10/21/Calc 3 Exam 2 → Oct 24

12.1 - 12.7

13.1 - 13.6

Quiz 13

1. $(1,1,1)$ on

$g = x^3 + x^2y^3 + 2z^3y + z^3 = 5$

$\nabla g = (3x^2 + 2xy^3, 3x^2y^2 + 8z^3y^3,$

$6z^2y + 3z^2)$

$\nabla g(1,1,1) = (5, 11, 9)$

$5(x-1) + 11(y-1) + 9(z-1) = 0$

$5x + 11y + 9z = 25$

2. $(0,2,5)$ T

$z = \dfrac{4x + e^{2xy} + y^2}{4x + e^{2xy} + y^2 - z = 0}$

cont ↓

$\langle 4+2ye^{2xy}, 2xe^{2xy}+2y, -1 \rangle$

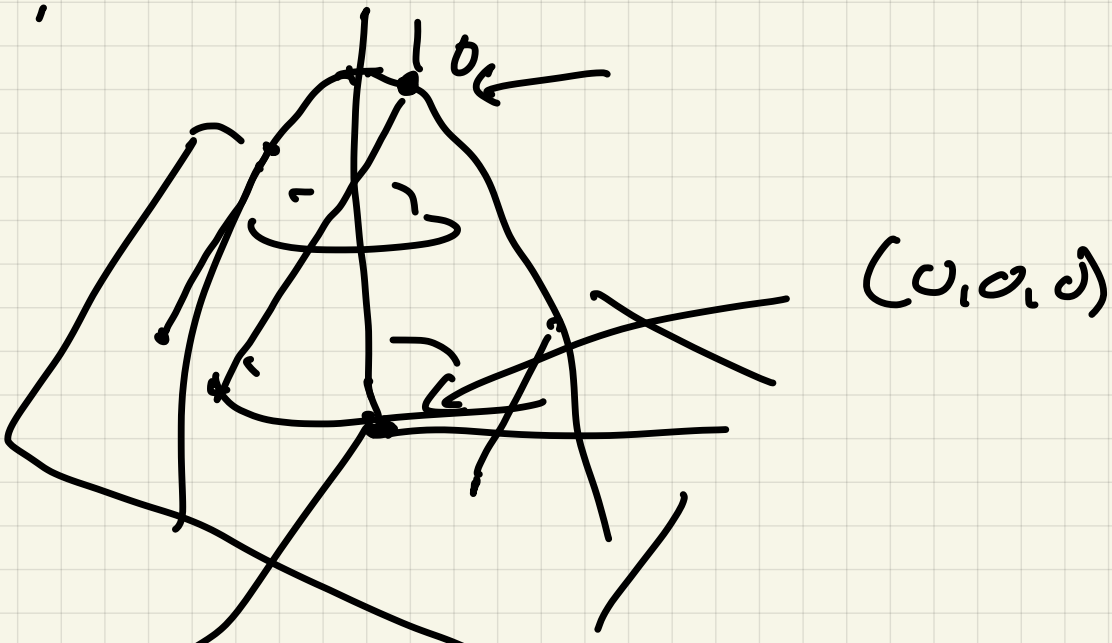
$\langle 8, 4, -1 \rangle$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8t \\ 2+4t \\ 5-t \end{pmatrix}$$

Ex1 Among all points on graph of $z = 10 - x^2 - y^2$ that lie above plane

$x+2y+3z = 0$ ← $n = \langle 1, 2, 3 \rangle$

which is farthest from plane?



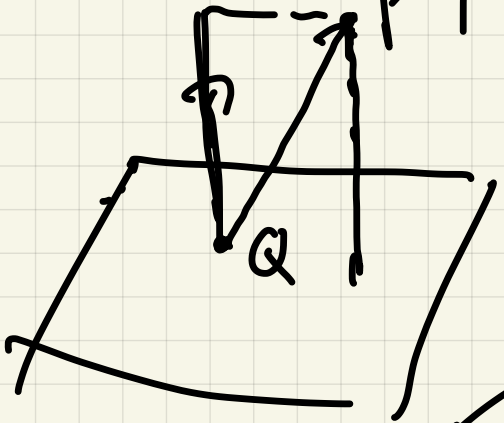
Ch 11: If n normal to plane, P is point dist plane to P

$$\left| \text{Proj}_{\vec{n}} (P - (0,0,0)) \right| =$$

$$\frac{P - \langle 0,0,0 \rangle \cdot n}{|n|}$$

so if $P = (x,y,z)$

$$\frac{(x,y,z) \cdot \langle 1,2,3 \rangle}{|\langle 1,2,3 \rangle|}$$



but

$\text{Proj}_n QP$ $z = 10 - x^2 - y^2$

$$\frac{\langle x,y,10-x^2-y^2 \rangle \cdot \langle 1,2,3 \rangle}{\sqrt{14}}$$

4

$$\frac{x+2y+3(10-x^2-y^2)}{\sqrt{14}}$$

$$f = \frac{30 - 3x^2 - 3y^2 + x + 2y}{\sqrt{14}}$$

maximize:

$$\nabla f = \left\langle \frac{-6x+1}{\sqrt{14}}, \frac{-6y+2}{\sqrt{14}} \right\rangle = \langle 0, 0 \rangle$$

at $(x, y) = \left(\frac{1}{6}, \frac{2}{6}\right)$

$$z = 10 - x^2 - y^2 \Rightarrow$$

$$(x, y, z) = \left(\frac{1}{6}, \frac{1}{3}, \frac{355}{36}\right)$$

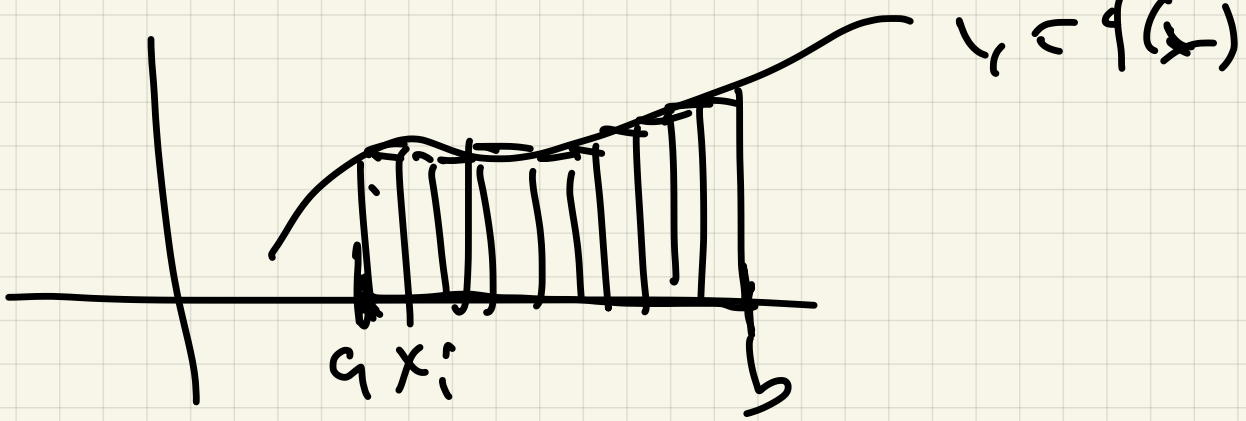
Ch 14 Integrals

§14.1 - 14.2

Calc I & Calc 2

$$\int_a^b f(x)\,dx = \text{signed area under curve}$$

estimate $\sum_{i=1}^{n} f(x_i)\,\Delta x_i$ (height × width)



$$\int_a^b f(x)\,dx = \lim_{|\Delta x| \to 0} \sum_{i=1}^{n} f(x_i)\,\Delta x_i$$

Riemann sum

To compute:

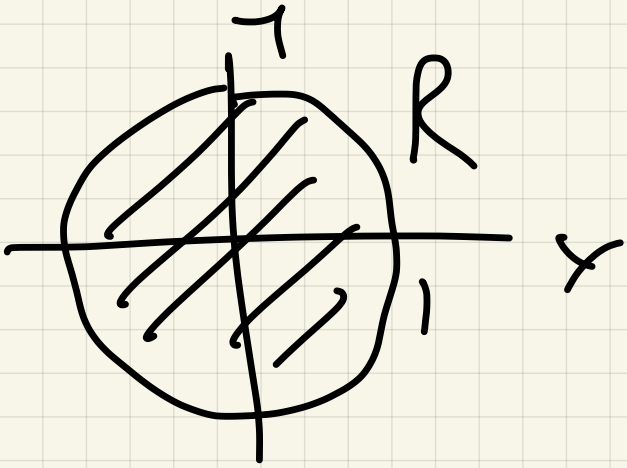
FTC: $\int_a^b f(x)\,dx = F(b) - F(a)$

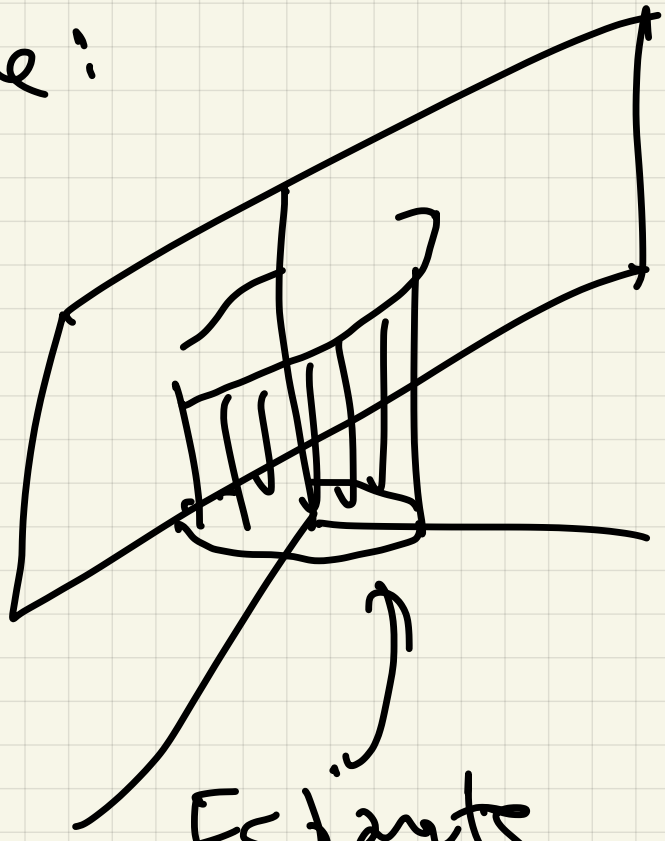
$F'(x) = f(x)$

Similar story:

Want to define:

$$\iint_R f(x,y)\,dA$$





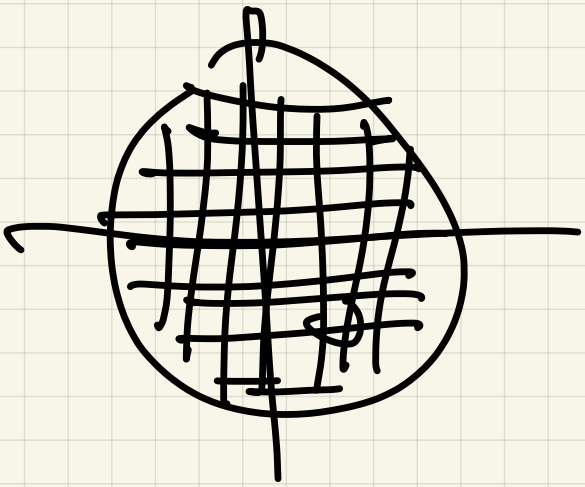
Estimate volume

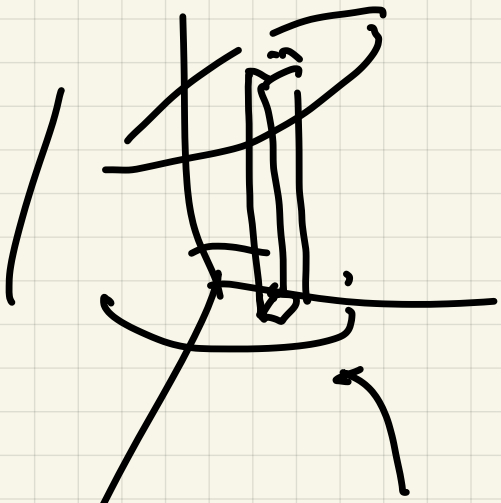
Way to estimate,

Partition R into little squares A_i

of Area ΔA_i

$|\Delta| =$ length of largest diagonal





~~Choose~~

Choose a point (x_i, y_i) in rectangle A_i

Volume $\approx \sum_{i=1}^{n} f(x_i, y_i) \cdot \Delta A_i$

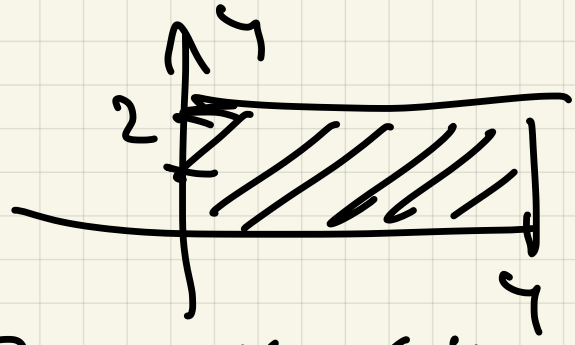
Total signed volume is

$$\lim_{|\Delta| \to 0} \sum_{i=1}^{n} f(x_i, y_i)\, \Delta A_i$$

Example:

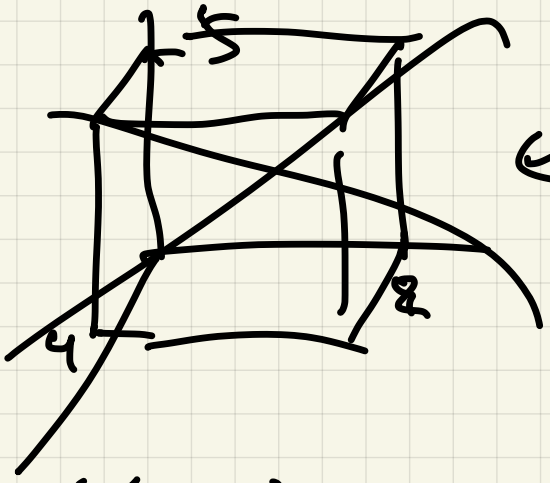
(a) $z = f(x,y) = 5$

R:

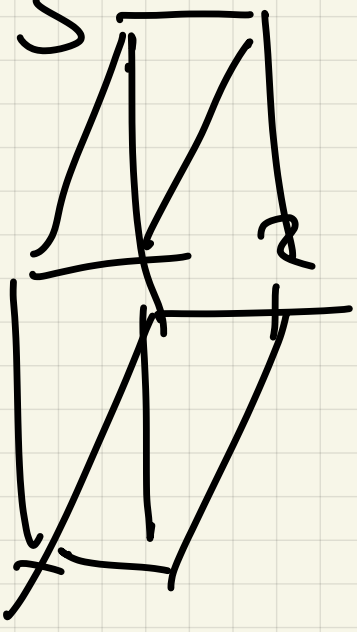


R: $0 \le x \le 4$
$0 \le y \le 2$

$$\iint_R 5\,dA = \text{Volume} = 40$$



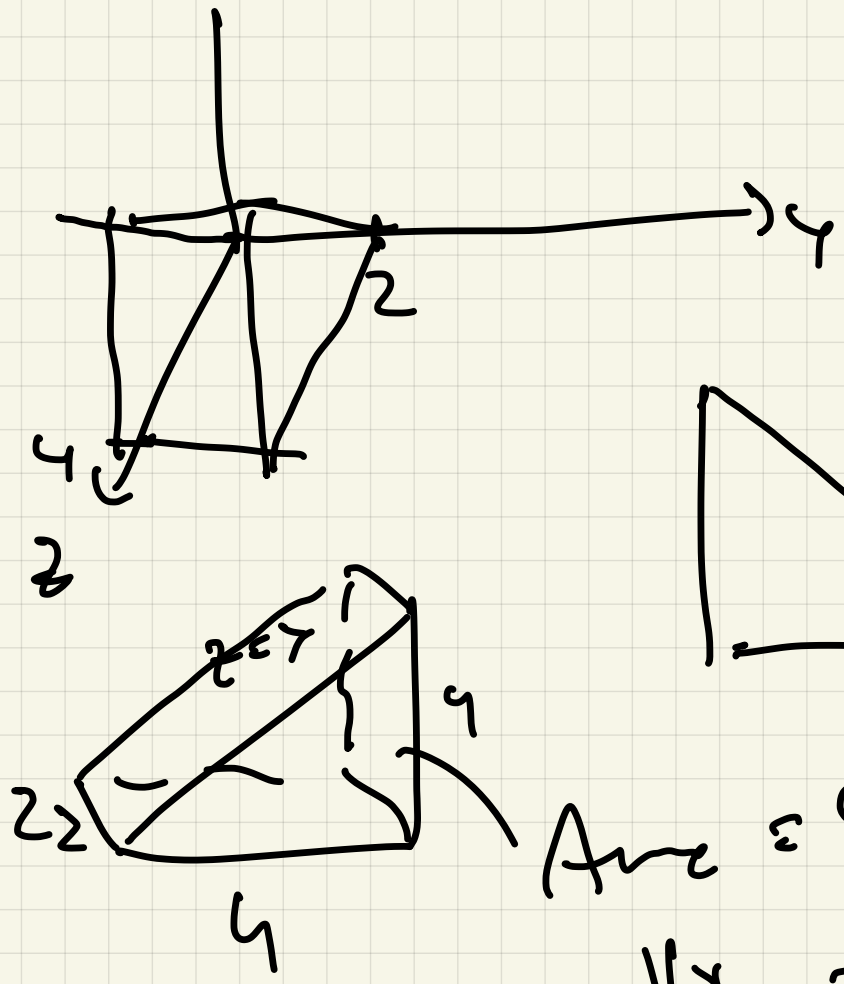
Volume

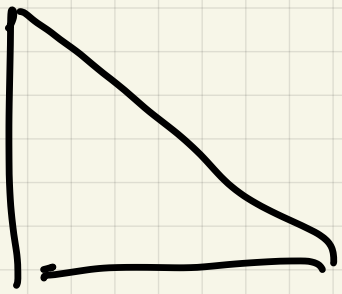


(b) $z = f(x,y) = x$

R same

$z = x$





$$Area = 8 \quad \frac{16}{2}$$

$$Vol. = 8 \cdot 2 = 16$$