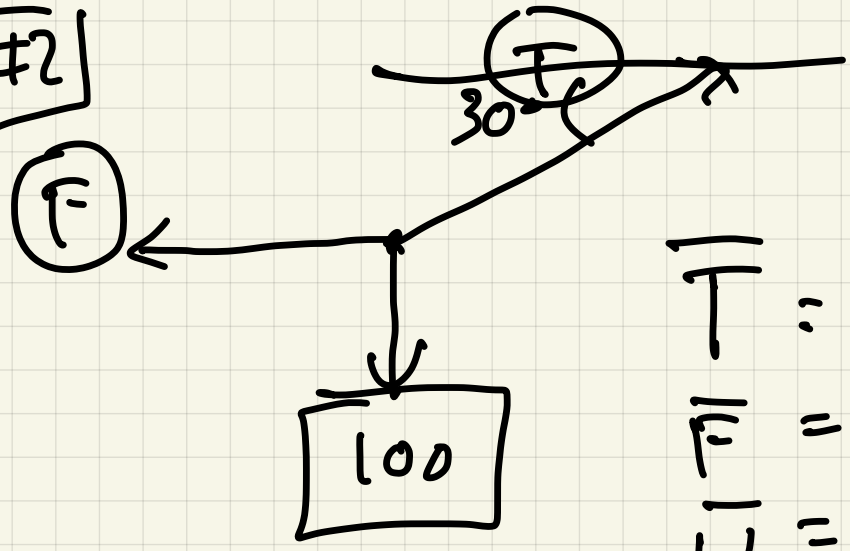
9/19 | Calc 3

Exam 1

avg 126 ≅ 84%

150	1
135	7
120	2
105	1
90	4
80	

#2



$\overline{T} = T(\cos 30, \sin 30)$

$\overline{F} = F\langle -1, 0\rangle$

$\overline{W} = \langle 0, -100\rangle$

$$\overline{T} + \overline{F} + \overline{W} = \overline{0}$$

$$T\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) + \langle -F, 0\rangle + \langle 0, -100\rangle = \langle 0, 0\rangle$$

$$\begin{cases} T\frac{1}{2} - 100 = 0 & y \\ T\frac{\sqrt{3}}{2} - F = 0 & x \end{cases}$$

$T = 200$ lbs

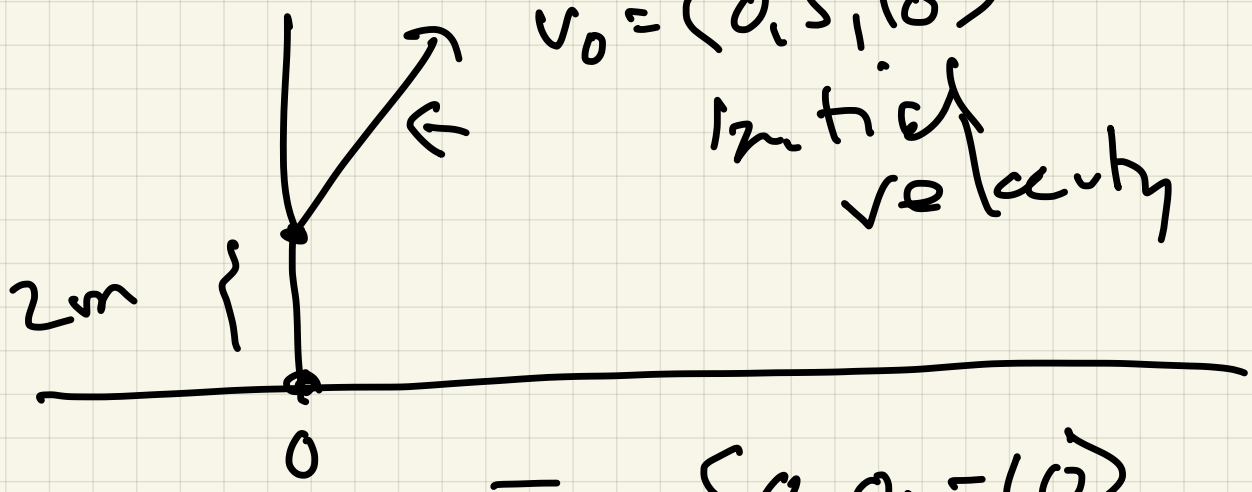
$$|\vec{F}| = 200 \frac{\sqrt{3}}{2} = 100\sqrt{3}$$

Last time: Integrals $\int \vec{v}(t)\,dt$

IVPs

Projectile motion:

Ex 0 A rock



$$\vec{g} = \langle 0, 0, -10 \rangle$$

$\vec{r}(t)$ = position of rock at time $t \geq 0$

$$\begin{cases} \vec{r}\,''(t) = \langle 0, 0, -10 \rangle \\ \vec{r}\,'(0) = \langle 0, 5, 10 \rangle \\ \vec{r}\,(0) = \langle 0, 0, 2 \rangle \end{cases}$$

By integrating twice,
we found

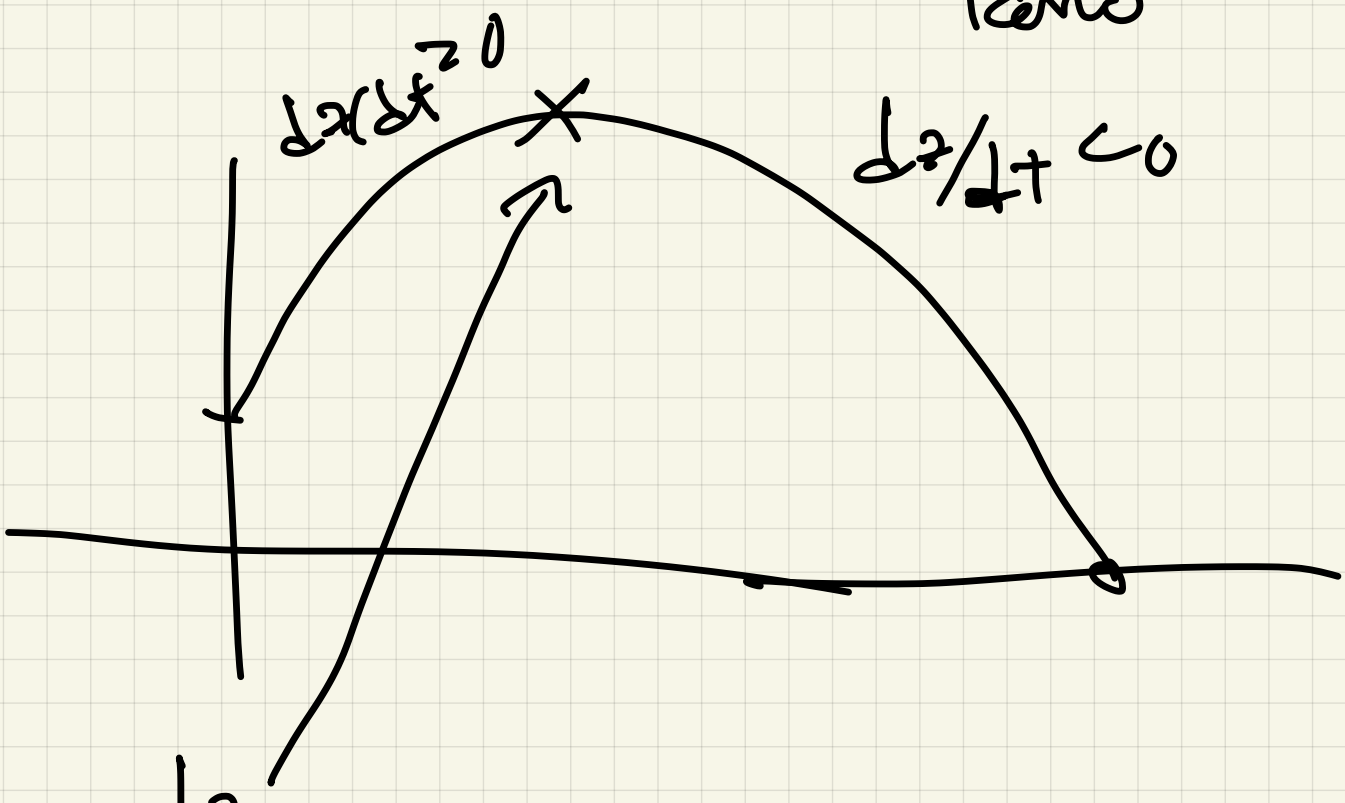
$$\vec{r}(t) = \langle 0,\ 5t,\ \boxed{-5t^2 + 10t + 2} \rangle \qquad t \geq 0$$

(x, y, z)

(a) What max height?

(b) When does rock land

(c) How far away does it land



$$\frac{dz}{dt} = 0$$

$$-10t + 10 = 0 \qquad t = 1$$

$z(1) = -5 + 10 + 2 = 7\ m$

(b) lands when $z = 0$

$$-5t^2 + 10t + 2 = 0$$

$\Downarrow$ quad formula

$t \neq 0$

$a \quad b \quad c$

$$5t^2 - 10t - 2 = 0$$

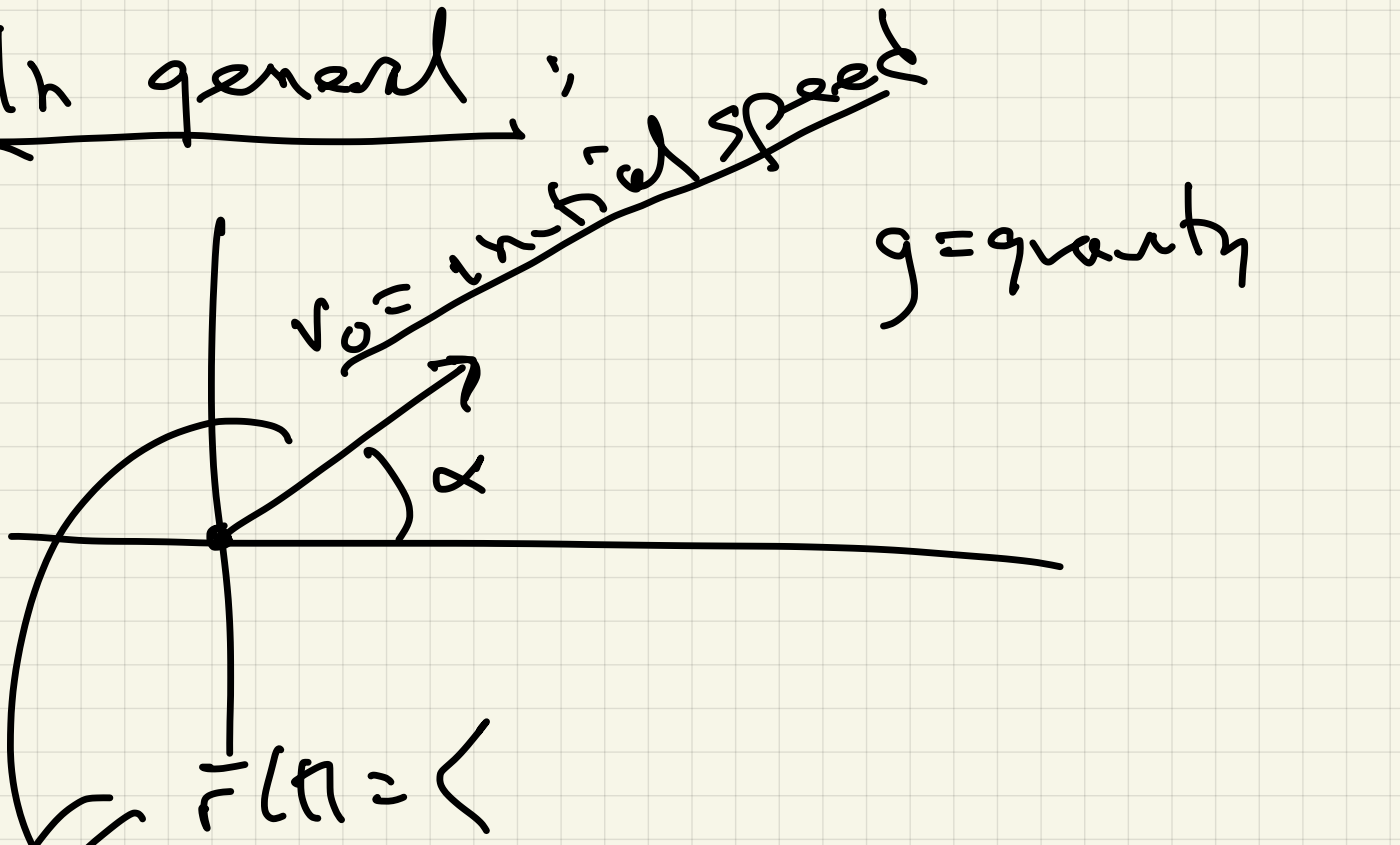
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{10 \pm \sqrt{100 + 40}}{10}$$

$$1 \pm \frac{\sqrt{140}}{10} = 2.183 \quad \cancel{-.183}$$

(c) dist = y-coord at $t = 2.183$

So $y = 5 - (2.183) = 10.916\ m$

In general :

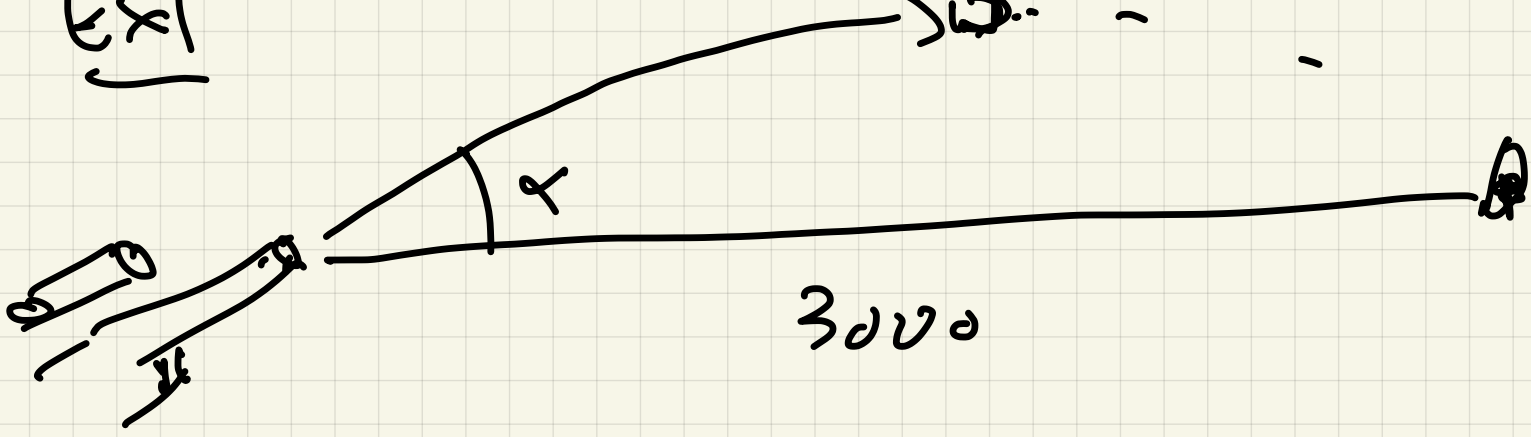


unit $\langle \cos\alpha, \sin\alpha \rangle$

initial velocity $v_0 \langle \cos\alpha, \sin\alpha \rangle$

$$\bar{r}(t) = \langle (v_0 \cos\alpha)\, t,\ (v_0 \sin\alpha)\, t - \frac{1}{2} g t^2 \rangle$$

Ex1



A shot is fired from a gun with muzzle velocity of 1200 ft/sec if to hit a target 3000 ft away. Determine minimal angle α of elevation to hit target

$g = 32\ ft/sec^2$

Motion

$\bar{r}(t) = \langle (1200\cos\alpha)t, (1200\sin\alpha)t - \frac{1}{2}\cdot 32\cdot t^2 \rangle$

x y

$\bar{r}_0 = 1200\langle \cos\alpha, \sin\alpha \rangle$

Shot hits target when $y = 0$

$y = 1200\sin\alpha\, t - 16t^2 = 0$

$1200\sin\alpha = 16t$

T = time of impact

$$\frac{1200\sin\alpha}{16} = 75\sin\alpha$$

At time $\boxed{T = \frac{75\sin\alpha}{}}$

want $x = 3000$

$$3000 = x = 1200\cos\alpha \cdot 75\sin\alpha$$

$$\frac{3000}{1200 \cdot 75} = \cos\alpha\sin\alpha$$

$$\frac{1}{30} = \cos\alpha\sin\alpha$$

Formula: $\sin 2\theta = 2\sin\theta\cos\theta$

$$\frac{1}{15} = 2\cos\alpha\sin\alpha = \sin 2\alpha$$

$$2\alpha = \sin^{-1}\frac{1}{15}$$

$$\alpha = \frac{1}{2}\sin^{-1}\frac{1}{15} =$$

$$\alpha = 1.911°$$

(Alc$_0$ $\alpha \approx 88.089°$)