10/30/ Calc3 Quiz 13

$$f = y^4 - 16xy + 8x^2$$

$$\nabla f = \langle -16y + 16x,\ 4y^3 - 16x \rangle$$

$$= 0$$

$$x = y \qquad 4x^3 - 16x = 0$$

$$4x(x^2 - 4) = 0$$

$$x = 0 \qquad x = \pm 2$$

$(0,0)$

$(2,2)$

$(-2,-2)$

$$d = \det \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$$

$$= \det \begin{pmatrix} 16 & -16 \\ -16 & 12y^2 \end{pmatrix}$$

$(0,0)$: $-256 < 0$ saddle

$(2,2)$ $16 \cdot 48 - 16 \cdot 16 = 512 > 0$

$f_{xx} = 16 > 0$ local min

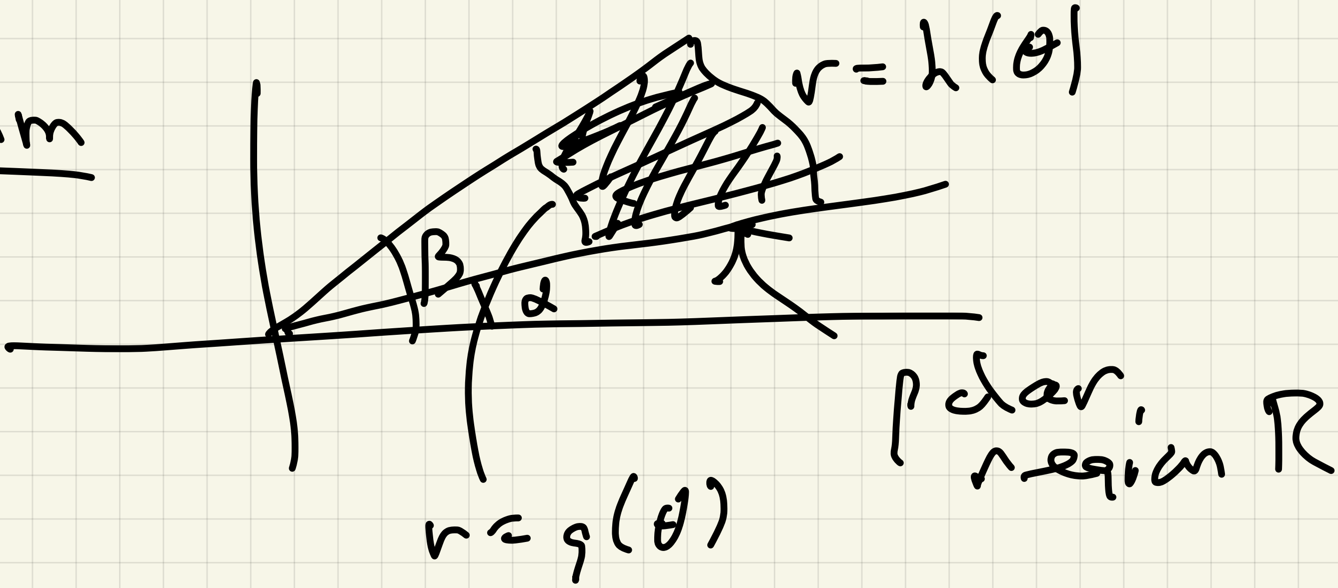
$(-2,-2)$ $d = 512 > 0$

$f_{xx} = 16$ local min

Last time $\iint_R f(x,y)\,dA = \int_{\circ}^{\circ}\int_{\circ}^{\circ} f(x,y)$ dydx / dxdy

Polar coordinates

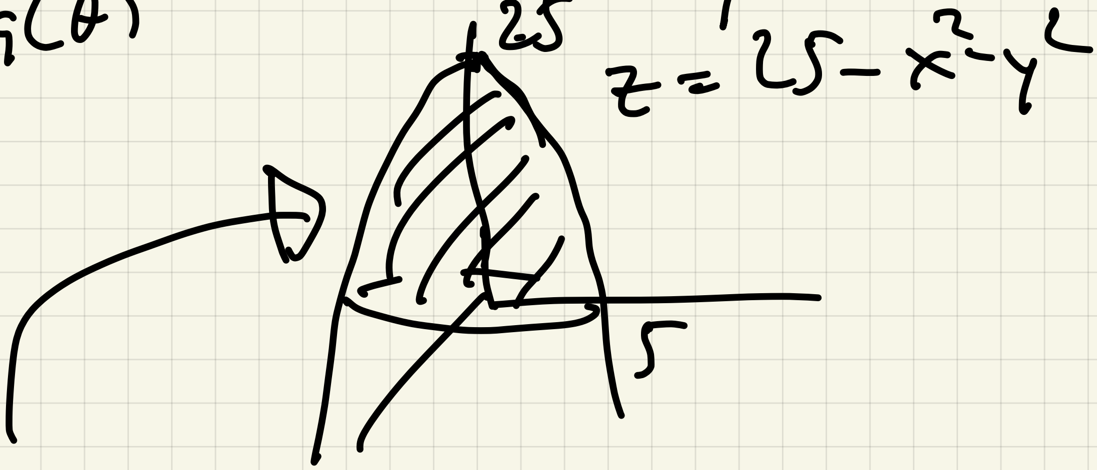
Thm



Then $\iint_R f(x,y)\,dA =$

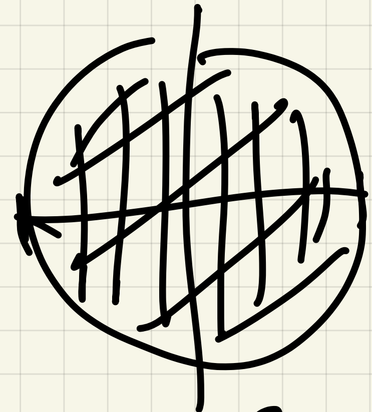
$$\int_\alpha^\beta \int_{g(\theta)}^{h(\theta)} f(r\cos\theta, r\sin\theta)\,\boxed{r}\,dr\,d\theta$$

Ex1



Find volume

$$\iint_R 25-x^2-y^2 \, dA$$

$R =$  5

$x^2+y^2=25$

Fubinis Thm

$$\int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} 25-x^2-y^2 \, dy\,dx$$

Can do this using Calc 2
(trig subs)

polar

$$\int_0^{2\pi}\int_0^{5} (25-r^2)\, r \, dr\,d\theta$$

$$\underbrace{r^2=x^2+y^2}$$

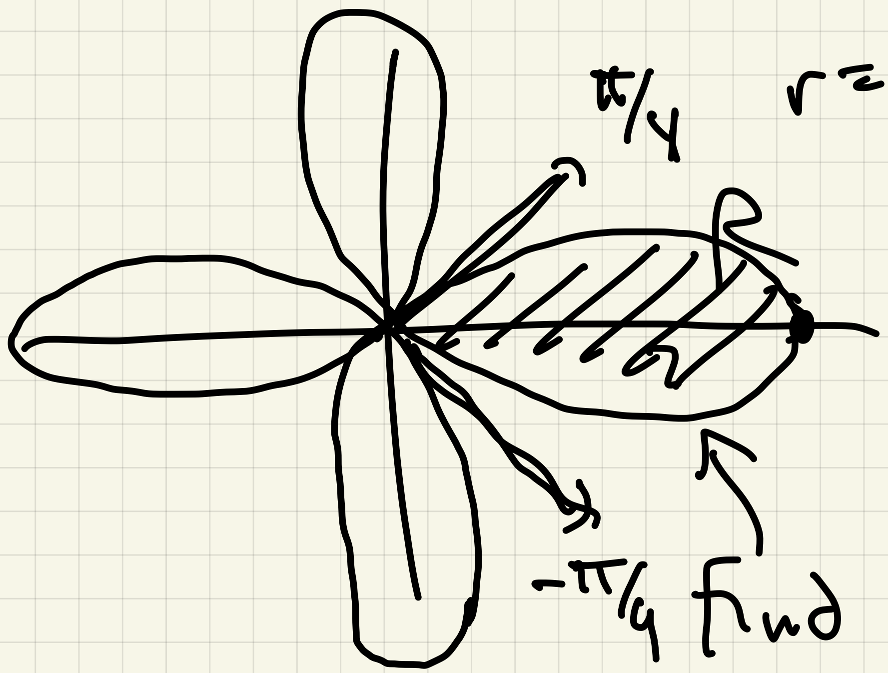
$$\int_0^5 25r - r^3 \, dr$$

$$\frac{25r^2}{2} - \frac{r^4}{4}\Big|_0^5 =$$

$$\left(\frac{625}{2} - \frac{625}{4}\right) = \int_0^{2\pi} \frac{625}{4}\, d\theta =$$

$$\frac{625\theta}{4}\Big|_0^{2\pi} = \frac{625\pi}{2}$$

<u>Ex2</u>



$r = 7\cos 2\theta$
$\parallel$
0

find Area :

$$\text{Area} = \iint_R 1\, dA = \int_{-\pi/4}^{\pi/4} \int_0^{7\cos 2\theta} r\, dr\, d\theta$$

$\parallel$

$$\frac{r^2}{2}\Big|_0^{7\cos 2\theta} = \frac{49\cos^2 2\theta}{2}$$

$$\text{Area} = \int_{-\pi/4}^{\pi/4} \frac{49\boxed{\cos^2 2\theta}}{2}\,d\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\int_{-\pi/4}^{\pi/4} \frac{49\left(\frac{1+\cos 4\theta}{2}\right)}{2}\,d\theta =$$

$$\frac{49}{4}\int_{-\pi/4}^{\pi/4} 1+\underline{\cos 4\theta}\,d\theta$$

$$\frac{49}{4}\left(\theta+\frac{1}{4}\sin 4\theta\right)\Big|_{-\pi/4}^{\pi/4} =$$

$$\frac{49}{4}\left(\frac{\pi}{4}-\left(-\pi/4\right)\right) =$$

$$\frac{49}{4}\left(\frac{\pi}{2}\right) = \frac{49\pi}{8}$$

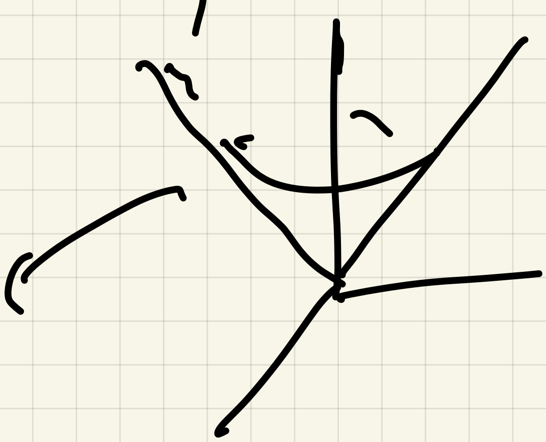
Ex 3 Find volume of the region bounded xy plane, the cone $z = \sqrt{x^2+y^2}$ and cylinder $x^2 - 6y + y^2 = 0$

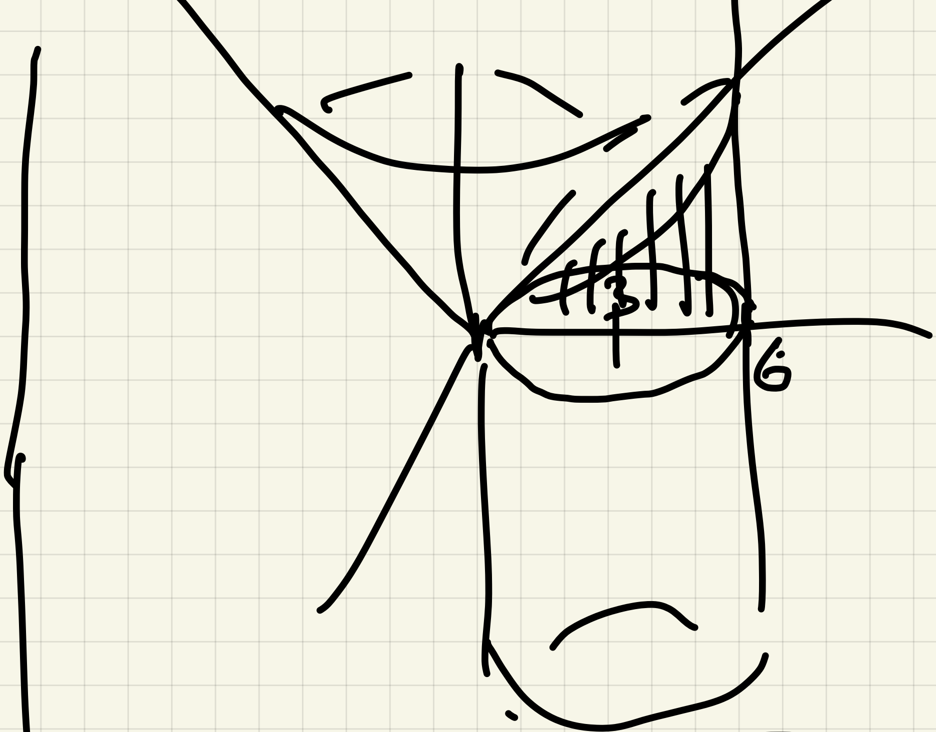
Try to visualize

$$x^2 - 6y + y^2 = 0$$

$$x^2 + y^2 - 6y + 9 = 9$$

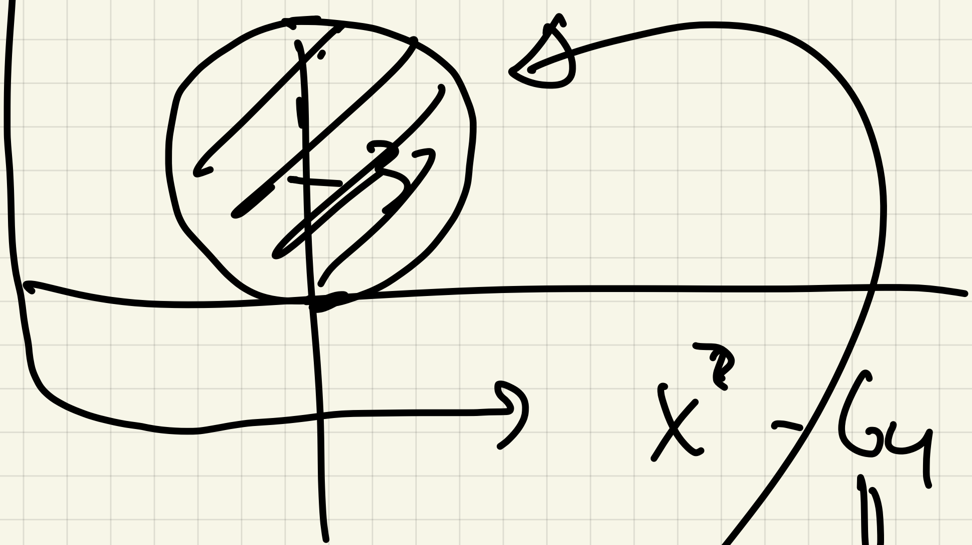
$$x^2 + (y-3)^2 = 9$$





$$V = \int_{0}^{6} \int_{-\sqrt{9-(y-3)^2}}^{\sqrt{9-(y-3)^2}} \boxed{\sqrt{x^2+y^2}}\, dx\, dy$$

$y \quad x$



$$x^2 - 6y + y^2 = 0$$

$$\Downarrow$$

$x = r\cos\theta$

$y = r\sin\theta$

$$r^2\cos^2\theta - 6r\sin\theta + r^2\sin^2\theta = 0$$

$$r^2 - 6r\sin\theta = 0$$

$$r(\underline{r - 6\sin\theta}) = 0$$

$$r = 6\sin\theta$$

$$V = \int_0^{\pi} \int_0^{6\sin\theta} \boxed{r \cdot r}\, dr\, d\theta$$

$$\boxed{\sqrt{x^2+y^2}}$$

$$\int_0^{\pi} \frac{r^3}{3} \Big|_0^{6\sin\theta} d\theta$$

$$\int_0^{\pi} \frac{216}{3} \sin^3\theta\, d\theta$$

$$72 \int_0^{\pi} \underline{\sin^3\theta}\, d\theta =$$

$$72\int_0^{\pi}(1-\cos^2\theta)\cdot\underline{\sin\theta}\, d\theta$$

$$u=\cos\theta$$

$$du=-\sin\theta\, d\theta$$

$$(-)du=\sin\theta\, d\theta$$

$$-72\int_{\theta=0}^{\theta=\pi}(1-u^2)\,du$$

$$-72\int_{u=1}^{u=-1}(1-u^2)\,du =$$

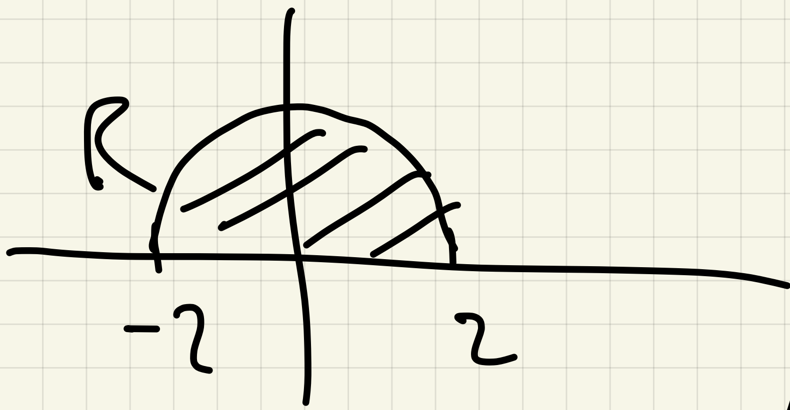
$$72\int_{-1}^{1}(1-u^2)\,du$$

$$72\left(\left(u-\frac{u^3}{3}\right)\Big|_{-1}^{1}\right.=$$

$$72\left(\,^2/_3-\left(-\,^2/_3\right)\right)=$$

$$72\left(\frac{4}{3}\right) = 96$$

Ex 4 $$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} \cos(x^2+y^2)\,dy\,dx$$



$$= \iint_R \cos(x^2+y^2)\,dA$$

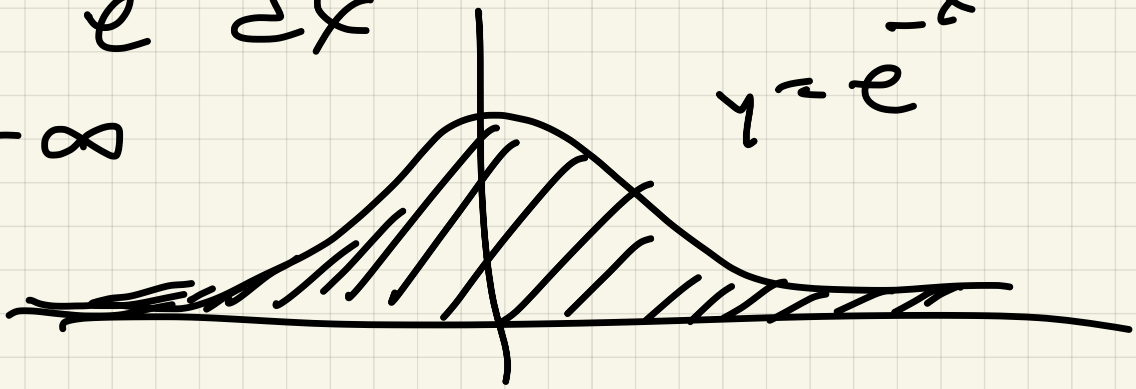
$$= \int_0^{\pi}\int_0^{2} \cos(r^2)\, r\,dr\,d\theta$$

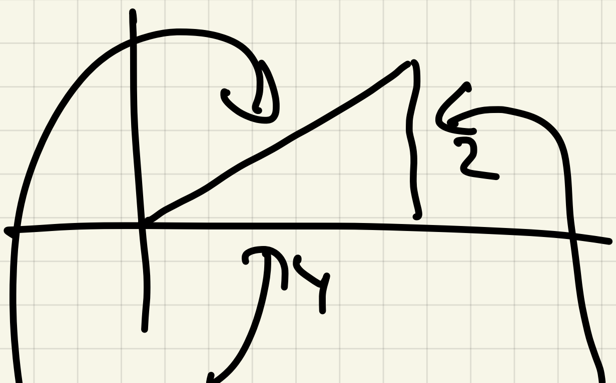
$u = r^2 \quad du = \frac{1}{2} r\,dr$

$$\int_0^{\pi} \frac{1}{2}\sin r^2 \Big|_0^2$$

$$\int_0^{\pi} \frac{1}{2}\sin 4 = \frac{\pi}{2}\sin 4 < 0$$

Stats applic $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\,dx$





$\theta = \phi \cos\theta = x = 4$

$$r = \frac{4}{\cos\theta} = 4\sec\theta$$

$\theta = \arctan \frac{1}{2}$

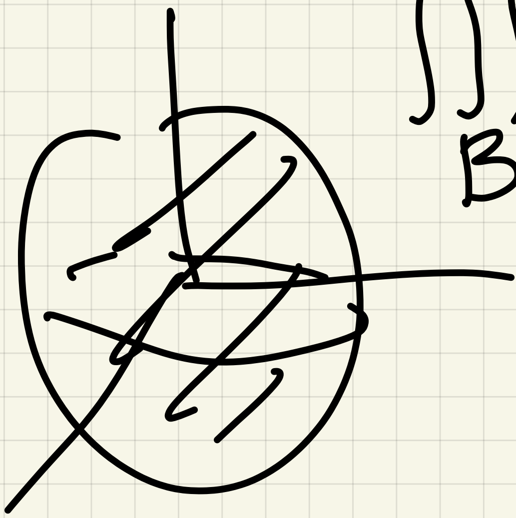
§14.6 Triple Integrals

Defn: If $f(x,y,z)$ is a function on a solid region B, the triple integral of $f(x,y,z)$

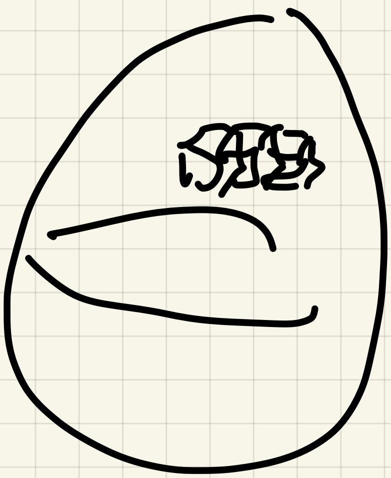
over B is

$$\iiint_B f(x,y,z)\,dV$$

$$= \lim_{|\Delta x| \to 0} \sum f(x_i, y_i, z_i)\,\Delta V_i$$



volume of i^{th} cube



(x_i, y_i, z_i) pt in i^{th} cube

What does it mean:

① $\iiint_B 1\,dV =$ Volume of B

② If $\rho(x,y,z)$ is density of B at (x,y,z) then

$$\iiint_B \rho(x,y,z)\, dV = \text{Mass of } B$$