10/7/ Calc 3

Quiz 10 avg 99%

1. $$\lim_{(x,y)\to(2,-1)} \frac{x+3y}{2x+y} = \frac{-1}{3}$$

2. Domain =
$$\{(x,y) : 2x+y \neq 0\}$$
$$y \neq -2x$$

3. $$\lim_{x\to 0} \frac{x}{2x} = \frac{1}{2}$$
$$\neq$$

4. $$\lim_{y\to 0} \frac{3y}{y} = 3$$

5. Lim DNE

yes

Last time

Chain Rule

Implicit Differentiation

Direction ~~d~~erivative

$z = f(x,y)$ pt (a,b)

$\partial z/\partial x$ $\partial z/\partial y$ direction:

$\bar{u}$ = unit vector
= $\langle u_1, u_2 \rangle$

$D_u f(a,b) =$

$$\lim_{\Delta t \to 0} \frac{f(a+u_1\Delta t, b+u_2\Delta t) - f(a,b)}{\Delta t}$$

$\parallel$ class

$$\frac{d}{dt} f(a+u_1 t, b+u_2 t)\Big|_{t=0}$$

Formula: $D_u f(a,b) = \nabla f(a,b) \cdot \bar{u}$

where $\nabla f(a,b) = \langle \partial f/\partial x (a,b), \partial f/\partial y (a,b) \rangle$

is gradient of $f(x,y)$ at (a,b)

$z = f(x,y) = 25 - x^2 - y^2$

(a) $\nabla f(3,4)$

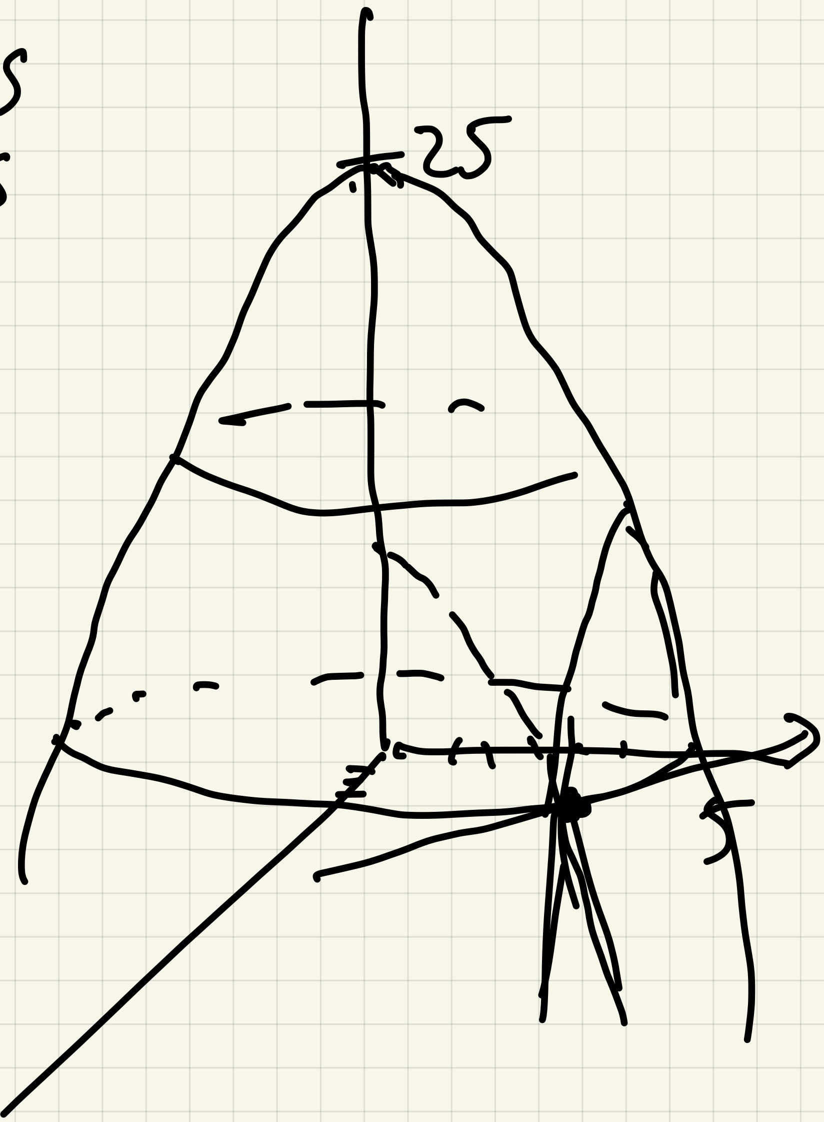
(b) $D_u f(3,4)$ in directions

pos x-axis

pos y-axis

$\langle 1,1 \rangle$

$\langle 4,-3 \rangle$



$\nabla f = \langle -2x, -2y \rangle$

$\nabla f = \langle -6, -8 \rangle$

pos x-direction !

$D_u f(3,4) = \langle -6,-8 \rangle \cdot \langle 1,0 \rangle = -6$

pos y-direction

$D_u f(3,4) = \langle -6, -8 \rangle \cdot \langle 0,1 \rangle = -8$

$\langle 1,1 \rangle$

$$D_u f(3,4) = \langle -6,-8 \rangle \cdot \frac{\langle 1,1 \rangle}{\sqrt{2}} = \frac{-14}{\sqrt{2}}$$

unit vector

$$= -9.899$$

$\langle 4,-3 \rangle$

$$D_u f(3,4) = \langle -6,-8 \rangle \cdot \frac{\langle 4,-3 \rangle}{5} = 0$$

<u>Notice</u> If θ is the angle between $\bar{u}$ and $\nabla f(a,b)$

Then $\cos\theta = \dfrac{\nabla f(a,b) \cdot \bar{u}}{|\nabla f(a,b)|\,|\bar{u}|}$

$\Downarrow$

$$D_u f(a,b) = |\nabla f(a,b)| \cos\theta$$

<u>Consequences</u>:

① Direction of maximal increase

is $\bar{u} = \dfrac{\nabla f(a,b)}{|\nabla f(a,b)|}$ $(\theta = 0)$

and $D_u f(a,b) = |\nabla f(a,b)|$

② Direction of maximal decrease $(\theta = \pi)$

is $\bar{u} = -\dfrac{\nabla f(a,b)}{|\nabla f(a,b)|}$

$D_u f(a,b) = -|\nabla f(a,b)|$

③ $\bar{u} \perp \nabla f(a,b) \Rightarrow \cos\theta = 0$

$D_u f(a,b) = 0$

In Ex | i we saw ③

In $\langle 4, -3 \rangle$

$-9{,}899$

$\shortparallel$

Directn $\langle 1,1 \rangle$ direct deriv $-\dfrac{14}{\sqrt{2}}$

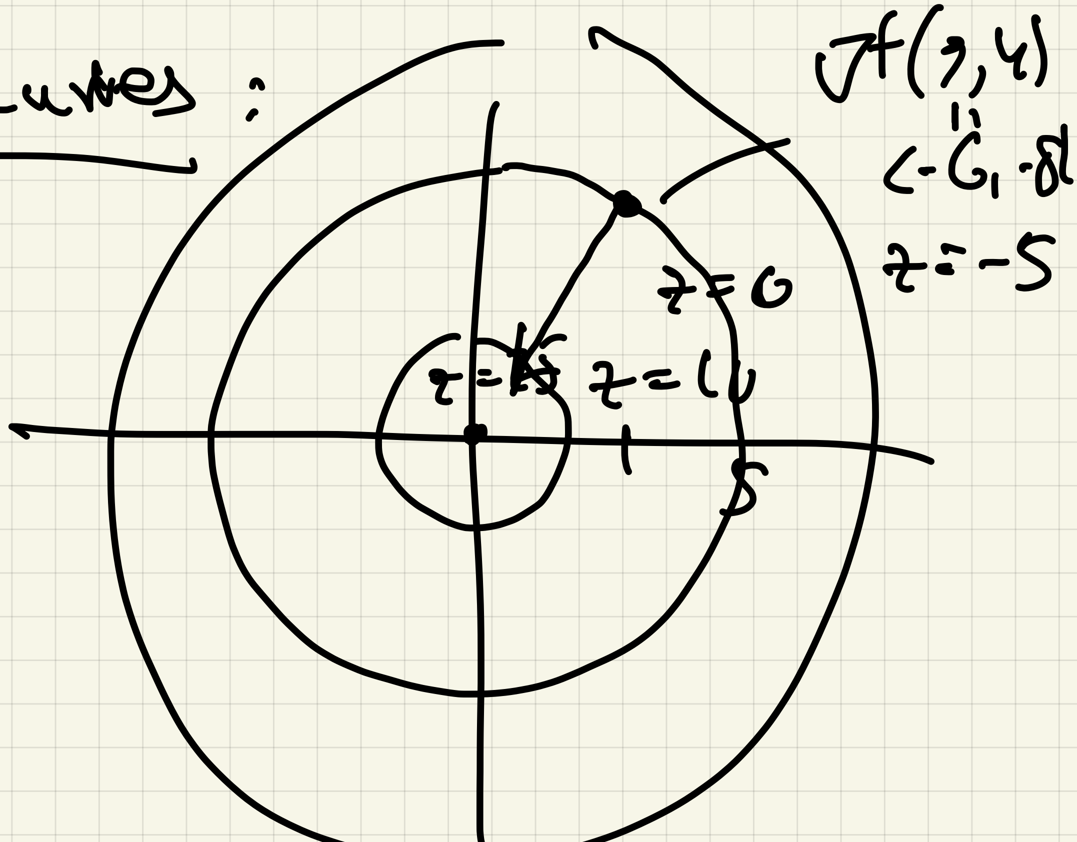
but direction of maximal decrease is

$\langle 6,8\rangle$

$$D_u f(3,4) = \langle -6,-8\rangle \cdot \frac{\langle 6,8\rangle}{10} =$$

$$-\frac{100}{10} = -10$$

<u>Level curves</u>:



If moving along a level curve $f(x,y) =$ const, directional derivative should be zero $= 0$

If $\bar{r}(t) = \langle x(t), y(t) \rangle$ is vector valued function and $f(\bar{r}(t)) = c =$ constant

(i.e. $r(t)$ lies on level curve $f(x,y) = c$)

$$c = f(\bar{r}(t)) \qquad \frac{d}{dt}$$

$\Downarrow$ Chain rule

$$0 = \frac{d}{dt} f(\bar{r}(t)) =$$

$$\underline{\partial f/\partial x} \cdot dx/dt + \underline{\partial f/\partial y} \cdot dy/dt$$

$$\nabla f(\bar{r}(t)) \cdot \bar{r}'(t) = 0$$

<u>Illustration:</u>

Ex $\quad z = g(x,y) = y - x^2$

$y - x^2 = g(x,y) = 0$

a) find ∇g

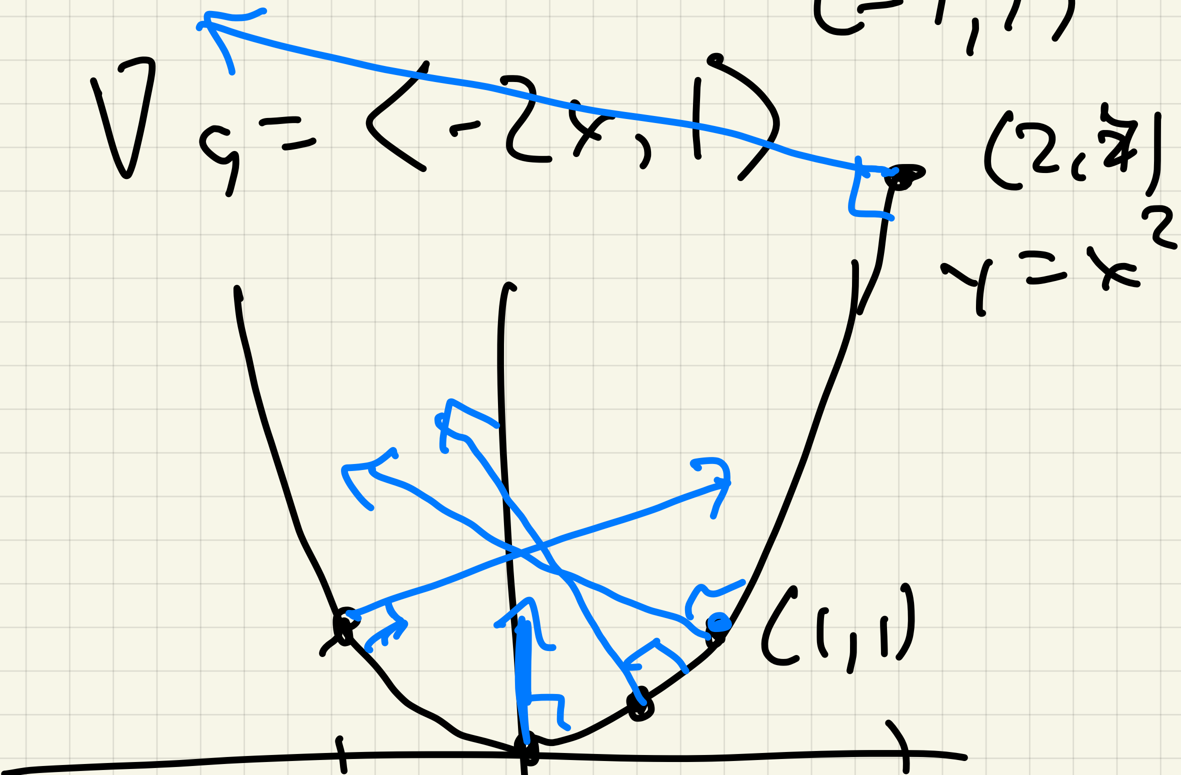
b) sketch level curve $g = 0$

c) Compute $\nabla g(a,b)$ at

$(0,0)$, $(1,1)$ $(2,4)$, $(\frac{1}{2}, \frac{1}{4})$, $(-1,1)$

a) $\nabla g = \langle -2x, 1 \rangle$

b)



$\boxed{\nabla g(0,0) = \langle 0,1 \rangle}$

$\boxed{\nabla g(1,1) = \langle -2,1 \rangle}$

$\boxed{\nabla g(2,4) = \langle -4,1 \rangle}$

$$\nabla g\left(\frac{1}{2}, \frac{1}{4}\right) = \langle -1, 1 \rangle$$

$$\nabla g(-1, 1) = \langle 2, 1 \rangle$$