10/17 Calc 3

Exam 2 → 10/24

Review sheet

Quiz 12 avg 88%

med 90%

1. $f = \underline{\underline{z}} + \underline{xy} + \underline{x}^2 \boxed{\ln(yz)}$ $= \ln y + \ln z$

$f_x = y + 2x\ln(yz)$

$f_y = x + \frac{x^2}{y}$

$f_z = 1 + \frac{x^2}{z}$

$\nabla f = \langle y + 2x\ln yz,\ x + \frac{x^2}{y},\ 1 + \frac{x^2}{z} \rangle$

$\nabla f(2,1,1) = \langle 1+0, 6, 5 \rangle =$

$\langle 1, 6, 5 \rangle$

2. $D_u f(2,1,1) = \langle 1, 6, 5 \rangle \cdot \frac{\langle 9, 2, 6 \rangle}{\sqrt{9^2 + 2^2 + 6^2}} =$

$$\frac{9+12+30}{11} = \frac{51}{11}$$

3. max incr $\frac{\nabla f}{|\nabla f|} = \frac{\langle 1,6,5\rangle}{\sqrt{62}}$

$|\nabla f| = \sqrt{62}$

$$\langle 1,6,5\rangle \cdot \frac{\langle 1,6,5\rangle}{\sqrt{62}} = \frac{62}{\sqrt{62}} = \sqrt{62}$$

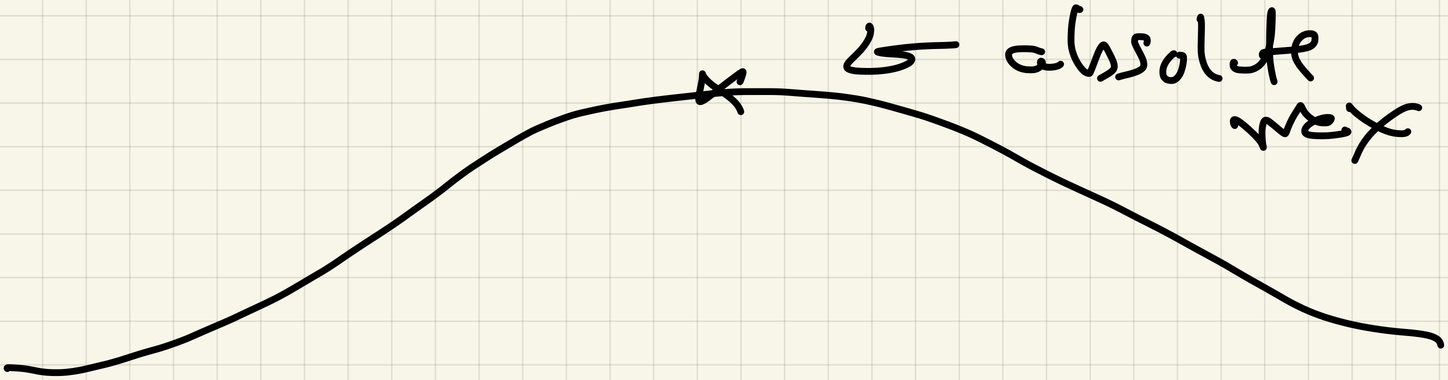
4. dcr $\frac{-\langle 1,6,5\rangle}{\sqrt{62}}$

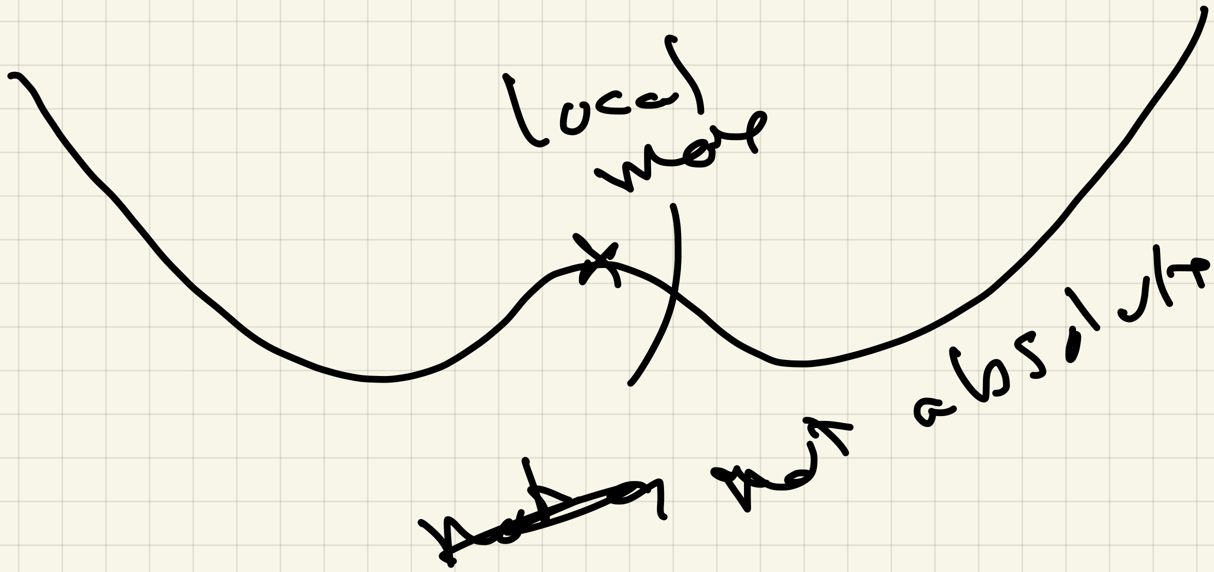
$-\sqrt{62}$

Last time $z = f(x,y)$

local max / min

Absolute max/min





<u>Thm 1</u> If $z = f(x,y)$ has local max/min (= extrema both) at (a,b) then

$$\boxed{\nabla f(a,b) = \langle 0,0 \rangle \quad \text{or} \quad \nabla f(a,b) \text{ DNE}}$$

(a,b) is a <u>critical point</u>

<u>Thm 2</u> If $\nabla f(a,b) = \langle 0,0 \rangle$

$(f_{xx}, f_{yy}, f_{xy}$ continuous$)$

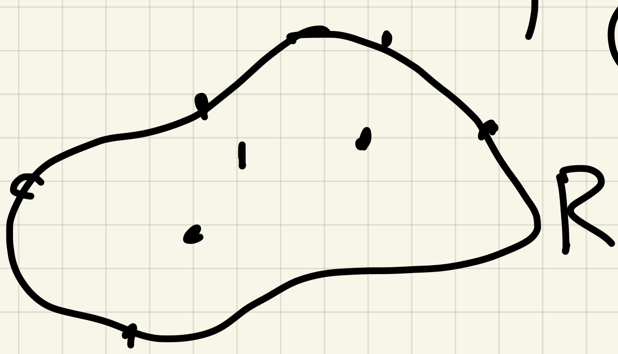
(near (a,b))

Set $d = \det\begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$

1. $d > 0$, $\boxed{f_{xx} > 0}$ $\Rightarrow$ local min
2. $d > 0$, $f_{xx} < 0$ $\Rightarrow$ local max
3. $d < 0$, saddle point

<u>Thm 3</u> If $z = f(x,y)$ continuous on closed bounded region R then f achieves its absolute max/min on R

<u>Where?</u>
1. Critical points inside ~~of~~ R
2. Boundary of R



~~Later~~ Note: For Thm 3

don't need Thm 2:

Just look z-values

Ex1 Find local max/min &

saddle pts for

$$z = f(x,y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$$

$$\nabla f = \langle \underbrace{3x^2 + 6x}_{\parallel}, \underbrace{3y^2 - 6y}_{\parallel} \rangle$$

$$3x(x+2) \qquad 3y(y-2)$$

$$\nabla f(x,y) = \langle 0, 0 \rangle \Longrightarrow$$

$$(x,y) = (0,0), (0,2)$$
$$(-2,0) \quad (-2,2)$$

$$f_{xx} = 6x+6, \; f_{yy} = 6y-6$$

$$f_{xy} = 0$$

crit pts $d = \det\begin{pmatrix} \underline{6x+6} & 0 \\ 0 & 6y-6 \end{pmatrix}$

$(0,0)$ $\quad d = \det\begin{pmatrix} 6 & 0 \\ 0 & -6 \end{pmatrix} = -36 < 0$

saddle pt

$(-2,2)$ $\quad d = \det\begin{pmatrix} -6 & 0 \\ 0 & 6 \end{pmatrix} = -36 < 0$

Saddle pts.

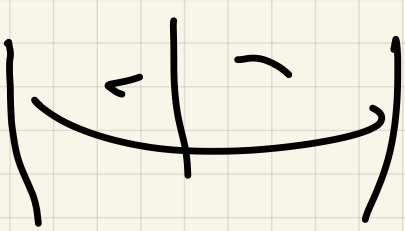
$(0,2)$ $\quad d = \det\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} = 36 > 0$

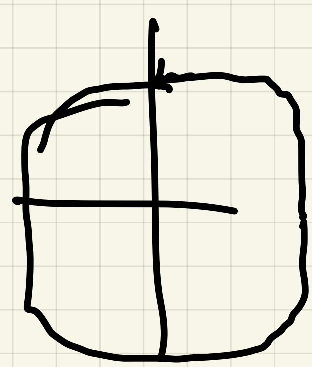
$f_{xx} = 6 > 0 \Rightarrow$ local min

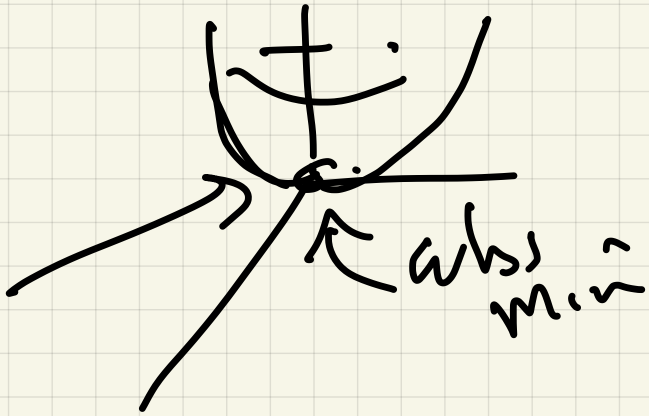
$(-2,0)$ $\quad d = \det\begin{pmatrix} -6 & 0 \\ 0 & -6 \end{pmatrix} = 36 > 0$

$f_{xx} = -6 < 0 \Rightarrow$ local max

(b) $\quad g(x,y) = x^4 + y^4$







Thm 2:

$$\nabla g = \langle 4x^3, 4y^3 \rangle = \langle 0, 0 \rangle \Rightarrow$$

$$x = y = 0$$

$(0,0)$ crit pt

g_{xx}	g_{yy}	g_{xy}
$12x^2$	$12y^2$	0

at $(0,0)$ = $g_{xx} = g_{yy} = g_{xy} = 0$

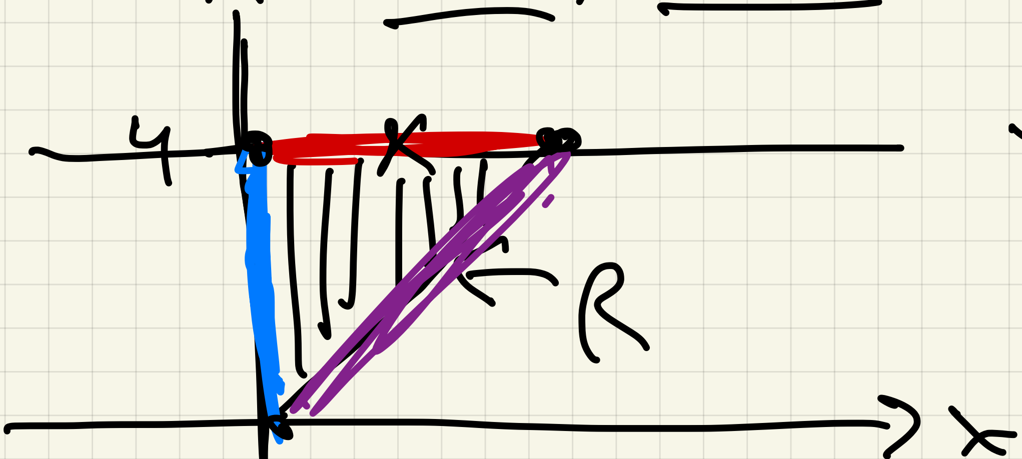
Thm 2 says nothing!

Ex 2 (#32)

Find absolute max/min of $\boxed{f(x,y) = x^2 - xy + y^2 + 1}$

on region R = triangle bounded by

by $\underline{x=0}$, $\underline{y=4}$, $x=y$.



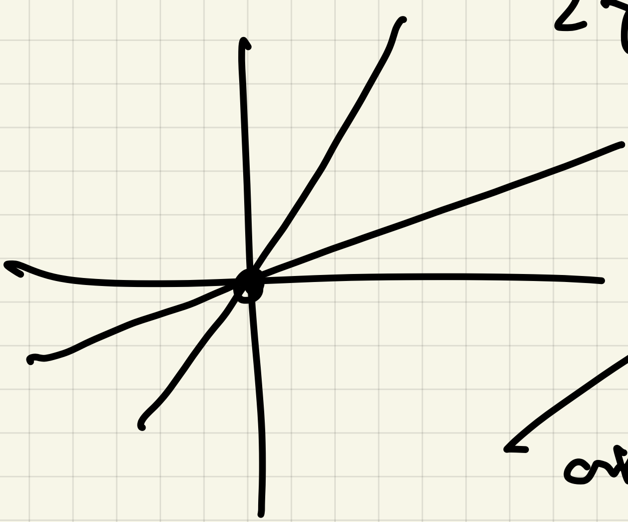
① Crit pts inside R

$$\nabla f = \langle 2x-y,\ 2y-x \rangle$$

solve $\boxed{2x-y=0}$ $y=2x$

$2y-x=0$ $y=\frac{1}{2}x$

$\Downarrow$



$(x,y)=(0,0)$

on boundary R

② Boundary of R.

3 sides:

left side $\quad x=0$:

$$f(x,y) = f(0,y) = \underline{y^2+1}$$

$$0 \le y \le 4$$

$$f'(y) = 2y = 0 \rightarrow y = 0$$

$$\boxed{\begin{array}{l} f(0,0) = 1 \\ f(0,4) = 17 \end{array}}$$

top $\quad$ Set $y=4$

$$f(x,4) = \underline{x^2 - 4x + 17}$$

$$0 \le x \le 4$$

$$f' = 2x - 4 = 0 \qquad x = 2$$

$$\boxed{\begin{array}{l} f(0,4) = 17 \\ f(2,4) = 13 \\ f(4,4) = 17 \end{array}}$$

diagonal $\quad x = y$

$f(x,y) = f(x,x) = x^2+1$

$0 \le x \le 4$

$f' = 2x = 0 \rightarrow x = 0$

$$\boxed{\begin{array}{l} f(0,0) = 1 \\ f(4,4) = 17 \end{array}}$$

<u>Conclusion</u>

Abs max 17 at $(0,4)$, $(4,4)$

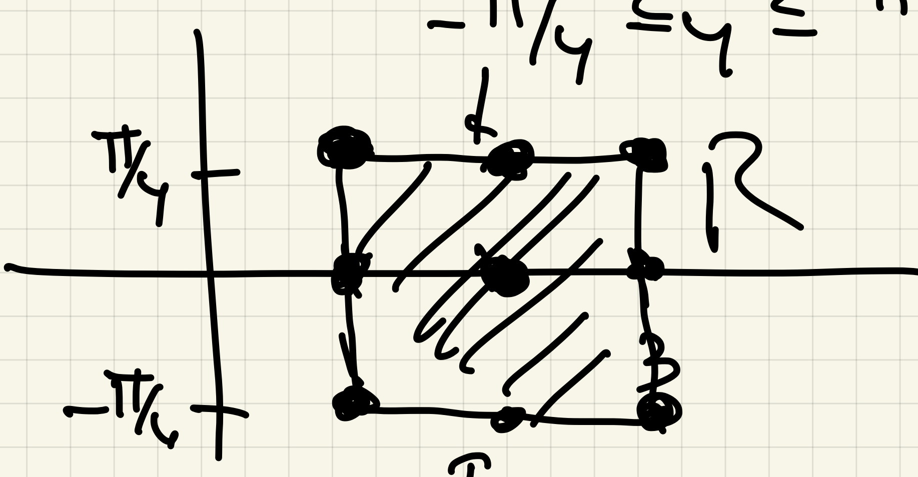
abs min 1 at $(0,0)$

(b) (#37)

$$\boxed{f(x,y) = \underline{(4x - x^2)} \cos y}$$

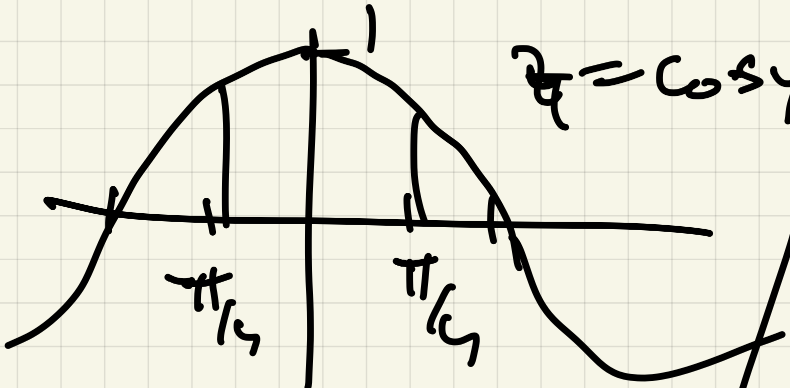
on R $\quad 1 \le x \le 3$

$-\pi/4 \le y \le \pi/4$



① Crit pt inside R

$\nabla f = \langle (4-2x)\cos y, -(4x-x^2)\sin y \rangle$



$\cos y \neq 0$

$-\pi/4 \le y \le \pi/4$

so

$(4-2x)\cos y = 0 \Rightarrow$

$x = 2, y = 0$

$(2,0)$

$\boxed{f(2,0) = 4}$

② Boundary:

left side: $x = 1$

$f(1,y) = 3\cos y \quad -\pi/4 \le y \le \pi/4$

$y = 0 \quad f(1,0) = 3$

$f(1,-\pi/4) = 3/\sqrt{2}$

$$\boxed{f(1, \pi/4) = 3/\sqrt{2}}$$

right side

$$f(3, y) = 3\cos y$$

$$f(3, 0) = 3$$

$$f(3, -\pi/4) = 3/\sqrt{2}$$

$$f(3, \pi/4) = 3/\sqrt{2}$$

top $y = \pi/4$

$$f(x, \pi/4) = (4x - x^2)\frac{1}{\sqrt{2}} =$$

$$f = \frac{1}{\sqrt{2}}(4x - x^2)$$

$$f' = \frac{1}{\sqrt{2}}(4 - 2x) = 0 \text{ at } x = 2$$

$$\boxed{f(2, \pi/4) = 4/\sqrt{2}}$$

$f(0, \pi/4) \leftarrow$

$f(3, \pi/4) \leftarrow$

bottom $\boxed{f(2, -\pi/8) = \frac{4}{\sqrt{2}}}$

4, 3, $3/\sqrt{2}$ $4/\sqrt{2}$

abs ~~max~~ is 4 at $(2, 0)$

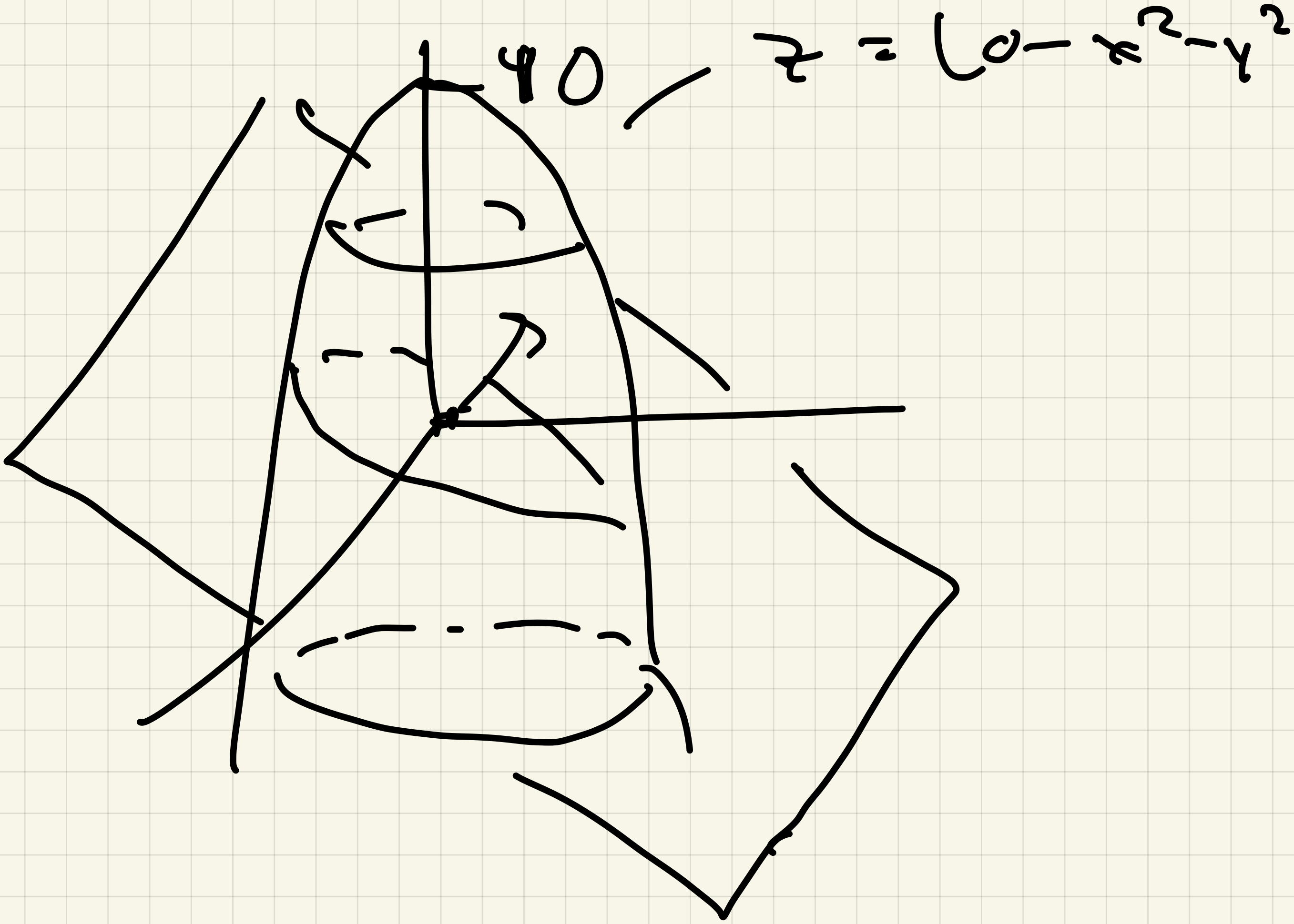
abs min is $3/\sqrt{2}$ at

$(1, \pi/4)$, $(1, -\pi/4)$

$(3, \pi/4)$ $(3, -\pi/4)$

(#49)

Among all points on graph of $z = 10 - x^2 - y^2$ that lie above plane $x + 2y + 3z = 0$ find the point that is furthest from plane


40
$z = 60 - x^2 - y^2$