10/23 | Calc3

Exam 2

avg 83

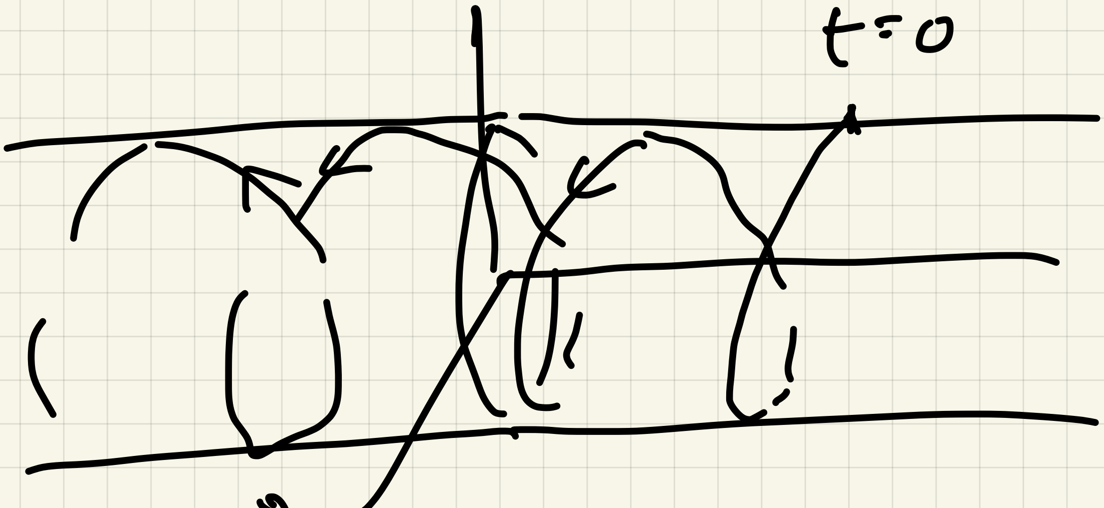
150	1
135	5
120	4
105	0
	4

#1

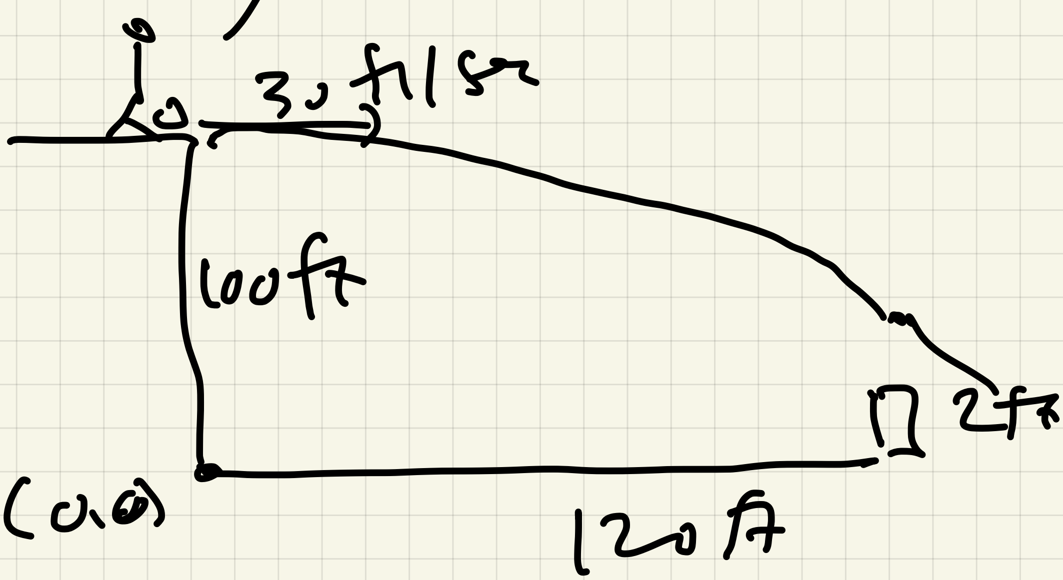
$r(t) = \langle 3\sin t,\ 5-2t,\ 3\cos t \rangle$

$\quad\quad x \quad\quad y \quad\quad z$

$x^2 + z^2 = 9$



#2



$a = g = \langle 0, -32 \rangle$

12

$v = \int \langle 0, -12 \rangle \, dt$

$v(t) = \langle 0, -12t \rangle + \langle \overset{30}{\overset{\shortparallel}{c_1}}, \overset{0}{\overset{\shortparallel}{c_2}} \rangle$

$v(0) = \langle 30, 0 \rangle \Downarrow$

(a) $v(t) = \langle 30, -12t \rangle$

(b) $r(t) = \int \langle 30, -12t \rangle \, dt$

$= \langle 30t, -6t^2 \rangle + \langle \overset{0}{\overset{\shortparallel}{d_1}}, \overset{100}{\overset{\shortparallel}{d_2}} \rangle$

$r(0) = \langle 0, 100 \rangle$

$r(t) = \langle 30t, 100 - 6t^2 \rangle$

(c) $\underset{\substack{\shortparallel \\ 120}}{x}$ $\quad y$

$t = 4$ $\quad y(4) =$

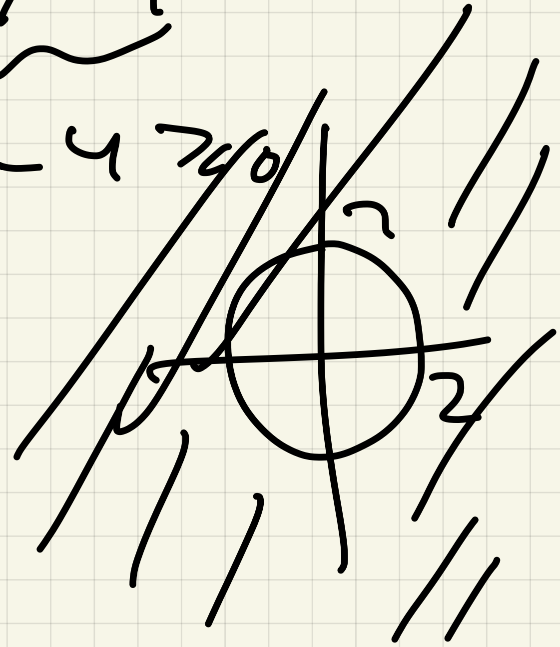
$100 - 6(4)^2 =$

$100 - 96 = 4 > 0$

$\boxed{\#3}$ ✓

$\boxed{\#4}$ $z = -\sqrt{\underbrace{x^2+y^2-4}}$

$x^2+y^2-4 \geq 0$



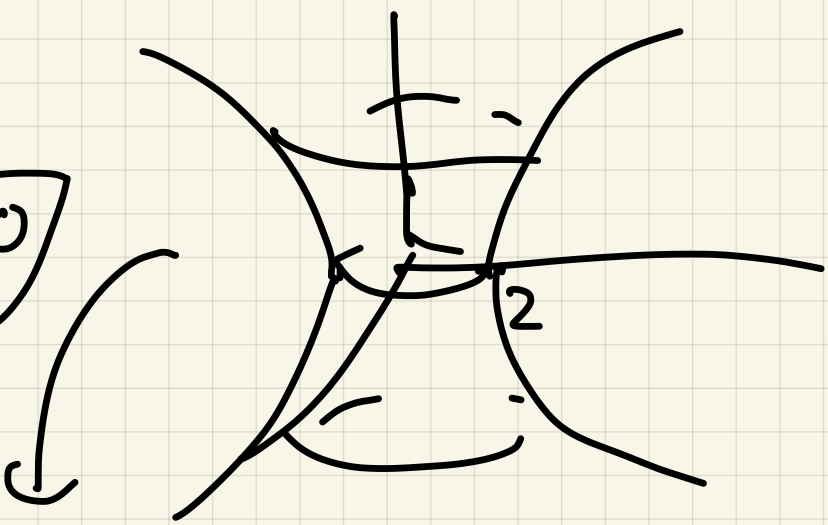
Dom $x^2+y^2 \geq 4$

(a)

(b)

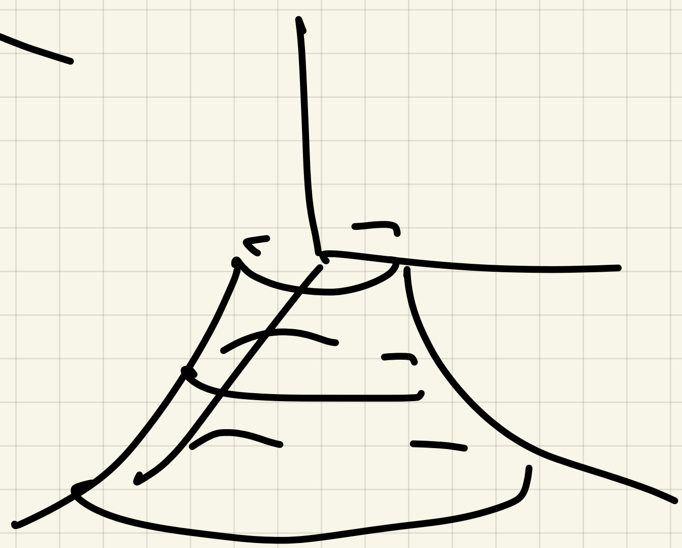
$z^2 = x^2+y^2-4$

$z^2+4 = x^2+y^2$



$\boxed{z \leq 0}$

range $(-\infty, 0]$



#5 ✓ #6 ✓

Last time $z = f(x,y)$

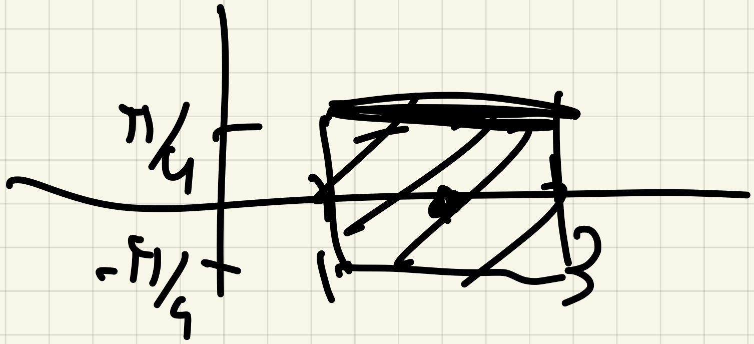
Thm: $f(x,y)$ continuous on closed bounded region $R \subset \mathbb{R}^2$,
$\Rightarrow$ f achieves max/min on R

(1) crit pt inside R

(2) Boundary

Ex 1) $f(x,y) = (4x - x^2)\cos y$

find max/min values on rectangular plate $1 \le x \le 3$, $-\pi/4 \le y \le \pi/4$



① crit pt inside R is $(2,0)$

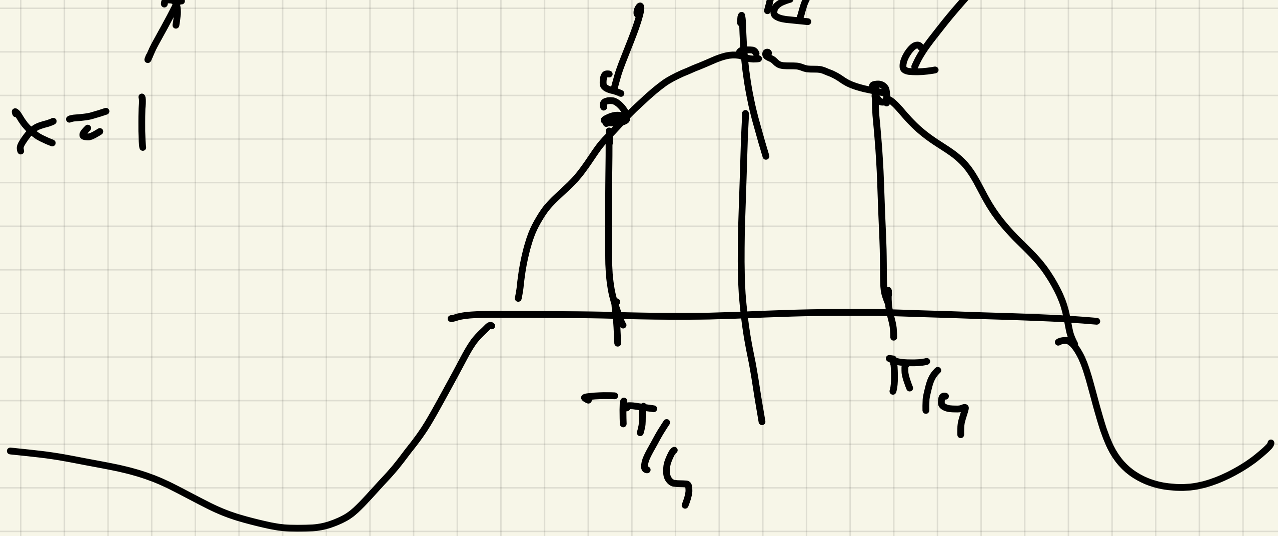
$$\boxed{f(2,0) = 4}$$

② Boundary:

Left $x = 1$

$$f(1,y) = 3\cos y$$

$x=1$



$$\boxed{\begin{aligned} f(1,0) &= 3 \\ f(1,\pi/4) &= 3/\sqrt{2} \\ f(1,-\pi/4) &= 3/\sqrt{2} \end{aligned}}$$

right $x=3$ $f(3,y) = 3\cos y$

$-\pi/4 \le y \le \pi/4$

$$\boxed{\begin{aligned} f(3,0) &= 3 \\ f(3,\pi/4) &= 3/\sqrt{2} \\ f(3,-\pi/4) &= 3/\sqrt{2} \end{aligned}}$$

Top: $y = \pi/4$ $f(x, \pi/4) =$

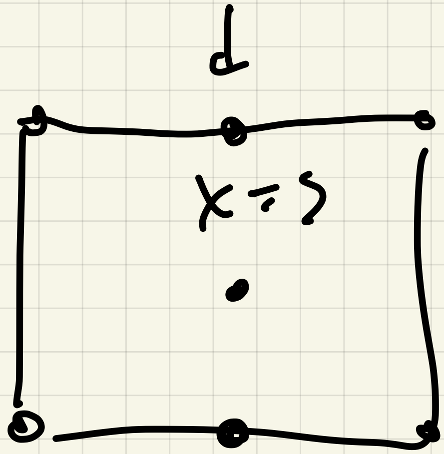
$$f = \frac{1}{\sqrt{2}}(4x - x^2)$$

$$f'(x) = \frac{1}{\sqrt{2}}(4-2x) = 0$$

$\Downarrow$

$x=2$

$$\boxed{f(2,\pi/4) = \frac{1}{\sqrt{2}} 4 = \frac{4}{\sqrt{2}}}$$



bottom $f(x, -\pi/4) = \frac{1}{\sqrt{2}}(4x - x^2)$

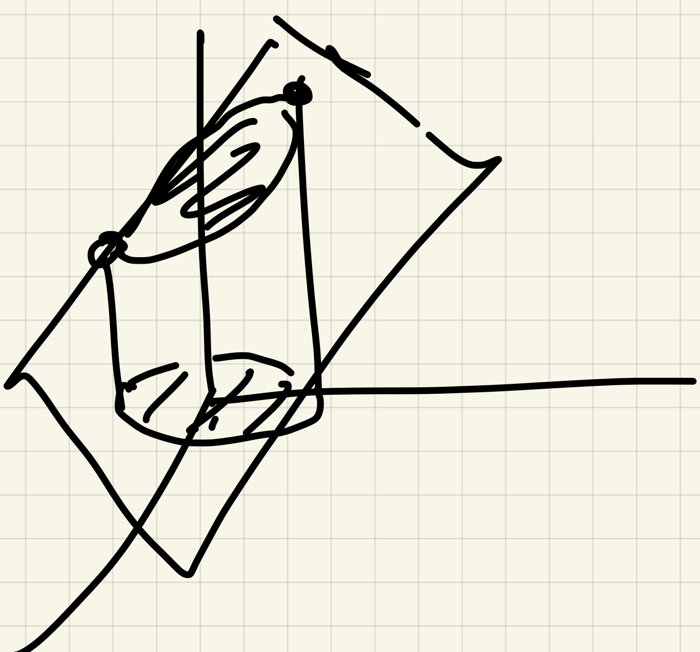
$x=2 \Rightarrow$ $\boxed{f(2, -\pi/4) = 4/\sqrt{2}}$

abs **Max** 4 at $(2, 0)$

abs **min** $3/\sqrt{2}$ at $(1, \pm \pi/4)$
$(3, \pm \pi/4)$

Ex) $f(x,y) = 2x + y + 1$

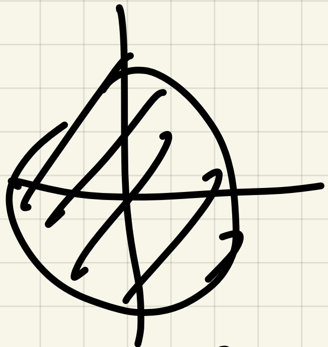
$R = \{(x,y) : x^2 + y^2 \leq 4\}$



(1) $\nabla f = \langle 2, 1 \rangle \neq \langle 0, 0 \rangle$

no crit pts inside R

②



boundary is circle

$x^2+y^2=4$ — $y=\pm\sqrt{4-x^2}$

top circle $y=\sqrt{4-x^2}$

bot $y=-\sqrt{4-x^2}$

WORKS

But easier:

$$\boxed{\begin{aligned} x&=2\cos\theta \\ y&=2\sin\theta \end{aligned}}$$

$x^2+y^2=4$

$0\le\theta\le 2\pi$

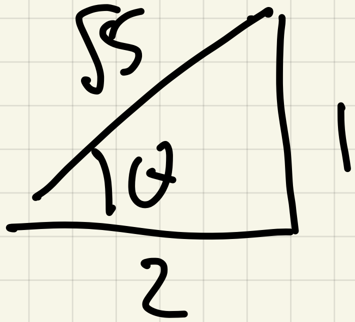
$f(x,y) = 2x+y+1$

$f(\theta) = 4\cos\theta+2\sin\theta+1$

$f'(\theta) = -4\sin\theta+2\cos\theta=0$

$4\sin\theta=2\cos\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2}{4} = \frac{1}{2}$



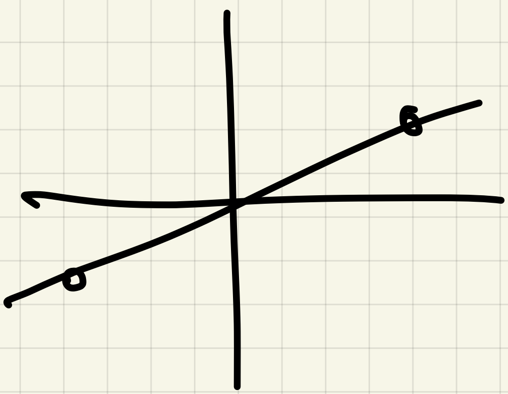
$\sin\theta = \frac{1}{\sqrt{5}}$

$\cos\theta = \frac{2}{\sqrt{5}}$

$\frac{8}{\sqrt{5}} + \frac{2}{\sqrt{5}} + 1 = \frac{10}{\sqrt{5}} + 1$

$2\sqrt{5}+1$ max

$-2\sqrt{5}+1$



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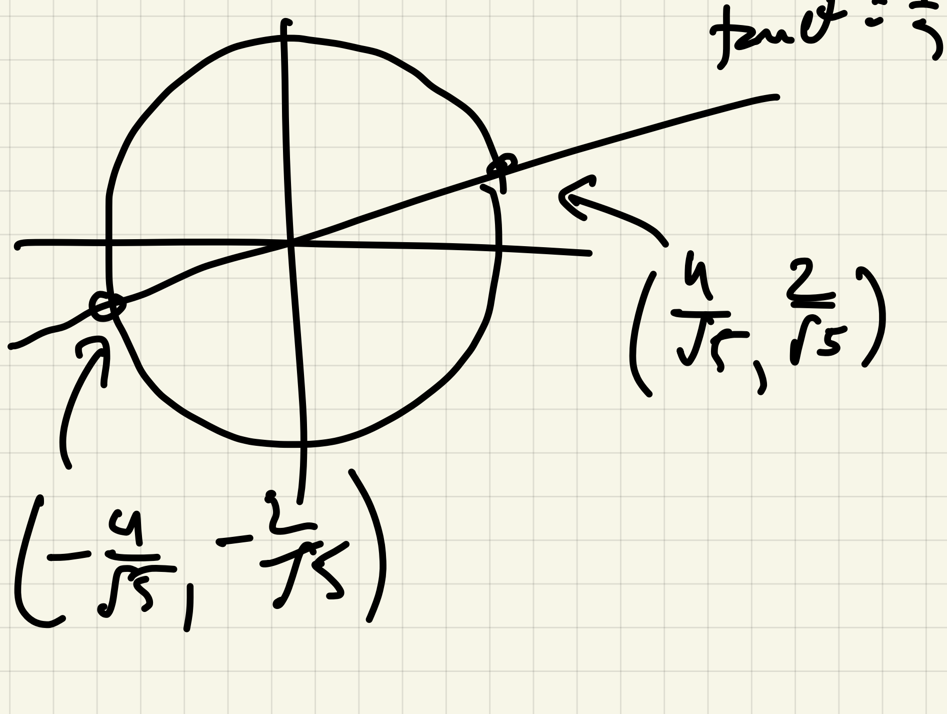
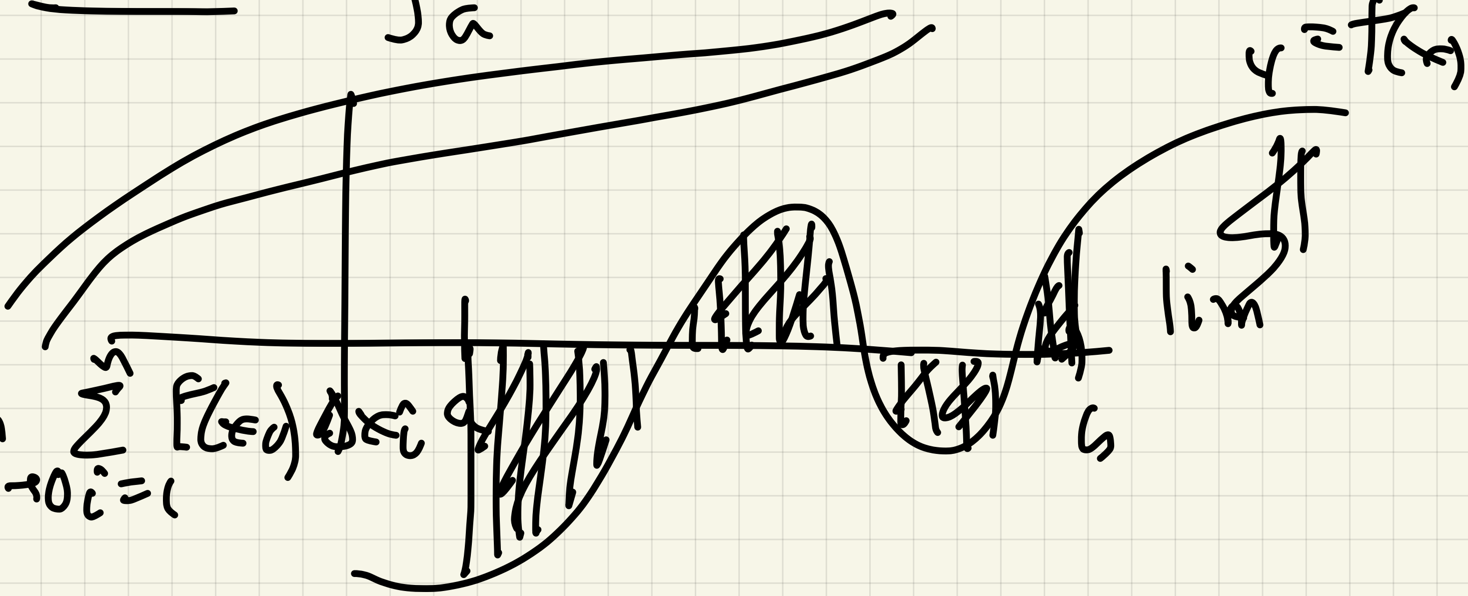


Ch 14: Integration:

Calc I: $\int_a^b f(x)\,dx$ = signed area under curve



$$\lim_{|\Delta x| \to 0} \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$$

FTC: $\int_a^b f(x)\,dx = F(b) - F(a)$

where $F' = f$

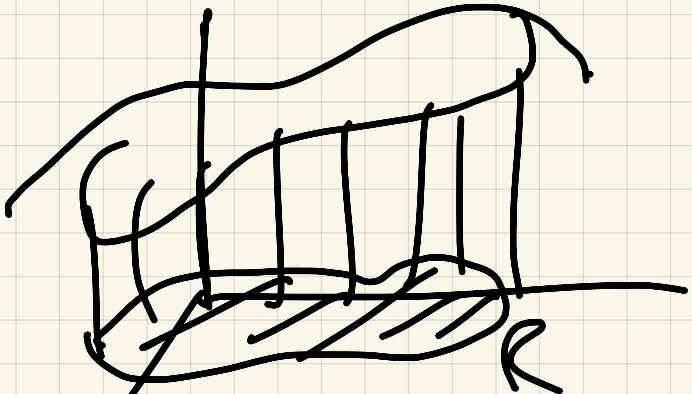
Calc 3: $z = f(x,y)$

$R \in \mathbb{R}^2$ region

Double integral $\iint_R f(x,y)\,dA$

= signed volume under graph of

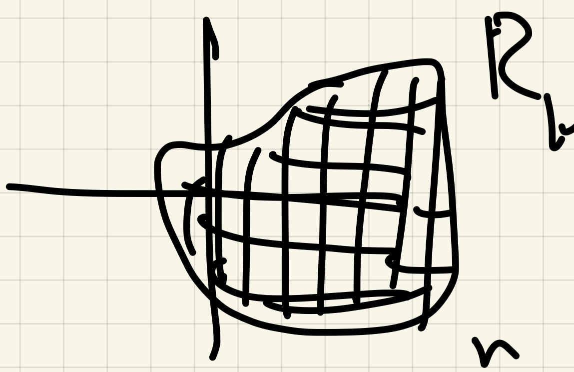
$z = f(x,y)$



Aside

Can estimate

$$\iint_R f(x,y)\,dA \approx$$



A_i little rectangles

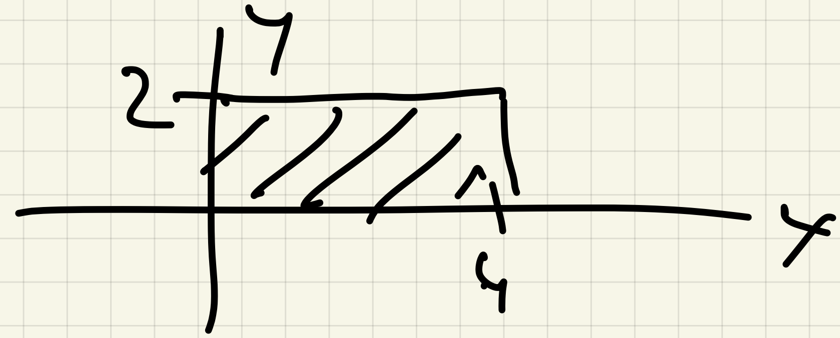
(x_i, y_i) in A_i

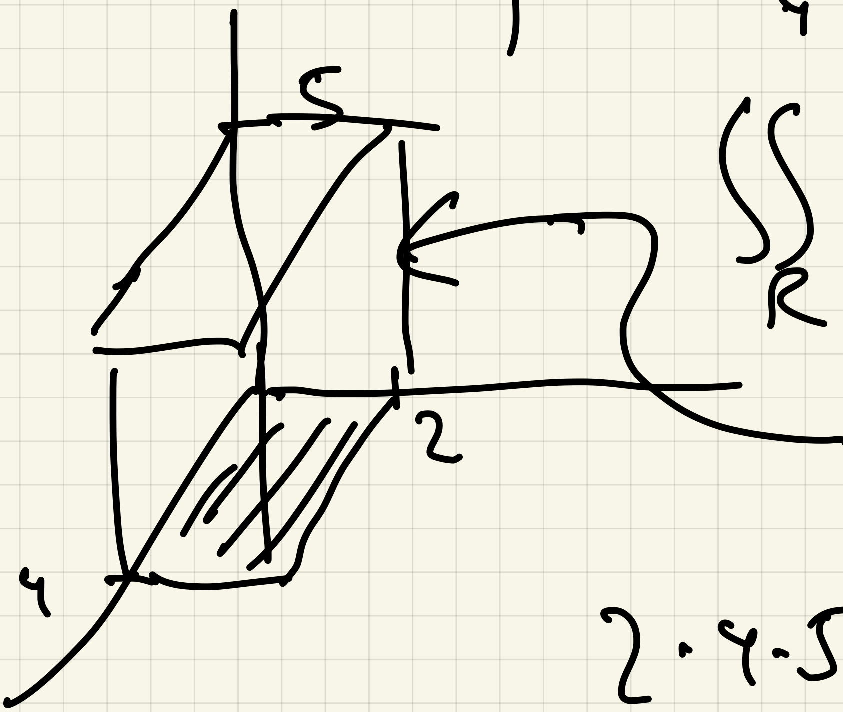
$$\sum_{i=1}^{n} f(x_i, y_i)\,(\text{Area } A_i)$$

Riemann sum

Ex1 (a) $z = f(x,y) = 5$

$R =$





$$\iint_R 5\,dA$$

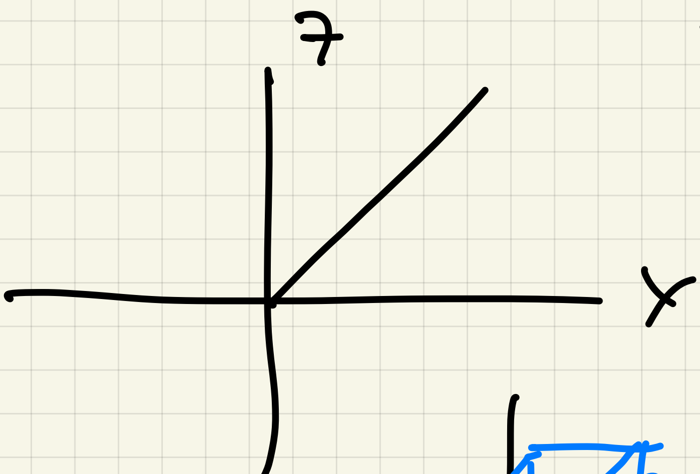
$=$

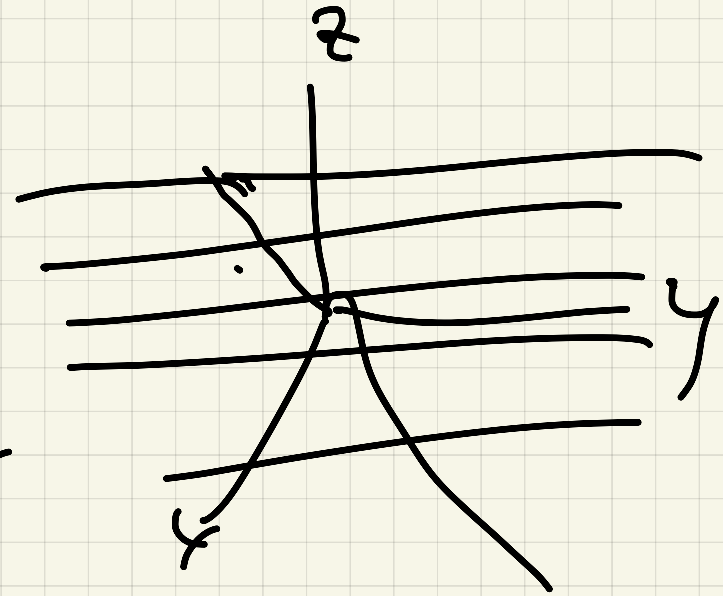
volume

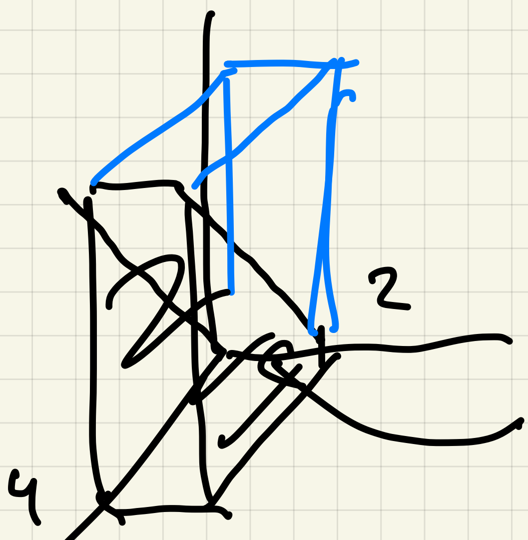
$=$

$$2\cdot 4\cdot 5 = 40$$

(b) $z = x$







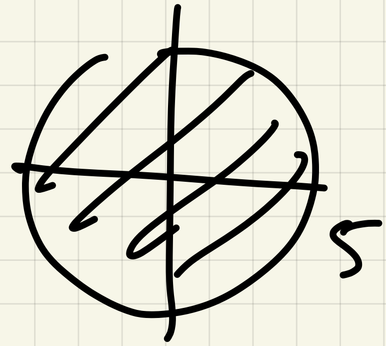
$$\iint_R x\,dA$$

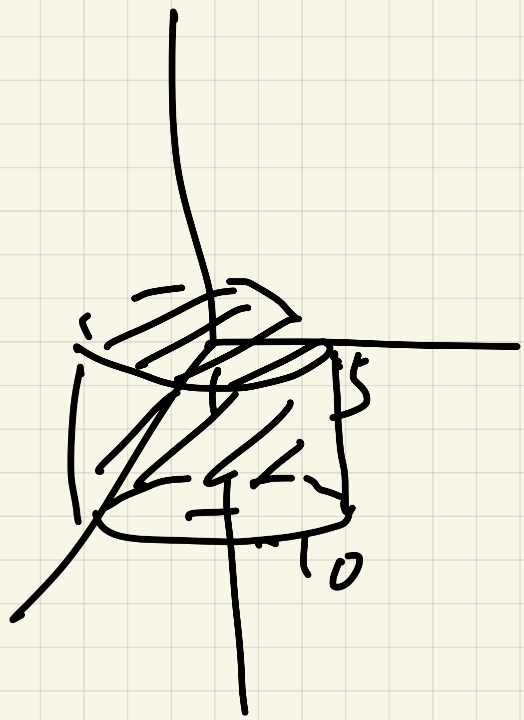
$=$

volume

$=$

$$\frac{1}{2}(2\cdot 4\cdot 4) = 16$$

(c) $z = -10$ $\quad R =$ 



$$\iint_R -10\, dA$$

$$\parallel$$

$$-\left(\begin{matrix}\text{Volume}\\ \text{cylinder}\end{matrix}\right)$$

$$\parallel$$

$$-(25\pi)(10) = -250\pi$$

How to <u>compute</u> $\iint_R f(x,y)\, dA$??

<u>Answer:</u> <u>Iterated integrals</u>

<u>Ex2</u>: $\int (x+y^3)\, dx = \frac{1}{2}x^2 + y^3 x + C(y)$

$$\int (x+y^3)\,dy = xy + \frac{1}{4}y^4 + C(x)$$

$$\int_{x=0}^{x=2} (x+y^3)\,dx = \frac{1}{2}x^2 + xy^3 \Big|_{x=0}^{x=2} = 2 + 2y^3$$

$$\int_{x=y}^{x=y^2} (x+y^3)\,dx = \frac{1}{2}x^2 + xy^3 \Big|_{x=y}^{x=y^2} = y^5 - \frac{1}{2}y^4 - \frac{1}{2}y^2$$