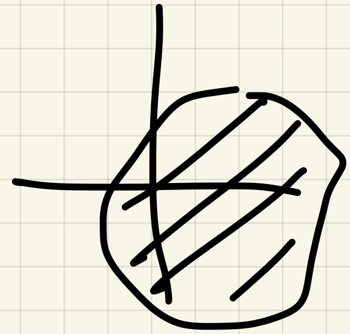
10/24 | Calc 3

Last time

$z = f(x,y)$

$R \subset \mathbb{R}^2$



$$\iint_R f(x,y)\,dA = \text{signed volume under graph}$$

How to calculate?

(A) (a) $\displaystyle\int (x+y^3)\,dx = \frac{x^2}{2} + y^3 x + C(y)$

(y) $\displaystyle\int (x+y^3)\,dy = xy + \frac{y^4}{4} + C(x)$

(B) Endpoints

$$\int_{x=y}^{x=y^2} (x+y^3)\,dx = \frac{1}{2}x^2 + xy^3 \Big|_{x=y}^{x=y^2} =$$

$$\underline{\left(\tfrac{1}{2}y^4\right.} + y^5\Big) - \left(\tfrac{1}{2}y^2 + \underline{y^4}\right)$$

$$y^5 - \tfrac{1}{2}y^4 - \tfrac{1}{2}y^2$$

(c) Can iterate process:

Ex 3 $$\int_0^1 \underbrace{\left[\int_0^2 (x+y^3)\,dx\right]} dy$$

$$\int_0^1 \underbrace{\frac{x^2}{2} + xy^3 \Big|_0^2} dy$$

$$\int_0^1 (2+2y^3) - (0)\ dy$$

$$2y + \frac{1}{2}y^4 \Big|_0^1 = 2 + \frac{1}{2} = 5/2$$

$$\int_0^2 \underbrace{\int_0^1 (x+y^3)\,dy}\,dx$$

$$xy + \frac{y^4}{4}\Big|_{y=0}^{y=1}$$

$$\int_0^2 x + \frac{1}{4}\,dx = \frac{x^2}{2} + \frac{1}{4}x\Big|_0^2 =$$

$$2 + \frac{1}{2} = 5/2$$

Connection: (Fubini's Theorem)

If the end points $\int_{\bigcirc}^{\bigcirc}\int_{\bigcirc}^{\bigcirc} f(x,y)\,dx\,dy$

correspond to region $R \subset \mathbb{R}^2$,

then

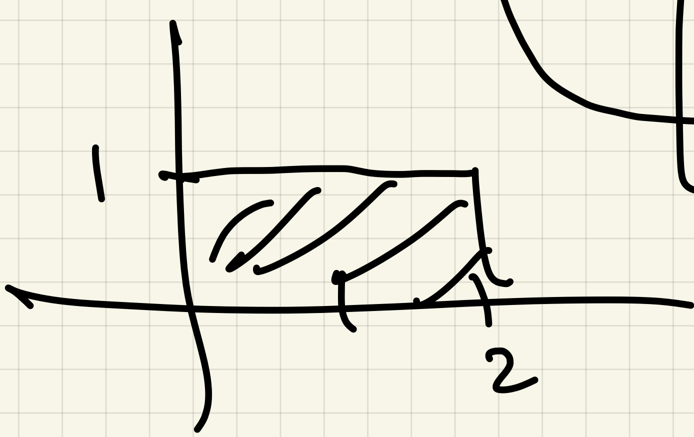
$$\int_{\bigcirc}^{\bigcirc}\int_{\cup}^{\cup} f\,dx\,dy = \iint_R f(x,y)\,dA$$

What does this mean?

Ex 3

$$\int_0^1\int_0^2 (x+y^3)\,dx\,dy$$

$$\boxed{\begin{array}{l} 0 \le x \le 2 \\ 0 \le y \le 1 \end{array}}$$



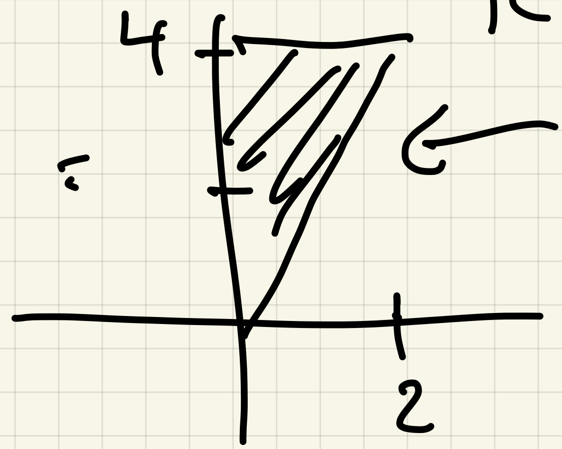
$$= \iint_R (x+y^3)\,dA$$

Ex 4 Find $\iint_R 2xy\, dA$

$y = 2x$

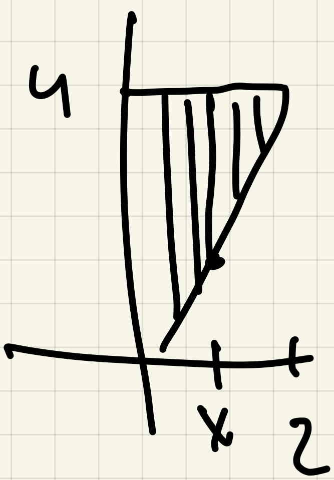
$x = \frac{1}{2}y$

R :



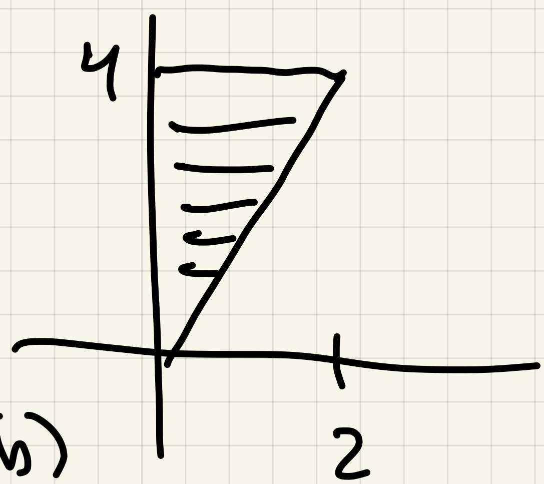
triangle ;

describe with ranges on x/y (endpoints)



(a) $0 \le x \le 2$

$2x \le y \le 4$



(b) $0 \le y \le 4$

$0 \le x \le y/2$

(a) $\int_0^2 \int_{2x}^4 \underbrace{2xy\, dy}\, dx =$

$$\int_0^2 xy^2\Big|_{y=2x}^{y=4} = \int_0^2 \underline{16x - 4x^3 dx}$$

$$8x^2 - x^4\Big|_0^2 = 32 - 16 = 16$$

(b) $$\int_0^4 \underbrace{\int_0^{y/2} 2xy \, dx}_{} \, dy$$

$$x^2 y\Big|_{x=0}^{x=y/2} =$$

$$\frac{y^2}{4} \cdot y = \int_0^4 \frac{y^3}{4} \, dy =$$

$$\frac{y^4}{16}\Big|_0^4 = \frac{4^4}{16} = 16 \checkmark$$

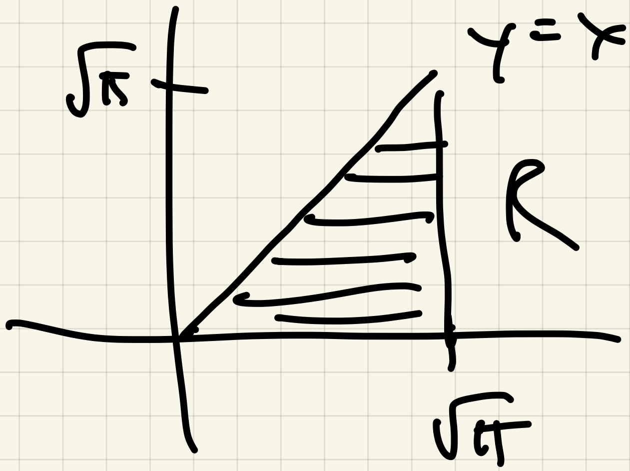
<u>Ex2</u> $$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \underline{\sin(x^2) \, dx \, dy}$$

???

$$\begin{matrix} y & x \end{matrix}$$

$$0 \leq y \leq \sqrt{\pi}$$

$y \le x \le \sqrt{\pi}$



Fubini

$$\iint_R \sin(x^2)\, dA$$

$0 \le x \le \sqrt{\pi}$

$0 \le y \le x$

Fubini

$$\int_0^{\sqrt{\pi}} \int_0^{x} \underbrace{\sin(x^2)\, dy}\, dx$$

$$(\sin x^2)\, y \Big|_{y=0}^{y=x}$$

$$\int_0^{\sqrt{\pi}} (\sin(x^2))\, x\, dx$$

$u = x^2$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$$\frac{1}{2}\int_{u=0}^{u=\pi} \sin u\, du =$$

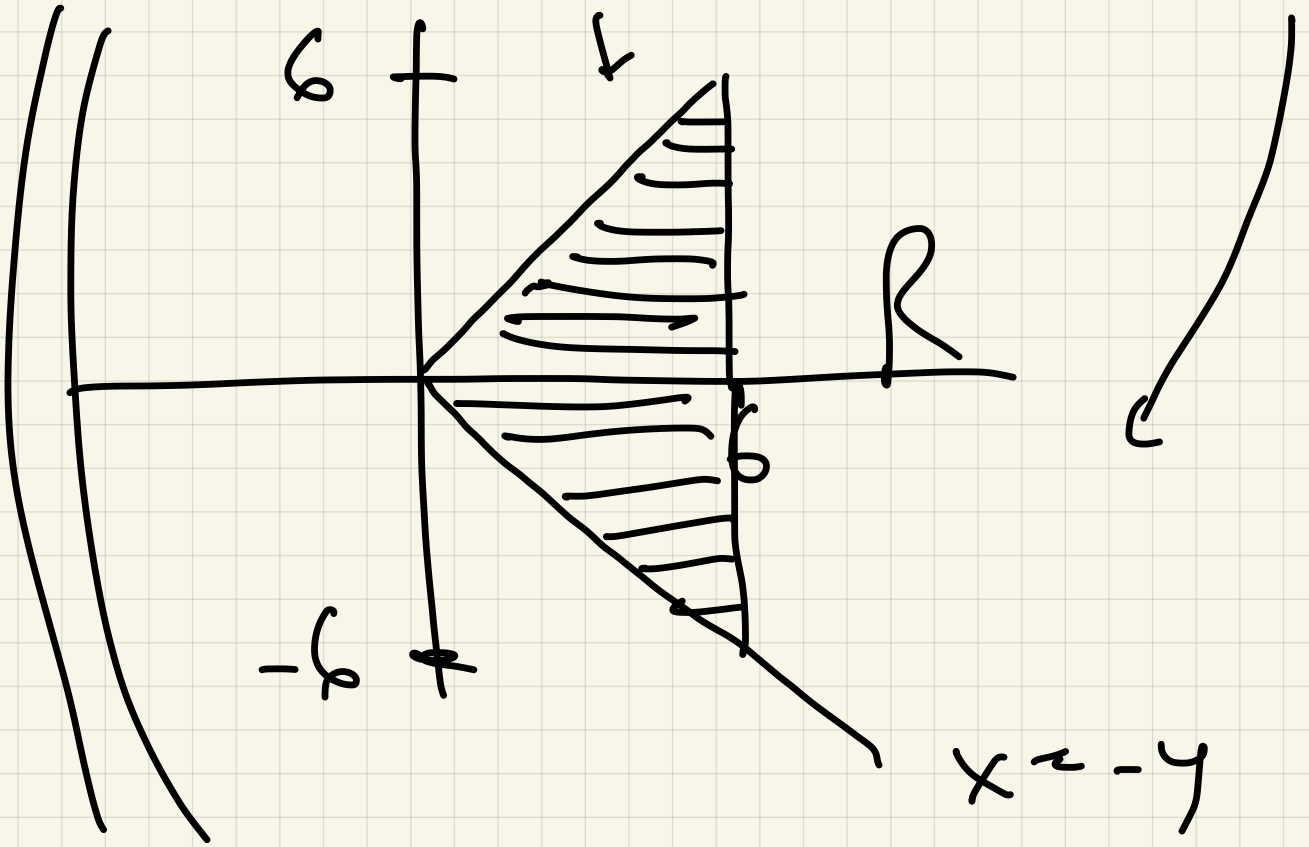
$$-\frac{1}{2}\cos u \Big|_0^{\pi} =$$

$$-\frac{1}{2}(-1) - \left(-\frac{1}{2}\right)(1)$$

$$\frac{1}{2} + \frac{1}{2} = 1.$$

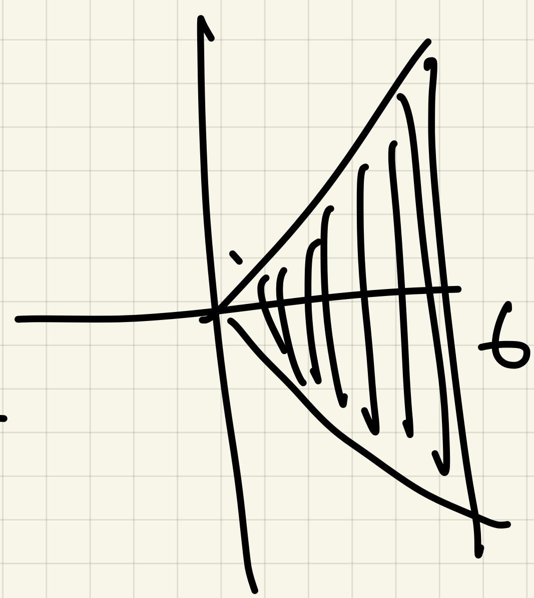
Ex

$$= \int_0^6 \int_y^6 1\, dx\, dy + \int_{-6}^0 \int_{-y}^6 1\, dx\, dy$$



$$\iint_R 1\, dA = \text{Area of } R$$

$$= \int_0^6 \underbrace{\int_{-x}^{x} 1\, dy}_{x}\, dx$$



$$y\Big|_{-x}^{x}$$

$$x - (-x) = \int_0^6 2x \, dx =$$

$$x^2 \Big|_0^6 = 36$$