10/21 Calc3 Quiz 9

$\boxed{\#1}$ $\lim_{(x,y)\to(2,3)} \frac{x-3y^2}{2x+y} = \frac{-25}{7}$

$\boxed{\#2}$ $f(x,y) = \frac{x-3y^2}{2x+y}$

continuous

$\{(x,y) : 2x+y \neq 0\}$

$y \neq -2x$

$\boxed{\#3}$ x-axis : $y=0$

$\lim_{x\to 0} \frac{x-0}{2x+0} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$

$\boxed{\#4}$ y-axis $x=0$

$\lim_{y\to 0} \frac{-3y^2}{y} = \lim_{y\to 0} \frac{-3y}{1} = 0$

$\|$

$\lim_{y\to 0} \frac{-6y}{1} = 0$

#5 lim DNE b/c $\frac{1}{2} \neq 0$

Last time: Partials

$f_x \quad f_y$

Higher partials

$f_{xx} \quad f_{xy} \quad f_{yy} \quad f_{yx}$

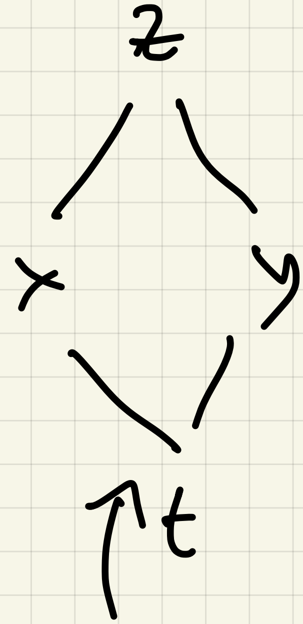
also $\quad g_{xyx} = g_{xxy} = g_{yxx}$

Chain Rule:

If $z = f(x,y)$, $\quad \left.\begin{array}{l} x = g(t) \\ y = h(t) \end{array}\right\} \Rightarrow$

z function of t

$$\boxed{\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = \frac{\partial z}{\partial x}\, g'(t) + \frac{\partial z}{\partial y}\, h'(t)}$$



$$\boxed{z_x\, x'(t) + z_y\, y'(t)}$$

Dependency diagram

Ex) $\boxed{z = x^2 + xy + y^3}$

$\boxed{x = t^2}$, $\boxed{y = t^3}$

(a) Find dz/dt with chain rule

(b) Check answer by substituting

$$\frac{dz}{dt} = \partial z/\partial x \cdot \frac{dx}{dt} + \partial z/\partial y\ \frac{dy}{dt}$$

$$(2x+y)\cdot 2t + (x+3y^2)\cdot 3t^2$$

$$(2t^2+t^3)\cdot 2t + (t^2+3t^6)\cdot 3t^2$$

$$4t^3 + \underbrace{2t^4 + 3t^4}_{} + 9t^8$$

$$4t^3 + 5t^4 + 9t^8$$

(b) Check: $z = x^2 + xy + y^3 =$

$$= (t^2)^2 + (t^2)(t^3) + (t^3)^3$$

$$= t^4 + t^5 + t^9$$

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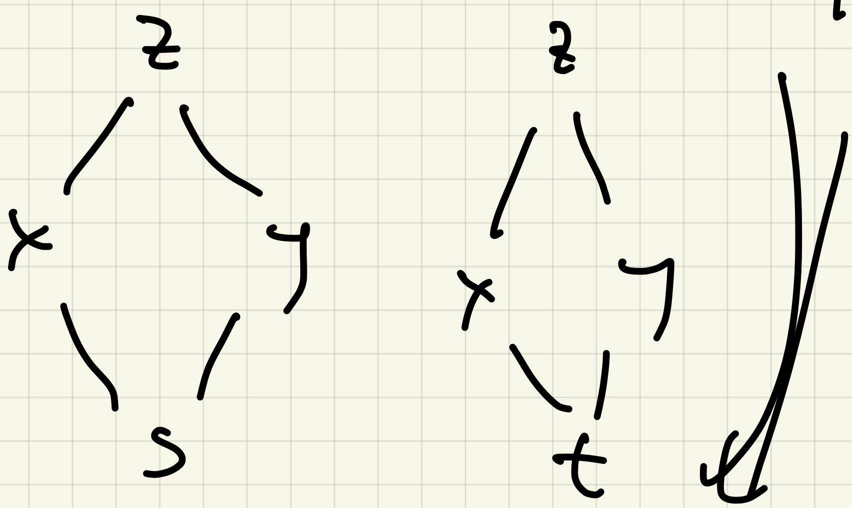
Ex $z = \arctan\left(\frac{y}{x}\right)$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

If $z = f(x,y)$, $\quad x = g(s,t)$

$\qquad\qquad\qquad\quad y = h(s,t)$



Then $\partial z/\partial s = \partial z/\partial x \cdot \partial x/\partial s + \partial z/\partial y \cdot \partial y/\partial s$

$\partial z/\partial t = \partial z/\partial x \cdot \partial x/\partial t + \partial z/\partial y \cdot \partial y/\partial t$

$$z = \arctan\left(\frac{y}{x}\right)$$

$$\boxed{x = r\cos\theta}$$
$$\boxed{y = r\sin\theta}$$

$$\partial z/\partial r = \boxed{\partial z/\partial x} \cdot \boxed{\partial x/\partial r} + \boxed{\partial z/\partial y} \cdot \boxed{\partial y/\partial r}$$

$$\left(\arctan' x = \frac{1}{1+x^2}\right)$$

$$= \boxed{\frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{-y}{x^2}} \cdot \cos\theta$$

$$+ \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} \cdot \sin\theta$$

$$= \frac{-y}{x^2+y^2}\cos\theta + \frac{x}{x^2+y^2}\sin\theta$$

$$= \frac{-r\sin\theta}{r^2}\cos\theta + \frac{r\cos\theta}{r^2}\sin\theta$$

$$= \frac{-r\sin\theta\cos\theta + r\cos\theta\sin\theta}{r^2} = 0$$

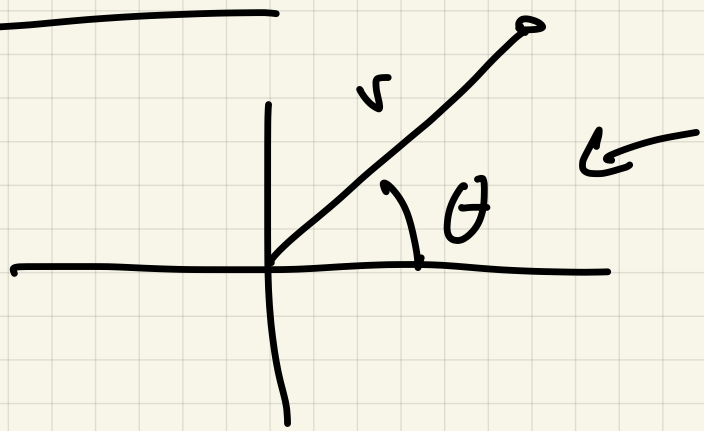
$$\partial z/\partial\theta = z_x x_\theta + z_y \cdot y_\theta$$

$$= \frac{-y}{x^2+y^2}(-r\sin\theta) + \frac{x}{x^2+y^2}(r\cos\theta)$$

$$= \frac{-\cancel{r}\sin\theta}{r^{\cancel{2}}}(-\cancel{r}\sin\theta) + \frac{r\cos\theta}{r^2}(r\cos\theta)$$

$$\sin^2\theta + \cos^2\theta = 1$$

Answer:



$$\tan\theta = \frac{y}{x}$$

$$\theta = \arctan\frac{y}{x}$$

Ex2 $w = x^2 + xy + z^2$

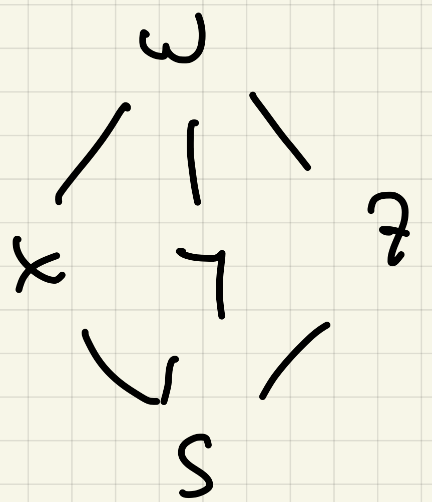
$x = s^3 + t^4$

$y = \sin(st)$

$z = e^{(s+3t)}$

$$w_s = w_x \cdot x_s + w_y \cdot y_s + w_z \cdot z_s$$

$$w_t = w_x \cdot x_t + w_y \cdot y_t + w_z \cdot z_t$$



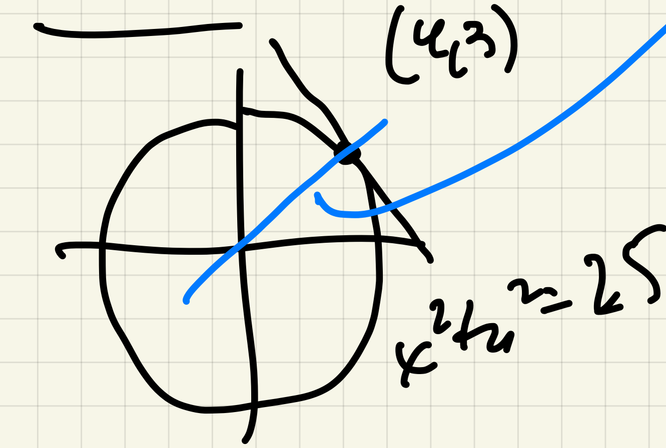
$$\frac{\partial w}{\partial s} = (2x+y) \cdot 3s^2 + (x)\, t \cos(st) + 2z\, e^{(s+3t+1)}$$

(replace $x/y/z$ with $s^3 + t^4$, $\sin(st)$, $e^{(s+3t+1)}$)

$$\partial w / \partial t = (2x+y) \cdot 4t^3 + x \cdot s \cdot \cos(st) + 2z \cdot 3e^{(s+3t)}$$

Implicit Derivatives

Calc1:



slope $= 3/4$

$\Rightarrow$ Ans $= -4/3$

Calc1: Explicit solution

$$x^2+y^2=25 = \quad y^2 = 25 - x^2$$

$$y = \pm\sqrt{25-x^2}$$

$$y = \sqrt{25-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}}(25-x^2)^{-\frac{1}{2}}(-\cancel{2}x)$$

$$= \left.\frac{-x}{\sqrt{25-x^2}}\right|_{x=4} = \frac{-4}{3} \checkmark$$

Implicit derivative: $\quad (y(x))^2$

$$\underline{x^2} + y^2 = 25$$

$$\frac{d}{dx}: \quad 2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

$$\text{so } m = -\frac{x}{y}\Bigg|_{\substack{x=4\\y=3}} = -\frac{4}{3} \checkmark$$

Ex $z = \sqrt{9-(x+1)^2-(y-2)^2}$

$$\partial z/\partial x(0,0) = -\frac{1}{2} \qquad \partial z/\partial y(0,0) = 1$$

(Visualized with picture of sphere)

Can also compute with implicit derivative

$$z^2 = 9 - (x+1)^2 - (y-2)^2$$

$\partial/\partial x$:

$$-2(x+1) = 2z \cdot \partial z/\partial x$$

$$\text{so } \partial z/\partial x = \frac{-2(x+1)}{2z} = \frac{-(x+1)}{z} \Big|_{\substack{x=0 \\ z=2}}$$

at $x=4$ $y=0 \to z=2$

$$-\frac{1}{2}$$

$\partial/\partial y$: $2z\frac{\partial z}{\partial y} = 0 - 0 - 2(y-2)^1$

$$= \frac{\partial z}{\partial y} = \frac{-2(y-2)}{2z} = \frac{-(y-2)}{z} \Big|_{\substack{y=0 \\ z=2}}$$

$$\frac{2}{2} = 1$$

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Ex 2 If $x^3 + xy + z^2 + yz^3 - 4 = 0$

defines a function $z = f(x,y)$

near $(1,1,1)$

Find $\partial z/\partial x$ and $\partial z/\partial y$ at $(1,1)$

$\partial/\partial x$: $3x^2 + y + 2z\frac{\partial z}{\partial x} + 3yz^2\frac{\partial z}{\partial x} = 0$

$$(2z + 3yz^2)\frac{\partial z}{\partial x} = -3x^2 - y$$

$$\frac{\partial z}{\partial x} = \left.\frac{-3x^2 - y}{2z + 3yz^2}\right|_{(1,1,1)} = \frac{-4}{5}$$

prod rule

$\partial/\partial y$: $0 + x + 2z\,\partial z/\partial y + z^3 + y3z^2\frac{\partial z}{\partial y} = 0$

$$\partial z/\partial y = \left.\frac{-x - z^3}{2z + 3yz^2}\right|_{(1,1,1)} = \frac{-2}{5}$$