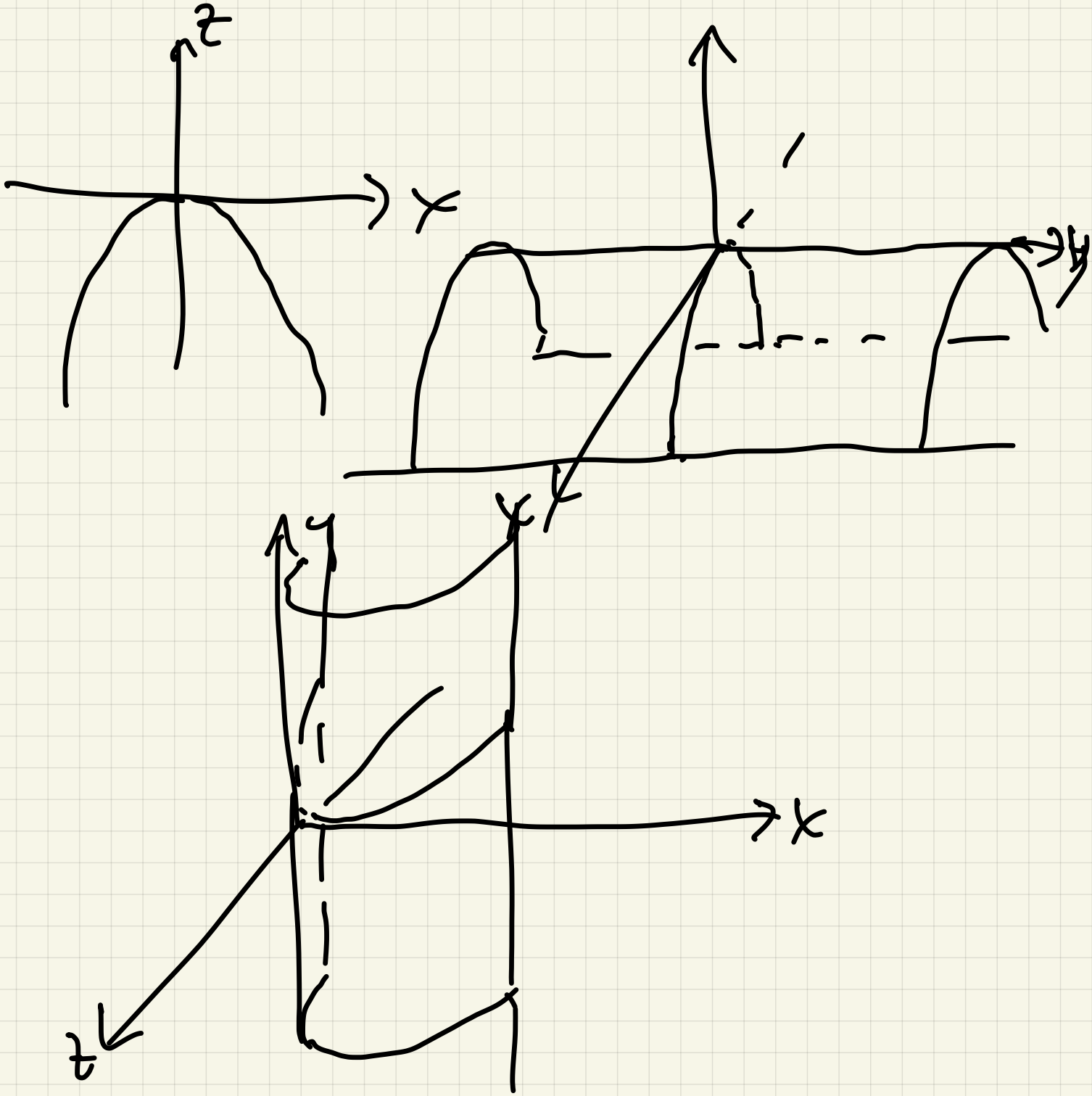
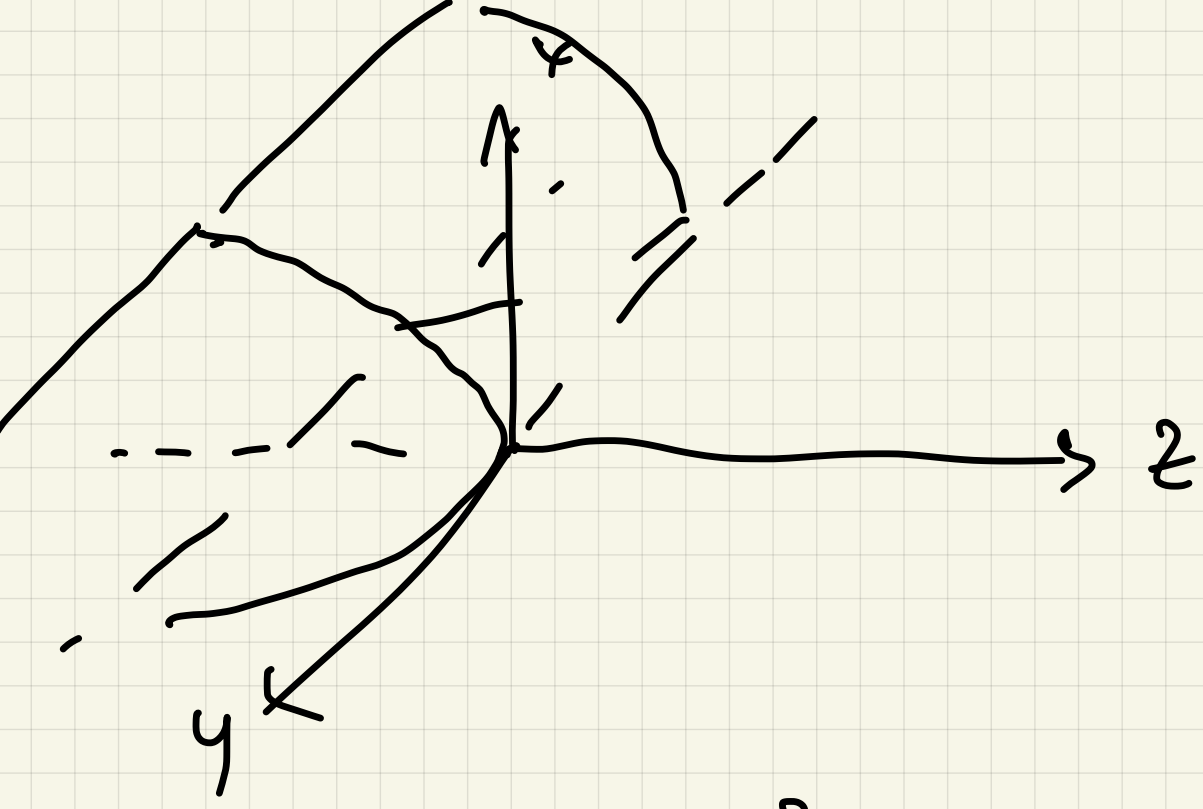
9/16/Calc3

Exam 1 → Thursday

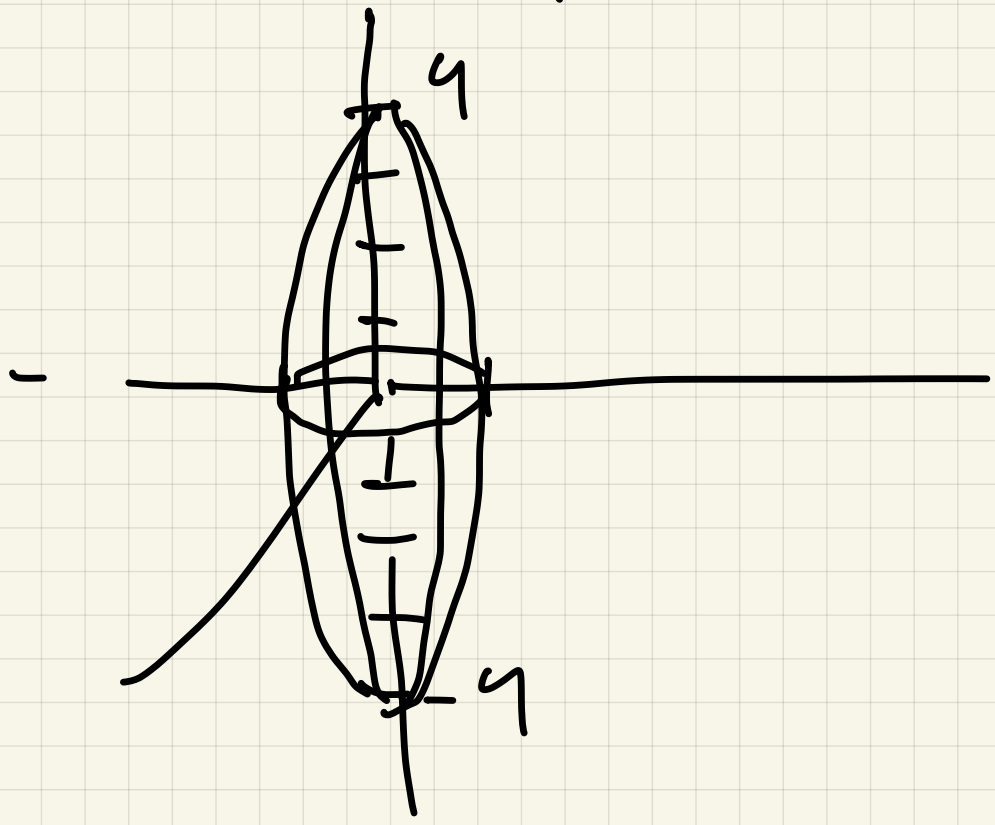
Quiz 6

1. $z = -x^2$





2. $x^2 + y^2 + \frac{z^2}{16} = 1$

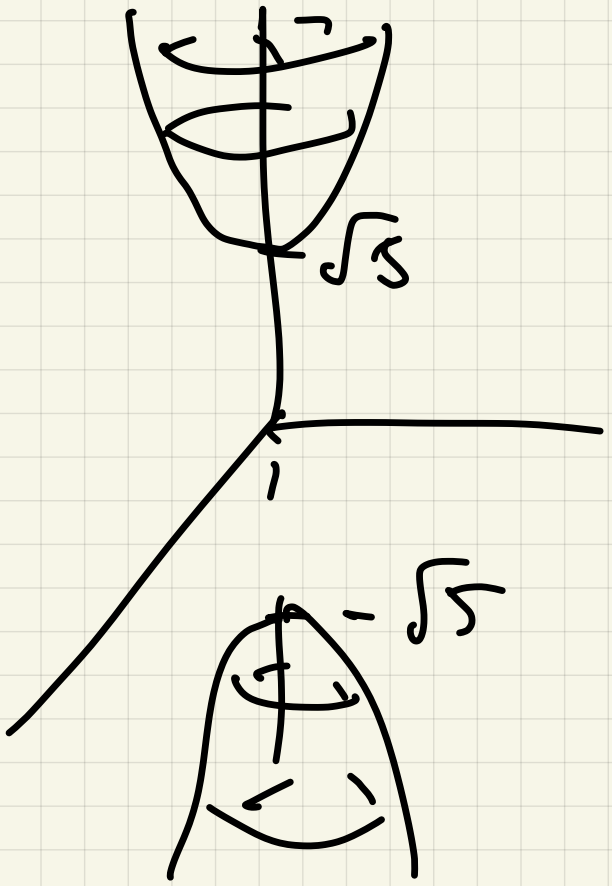


3. $x^2 + y^2 - z^2 + 5 = 0$

$x^2 + y^2 = z^2 - 5$ $\left(\begin{array}{c} z = c \\ \text{gives} \\ \text{motion} \end{array}\right)$

$z \geq \sqrt{5}$

$z \leq -\sqrt{5}$



Last time

Vector valued functions

Indefinite integrals

Definite integrals

IVPs = Initial value problems

$$\begin{cases} \vec{r}'(t) = \langle 5t^2, \cos t, 1 \rangle \\ \vec{r}(1) = \langle 0, 0, 0 \rangle \end{cases}$$

$$\vec{r}(t) = \int \langle 5t^2, \cos t, 1 \rangle$$

$$= \left\langle \frac{5t^3}{3}, \sin t, t \right\rangle + \langle c_1, c_2, c_3 \rangle$$

$$\vec{r}(1) = \left\langle \frac{5}{3}, \sin 1, 1 \right\rangle + \langle c_1, c_2, c_3 \rangle$$

$$= \langle 0, 0, 0 \rangle$$

$$c_1 = -5/3$$

$$c_2 = -\sin 1$$

$$c_3 = -1$$

$$\vec{r}(t) = \left\langle \frac{5t^3}{3} - \frac{5}{3}, \sin t - \sin 1, t - 1 \right\rangle$$

Projectile motion:

Ex 0 Rock is thrown at from $\langle 0, 0, 2 \rangle$ and initial velocity $\langle 0, 5, 10 \rangle$ and gravitation acceleration

$$\vec{a} = \langle 0, 0, -10 \rangle$$

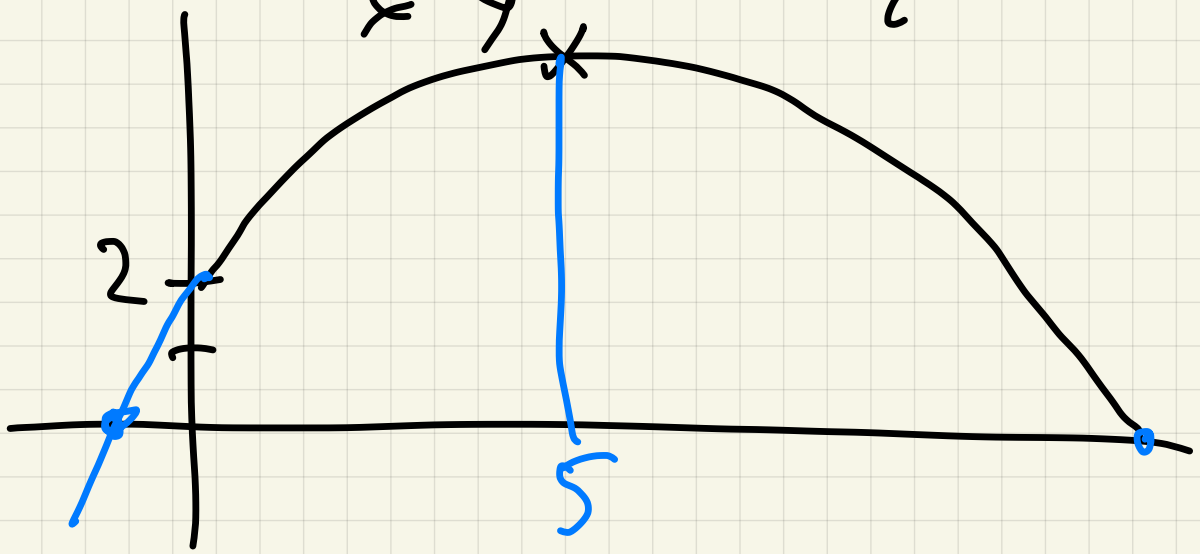
① $\vec{r}'(t) = \vec{v}(t) = \int \vec{a}\, dt$

$\vec{v}(t) = \langle 0, 5, -10t + (0) \rangle$

② $\vec{r}(t) = \int \vec{v}(t)$

$\Downarrow$

$\vec{r}(t) = \langle \underbrace{0}_{x}, \underbrace{5t}_{y}, \underbrace{-5t^2 + 10t + 2}_{z} \rangle$



Typical follow up:

a) What's the max height?

b) When does it land?

c) How far away?

max height occurs when

$$\frac{dz}{dt} = 0 \Rightarrow -10t + 10 = 0$$

when $\boxed{t = 1}$

∴ max ht is

$$z(1) = -5 + 10 + 2 = 7 \text{ m.}$$

b) Rock lands when

$$z = 0 \Rightarrow$$

$$-5t^2 + 10t + 2 = 0$$

↓ quad eqn

$$5t^2 - 10t - 2 = 0$$

$$\frac{10 \pm \sqrt{100 - 4(5)(-2)}}{2 \cdot (5)} =$$

$$\frac{10 \pm \sqrt{140}}{10} =$$

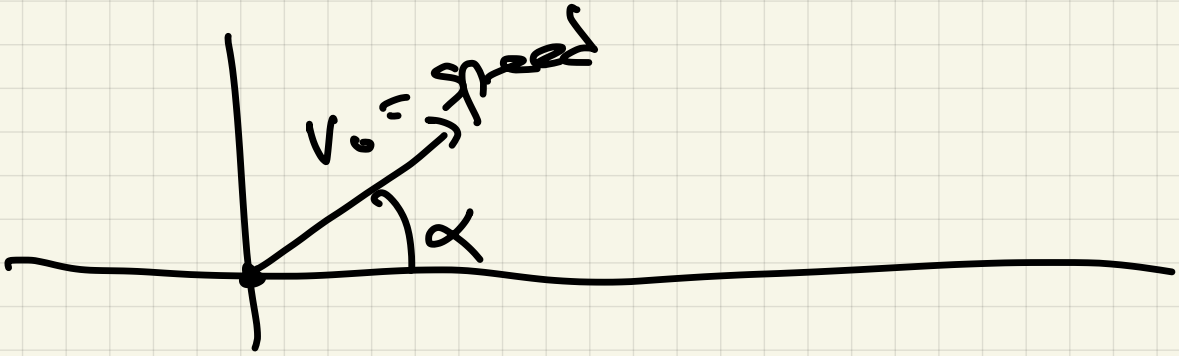
$$1 \pm \frac{\sqrt{140}}{10}$$

$$1 + \frac{\sqrt{140}}{10} \cong 2,183 \text{ s}$$

c) How far?

dist $5(2,183) = 10,916$ m.

General formula:

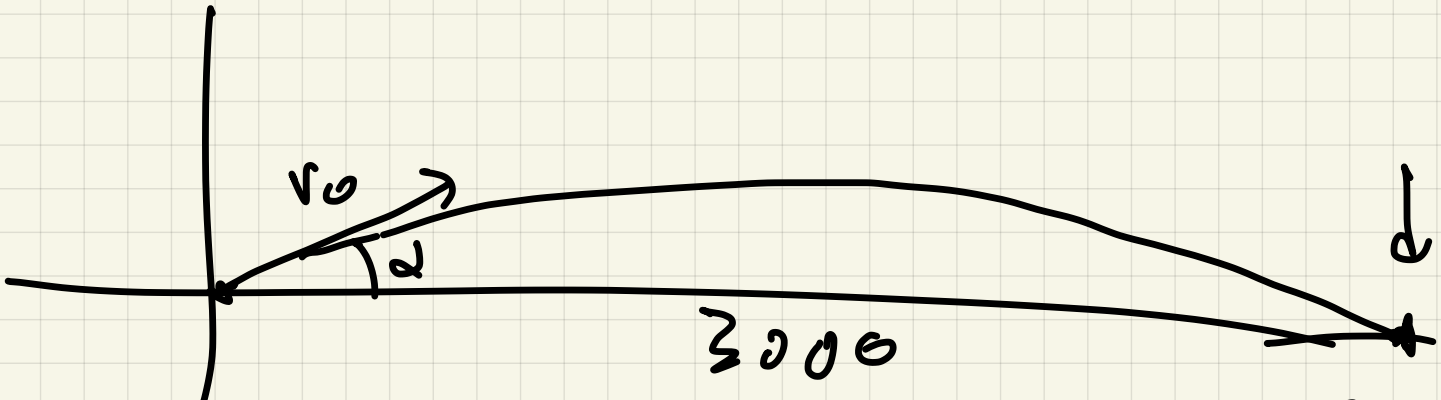


$\vec{V}_0 = \langle v_0 \cos\alpha, v_0 \sin\alpha \rangle$

$$\langle v_0 \cos\alpha t,\ v_0 \sin\alpha t - \tfrac{1}{2} g t^2 \rangle$$

g = grav accel.

Ex A shot is fired from a gun with muzzle velocity of 1200 ft/sec



Bullet hits a target 3000 ft away.

Find the (minimal) angle of elevation α.

$g = -32 \text{ ft/sec}^2$

$\vec{r}(t) = \langle 1200\cos\alpha t,\ 1200\sin\alpha t - 16t^2 \rangle$

When does bullet hit target?

$x = 3000$

$1200\cos\alpha t = 3000$

$y = 0$:

$1200\sin\alpha t - 16t^2 = 0$

$t = 0 \Rightarrow$ $t = \dfrac{1200\sin\alpha}{16}$

$= 75\sin\alpha$