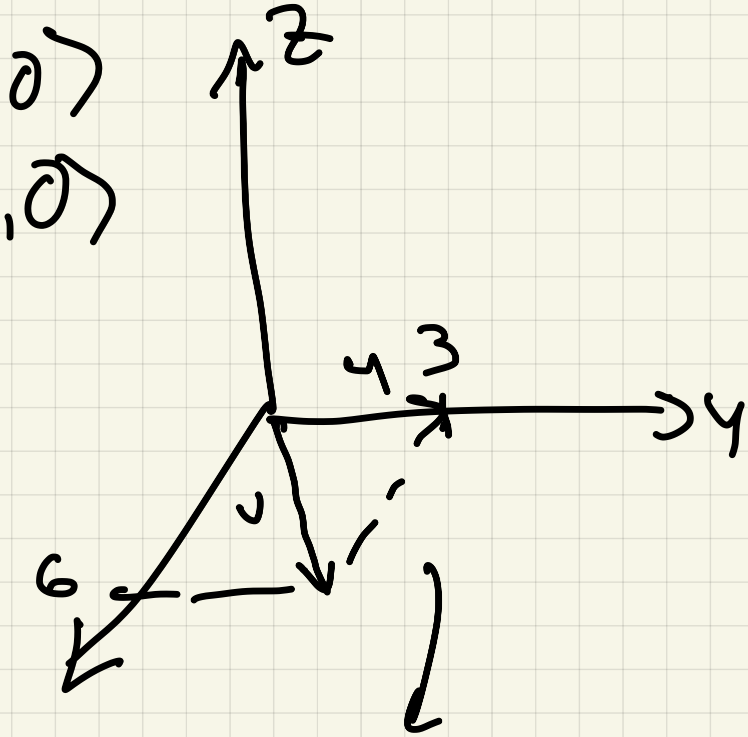


A $\bar{u}\times\bar{v} \perp \bar{u}, \bar{v}$ via right hand rule

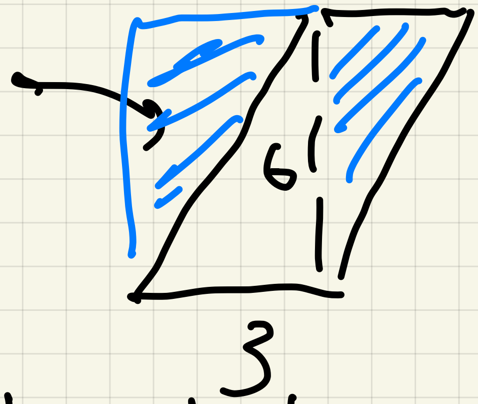
B Area $= |\bar{u}\times\bar{v}|$

Ex
$\bar{u} = \langle 0,3,0\rangle$
$\bar{v} = \langle 6,3,0\rangle$

$\bar{u}\times\bar{v}$ $\downarrow$ direction



$\bar{u} \times \bar{v} = \langle 0, 0, -18 \rangle$ Area = 18



<u>Algebra</u>:
$$\begin{vmatrix} i & j & k \\ 0 & 3 & 0 \\ 6 & 3 & 0 \end{vmatrix} =$$

$$0i - 0j - 18k = \langle 0, 0, -18 \rangle$$

<u>Defn</u>: <u>Triple</u> scalar product of $\bar{u}, \bar{v}, \bar{w}$, is $(\bar{u} \times \bar{v}) \cdot \bar{w}$

<u>Note</u> for easy calculation

$$(\bar{u} \times \bar{v}) \cdot \bar{w} = \bar{u} \cdot (\bar{v} \times \bar{w}) =$$

$$\langle u_1\ u_2\ u_3\rangle \cdot \begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

<u>Ex)</u>

$$\bar{u} = \langle 2, 0, 0\rangle$$
$$\bar{v} = \langle 0, -3, 0\rangle \quad \Rightarrow$$
$$\bar{w} = \langle 0, 0, 4\rangle$$

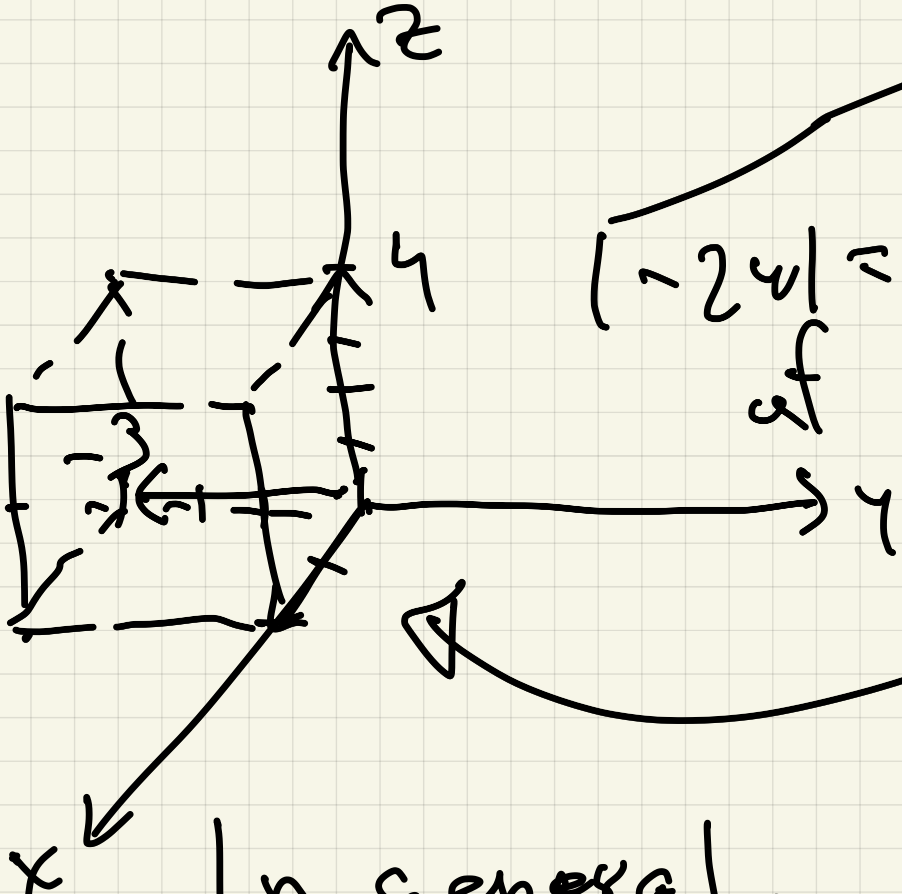
$$(\overline{u \times v}) \cdot \bar{w} = \begin{vmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{vmatrix}$$

$$2 \begin{vmatrix} -3 & 0 \\ 0 & 4 \end{vmatrix} - \underbrace{0\left(\quad \right| + 0 \left| \quad \right)}$$

$$=$$

$2 \cdot (-3) 4 = -24$

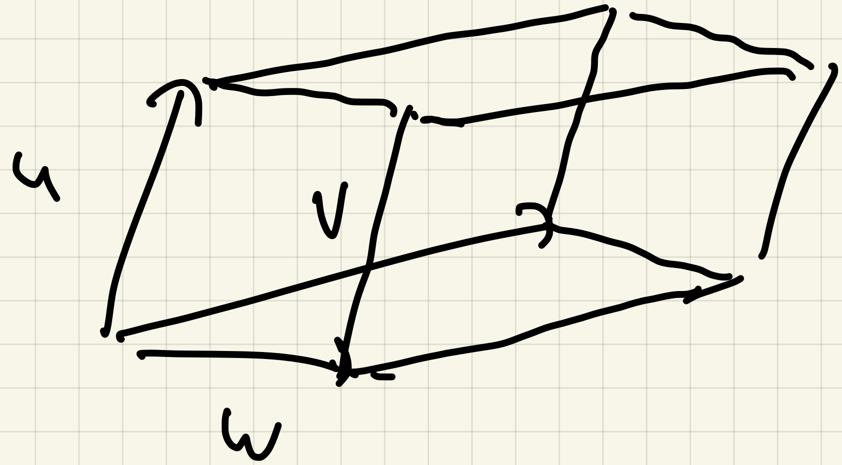
Geometric interpretation:



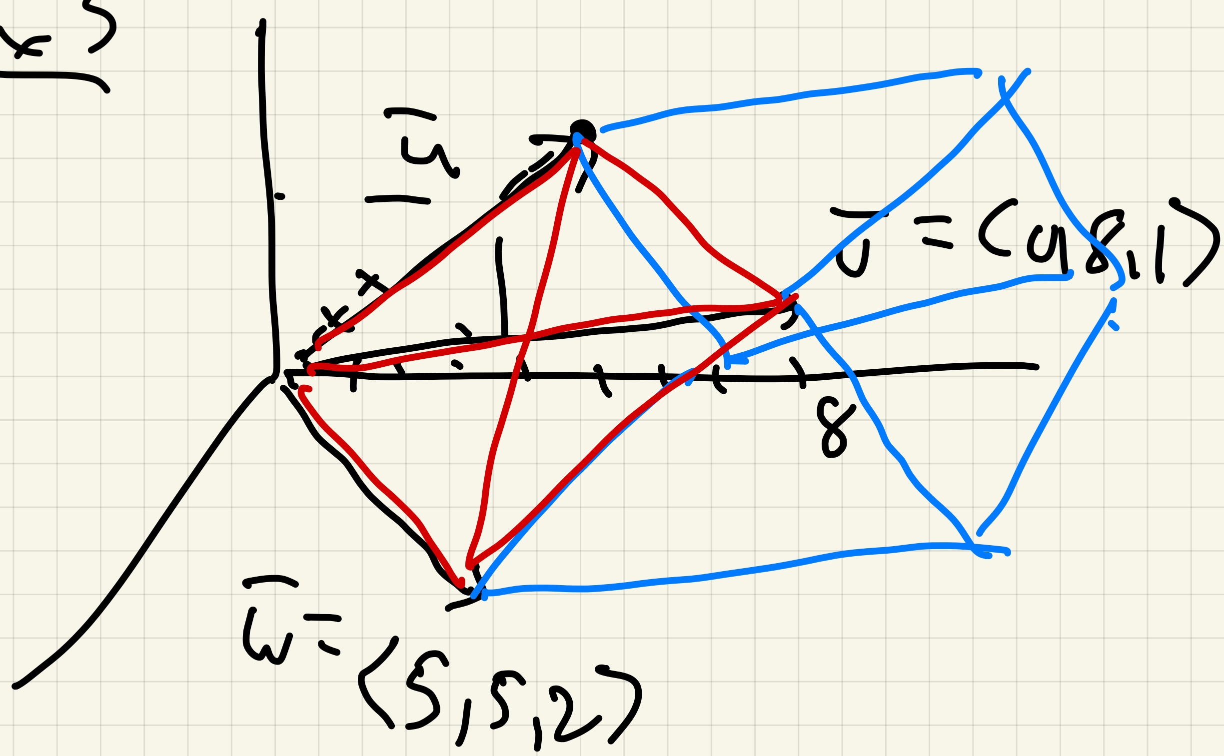
$|-24|$ = volume of box

In general,

$|(u \times v) \cdot w|$ = volume of the parallelepiped spanned by $\bar{u}, \bar{v}, \bar{w}$,



Ex 3



$$\text{Volume} = |(\bar{u} \times \bar{v}) \cdot \bar{w}| =$$

$$|\bar{u} \cdot (\bar{v} \times \bar{w})| =$$

$$\left| \begin{vmatrix} -2 & 4 & 3 \\ 0 & 8 & 1 \\ 5 & 5 & 2 \end{vmatrix} \right| =$$

$$\left| -2\underbrace{\begin{vmatrix}8 & 1\\ 5 & 2\end{vmatrix}}_{11} - 4\underbrace{\begin{vmatrix}0 & 1\\ 5 & 2\end{vmatrix}}_{-5} + 3\underbrace{\begin{vmatrix}0 & 8\\ 5 & 5\end{vmatrix}}_{-40}\right| =$$

$$\left| -22 + 20 - 120 \right| = |-122|$$

$$122$$

<u>Remark</u>: Volume of the tetrahedron spanned by $\vec{u}, \vec{v}, \vec{w}$ is $\frac{1}{6}$ (volume of parallelepiped)

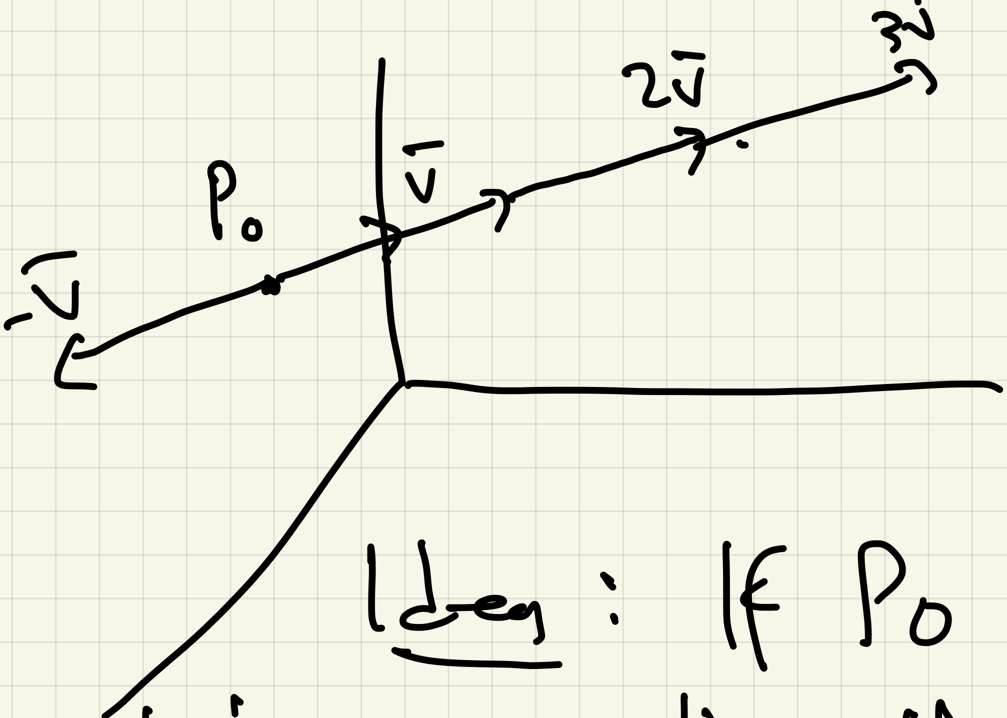
§11.5 <u>Lines and planes in $\mathbb{R}^3$</u>

<u>In $\mathbb{R}^2$</u> line $y = mx + b$

m slope

b y-intercept

<u>In $\mathbb{R}^3$</u> ???



Idea: If P_0 is on a line and direction is $\bar{v}$ we should add scalar multiples of $\bar{v}$ to P_0

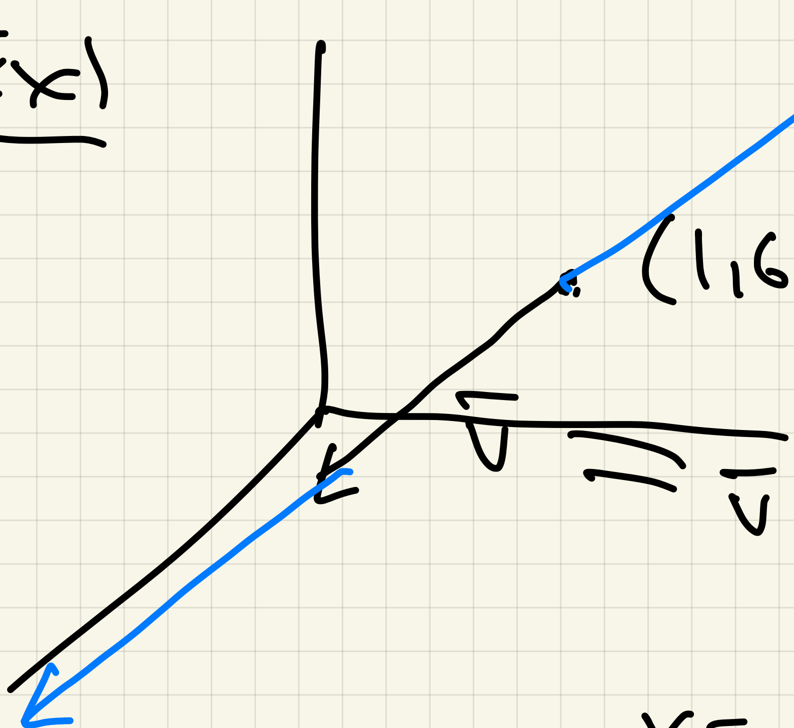
Parametric Equation for a line:

If line L is $//$ to $\bar{v} = \langle a, b, c \rangle$ and $P_0 = (x_0, y_0, z_0)$ is on L, Then L has parametric equations

$$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$$
$$= \overline{P_0} + t\bar{v}$$

OR $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 + ta \\ y_0 + tb \\ z_0 + tc \end{pmatrix}, \ t \in \mathbb{R}$

<u>Ex1</u>



$\bar{v} = \langle 2, -6, -1 \rangle$

<u>Equation</u>:

$$x = 1 + 2t$$
$$y = 6 - 6t \qquad t \in \mathbb{R}$$
$$z = 1 - t$$

<u>Remarks</u> ① Not like equation $y = mx + b$ due to variable t

② Many descriptions of same line:

$$\begin{matrix} x = \\ y = \\ z = \end{matrix} \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -10 \\ 30 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -6 \\ -1 \end{pmatrix}$$

$$x = 3 - 10t$$
$$y = 30t$$
$$z = 5t$$

same line as Ex 1:

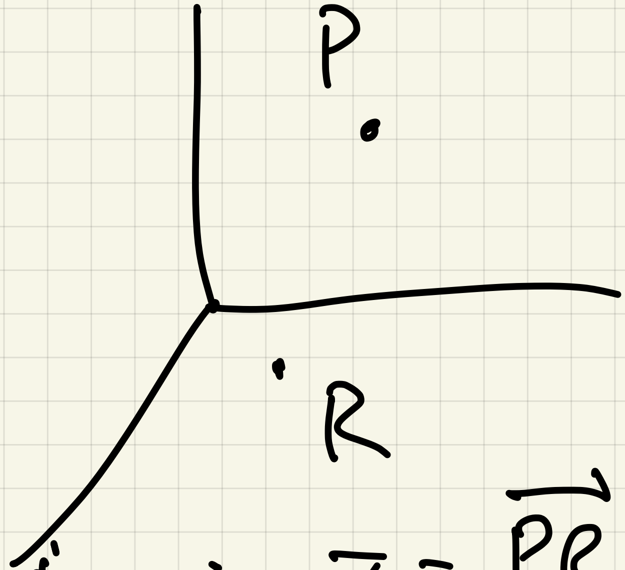
$t = 1 \Rightarrow (3,0,0)$ is on line

also $\quad -5\begin{pmatrix} 2 \\ -6 \\ -1 \end{pmatrix} = \begin{pmatrix} -10 \\ 30 \\ 5 \end{pmatrix}$

<u>Ex 2</u> Find line L_1

passing through $P = (1,2,4)$

and $R = (3,5,-1)$



direction : $\vec{v} = \overrightarrow{PR} = \langle 2, 3, -5 \rangle$

so $L_1 : \begin{cases} x = 1 + 2t \\ y = 2 + 3t \\ z = 4 - 5t \end{cases}$

(b) Is $Q = (-5, -7, 19)$ on L_1?

Yes: $t = 3 \Rightarrow \begin{pmatrix} -5 \\ -7 \\ 19 \end{pmatrix}$

(c) Does L_1 intersect the

$L_2 : \begin{cases} x = 3 - 4t \\ y = 7 - 6t \\ z = 1 + 10t \end{cases}$

Observe: $L_1 \parallel L_2$

$\langle 2, 3, -5 \rangle$ $\quad$ $\langle -4, -6, 10 \rangle$

So $L_1 = L_2$ or $L_1 \cap L_2 = \emptyset$

To test this,

$$\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{matrix} 3-4t \\ 7-6t \\ 1+10t \end{matrix} \qquad \begin{matrix} \to t = \frac{1}{2} \\ \to t = 5/6 \\ \to 3/10 \end{matrix}$$

NO

so $L_1 \parallel L_2$ don't meet.

(d) Show that L_1 does Intersect

L_3: $\begin{aligned} x &= -3+3t \\ y &= 13-4t \\ z &= -3+t \end{aligned}$

Must try to solve

$$\left.\begin{array}{lrcl} x & 1+2t & = & -3+3s \\ y & 2+3t & = & 13-4s \\ z & 4-5t & = & -3+s \end{array}\right\} \Longrightarrow$$

$$-3s+2t = -4$$
$$4s+3t = 11$$
$$-s-5t = -7$$

$$\begin{aligned} 4s+3t &= 11 \\ -4s-20t &= -28 \\ \hline -17t &= -17 \end{aligned}$$

$t=+1$

$s=2$

So

work

$\Downarrow$

$P = (3, 5, -1)$ on

L_1 and L_2