10/28/ Calc 3

Exam 2

avg 85%
med 90%

150

135 — 10

120 — 4

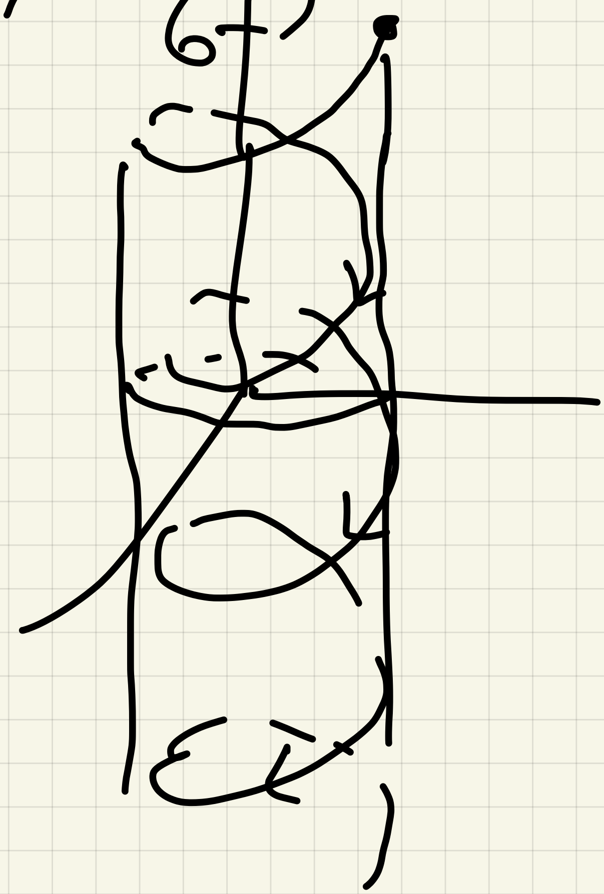
105 — 2

— 3

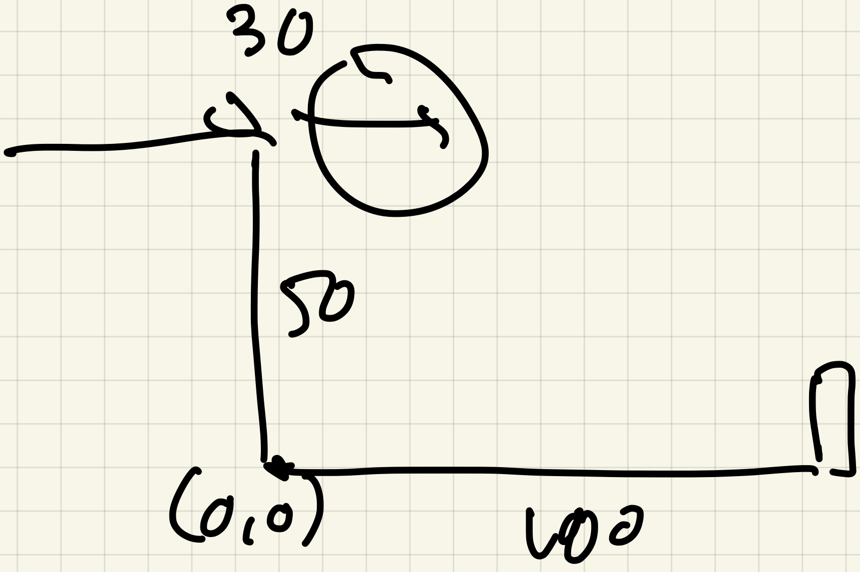
1. $\vec{r}(t) = \langle 2\sin t, 2\cos t, 6-5t \rangle$

$x \qquad y$

$x^2+y^2=4$,



2. $\bar{a}, \bar{v}, \bar{r}$



$\bar{a} = \langle 0, -6 \rangle$

$\bar{v}' = \bar{a}, \quad \bar{v} = \int \langle 0, -6 \rangle \, dt$

$\langle 0, -6t \rangle + \bar{C}$

$\bar{v}(0) = \langle 30, 0 \rangle$

$\bar{v}(t) = \langle 30, -6t \rangle$

$\bar{r}(t) = \int \bar{v}(t) = \int \langle 30, -6t \rangle =$

$\langle 30t, -3t^2 \rangle + \bar{D}$

$\bar{r}(0) = \langle 0, 50 \rangle$

$\bar{r}(t) = \langle \underset{x}{30t}, \underset{y}{50 - 3t^2} \rangle$

$\left[\bar{r}(t) = \langle v_0 \cos\alpha \, t, v_0 \sin\alpha \, t - \frac{1}{2} g t^2 \rangle \right.$

(c) $\quad x = 100$

$\Downarrow$

$30t = 100 \quad t = \frac{100}{30} =$

$t = \frac{10}{3}$

$50 - 3t^2 = 50 - 3\left(\frac{10}{3}\right)^2$

$50 - \frac{100}{3} =$

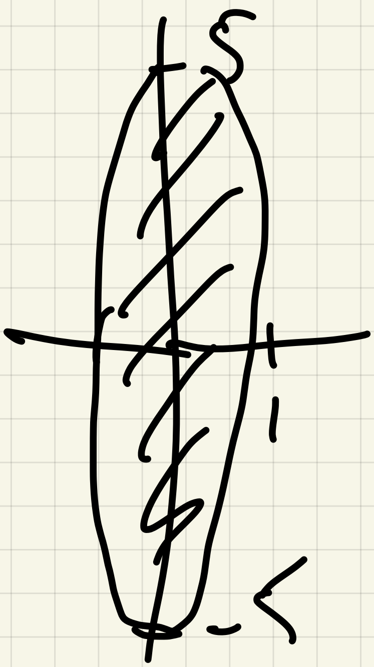
$16.\dot{6} > 10$

flies over

3. $z = -10\sqrt{1 - x^2 - y^2/25}$

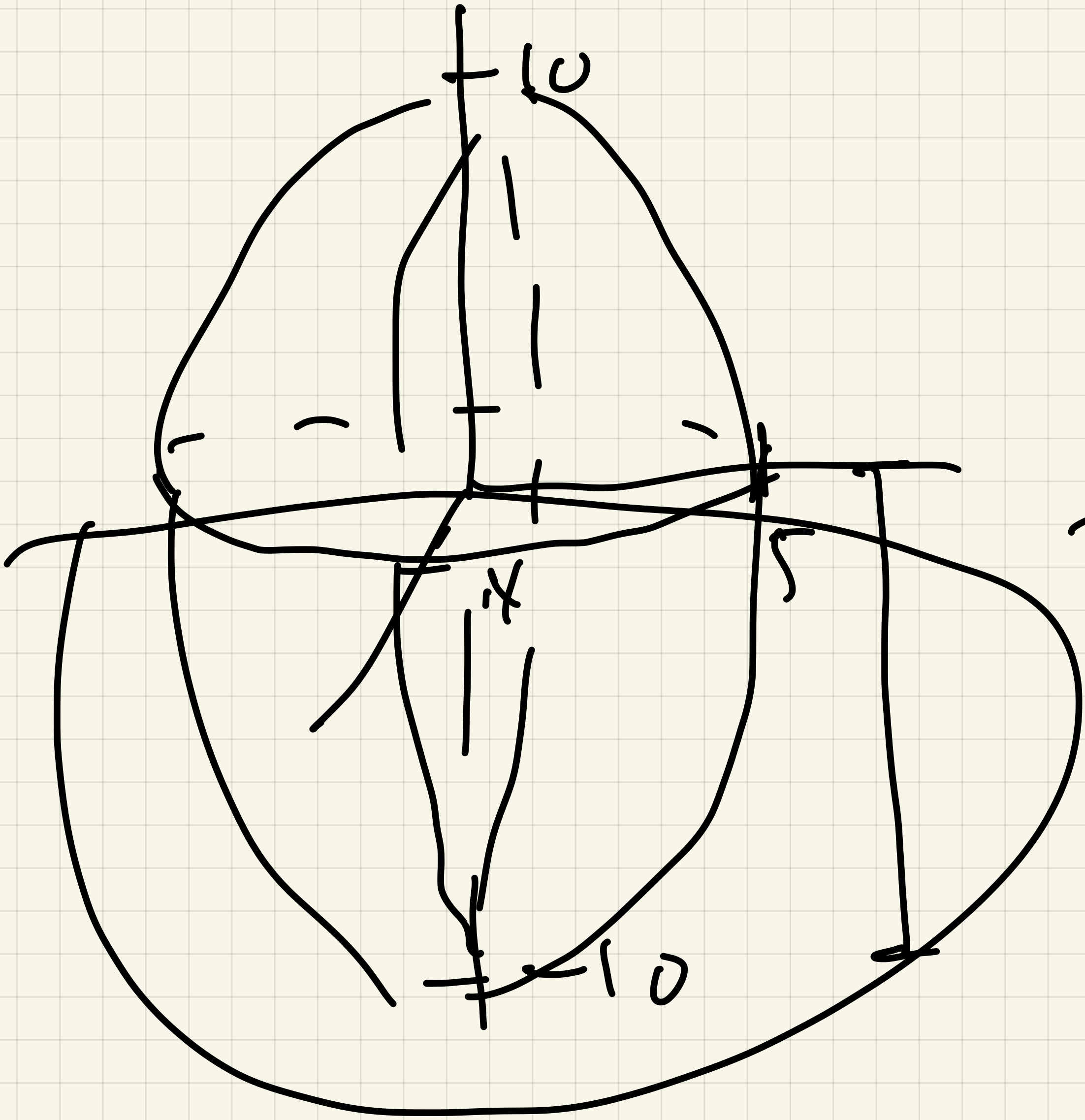
$1 - x^2 - y^2/25 \geq 0$

$1 \geq x^2 + y^2/25$



$\frac{z^2}{10^2} = 1 - x^2 - y^2/25$

$1 = x^2 + y^2/25 + z^2/10^2$



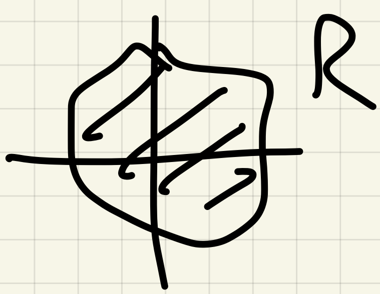
$z \leq 0$

$[-10, 0]$

Last Time

(A) $\iint_R f(x,y)\,dA$ = double integral = signed volume

Geometric R



(B) $\displaystyle\int_a^b \int_{h(x)}^{g(x)} f(x,y)\,dy\,dx \quad \left(\int_c^d \int \; dx\,dy \right)$

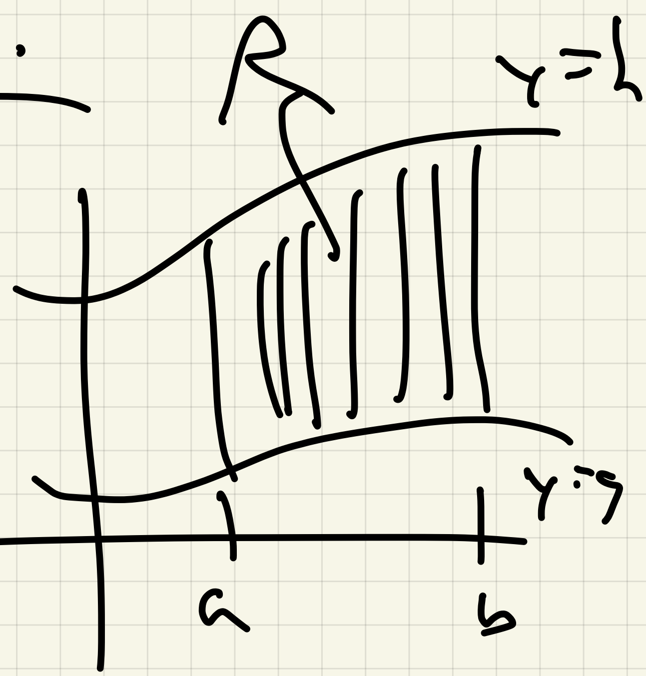
Iterated integral

Fubini's thm: (A) = (B), f

end points for (B) correspond to a region.

$a \le x \le b$

$g(x) \le y \le h(x)$



Sometimes can switch order $dx\,dy$ vs $dy\,dx$

Use Fubini's thm to evaluate

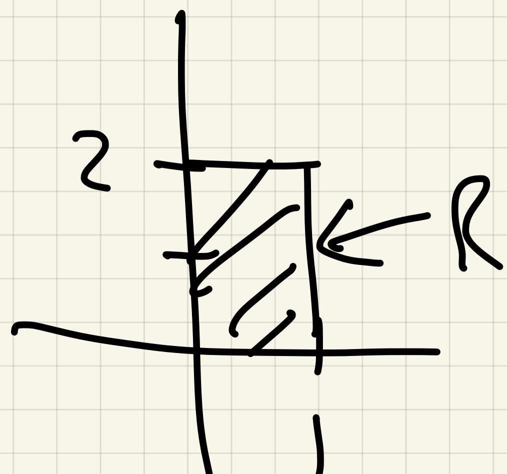
Ex1

$$\int_0^2 \underbrace{\int_0^1 \frac{x}{1+xy}\,dx\,dy}$$

can do division...

Fubini ‖

$$\iint_R \frac{x}{1+xy}\,dA$$



$$\boxed{\begin{array}{l} 0 \le x \le 1 \\ 0 \le y \le 2 \end{array}}$$

Fubini ‖

$$\int_0^1 \int_0^2 \frac{x}{1+xy}\,dy\,dx$$

d

$u = 1 + xy$

$du = x\,dy$

$$\int \frac{5}{1+5\cancel{xy}}\,dy$$

$$\int_0^2 \frac{x}{1+xy}\,dy = \int \frac{du}{u} =$$

$$\ln|u| = \ln|1+xy)\Big|_{y=0}^{y=2}$$

$$\ln(1+2x) - \underbrace{\ln 1}_{= 0}$$

$$\int_0^1 \underbrace{\ln(1+2x)}_{u}\,\underbrace{dx}_{dv} =$$

$du = \dfrac{2}{1+2x}$ $\quad v = x$

$uv - \int v\,du$ $\leftarrow$ Calc 2

$$x\ln(1+2x) - \int \frac{2x}{1+2x}$$

$$\frac{2x}{1+2x} = \overbrace{\frac{1+2x}{1+2x}}^{1} - \frac{1}{1+2x}$$

$$x\ln(1+2x) - \int 1 - \frac{1}{1+2x}$$

$$x\ln(1+2x) - x + \frac{1}{2}\ln(1+2x)\Big|_0^1$$

$$= \left(\ln 3 - 1 + \frac{1}{2}\ln 3\right) - (0)$$

$$= \frac{3}{2}\ln 3 - 1.$$

<u>Dealing with regions</u>

+ <u>endpoints</u>

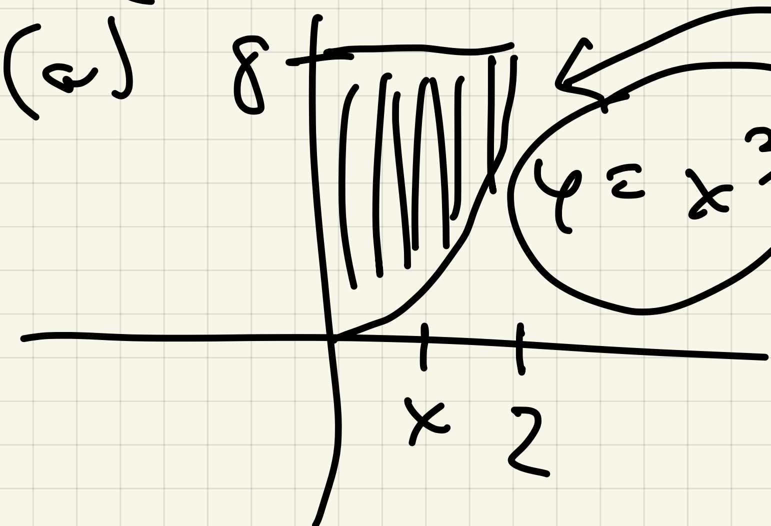
<u>Ex2</u> Write iterated integral

for $\iint_R dA$

for region R, in both

(#9) orders

(a)



$y = x^3 \quad = \quad x = \sqrt[3]{y}$

$$\int_0^2 \int_{x^3}^8 1 \, dy \, dx$$

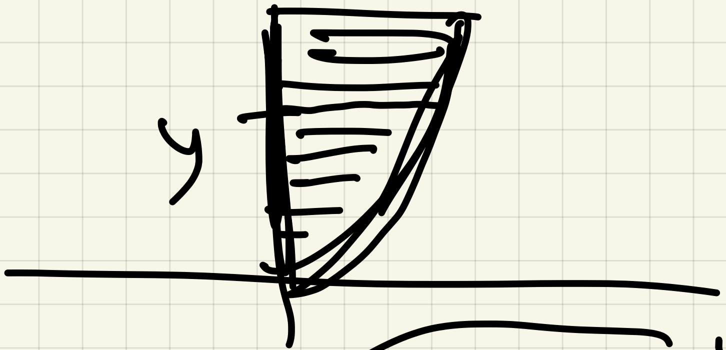
$0 \le x \le 2$

$x^3 \le y \le 8$

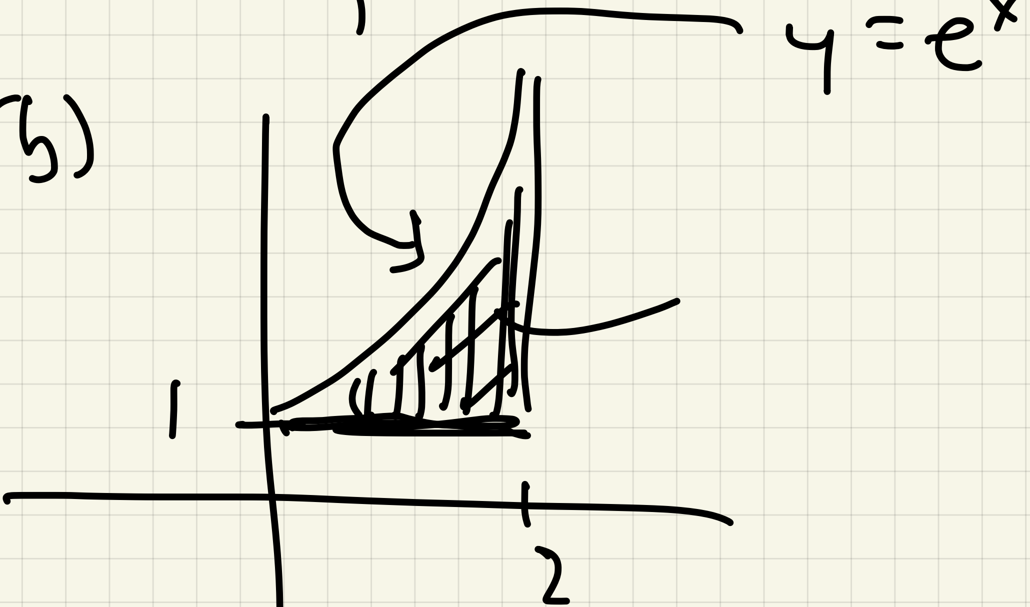
$$\int_0^8 \int_0^{\sqrt[3]{y}} 1 \, dx \, dy$$

$0 \le y \le 8$

$0 \le x \le \sqrt[3]{y}$



(b)

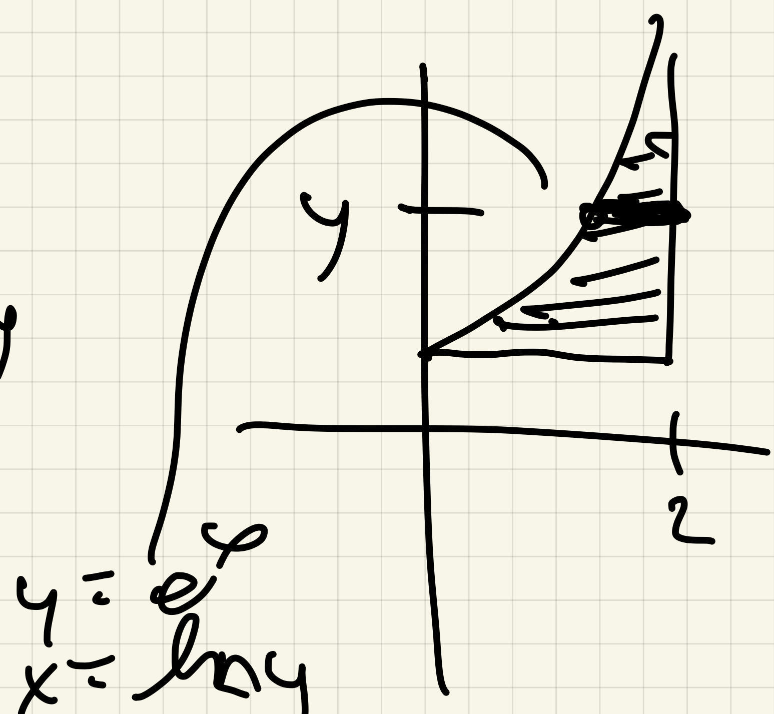


$$\int_0^2 \int_1^{e^x} 1 \, dy \, dx$$

$0 \le x \le 2$

$1 \le y \le e^x$

$$\int_1^{e^2} \int_{\ln y}^{2} 1 \, dx \, dy$$



$$0 \leq y \leq e^2$$

$$\ln y \leq x \leq 2$$