10/14 Calc3

Quiz 11

$h(x,y) = 5+3x^2+3y^2$

$x = s\cos t$

$y = s\sin t$

Find $\partial h/\partial s$

$\partial h/\partial s = \partial h/\partial x \; \partial x/\partial s + \partial h/\partial y \; \partial y/\partial s$

$6x\cos t + 6y\sin t$

1. $6s\cos t\cos t + 6s\sin t\sin t$

$6s(1) = 6s$

2. $\cancel{h}(s,t) =$

$5+3(s\cos t)^2+3(s\sin t)^2$

$= 5+3s^2(\underbrace{\cos^2 t+\sin^2 t}_{1})$

$= 5+3s^2$

$$\frac{\partial}{\partial s}(5+3s^2) = 6s \quad \checkmark$$

__Last time__ Gradient in 2D or 3D

Tangent lines to level curves

(13.5) $f(x,y) = \text{const}$

(13.6) Tangent plane to level surface

$$f(x,y,z) = c$$

__Ex1__ Find tangent plane to surface $S: z = \sqrt{x^2+y^2-4}$

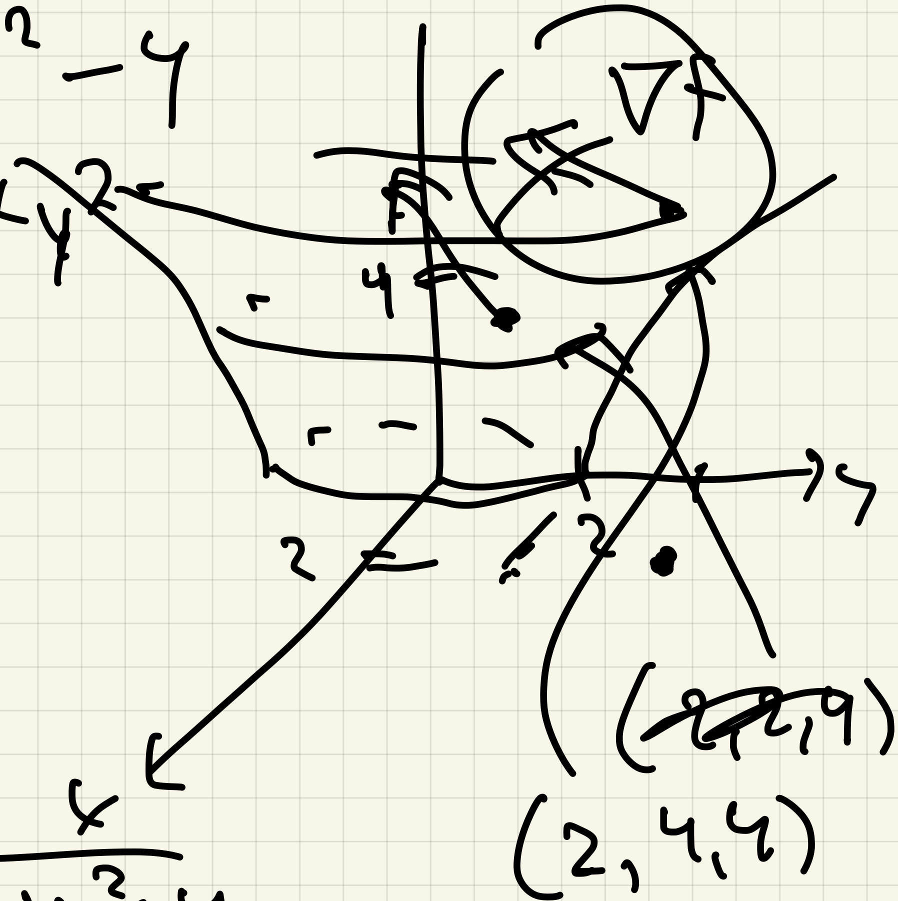
at $(2,4,4)$

rewrite

$$\underbrace{f(x,y,z) = z - \sqrt{x^2+y^2-4}}_{\text{level surface}} = 0$$

$$z^2 = x^2+y^2-4$$

$$z^2+4 = x^2+y^2$$



$$f = z - \sqrt{x^2+y^2-4},$$

$$\nabla f = \left\langle \frac{-x}{\sqrt{x^2+y^2-4}}, \frac{-y}{\sqrt{x^2+y^2-4}}, 1 \right\rangle$$

$$\nabla f(2,4,4) = \boxed{\left\langle -\frac{1}{2}, -1, 1 \right\rangle} = \bar{n}$$

$$P_0 = (2,4,4)$$

$$-\frac{1}{2}(x-2) - 1(y-4) + 1(z-4) = 0$$

$$-\frac{1}{2}x - y + z = -1 - 4 + 4 = -1$$

<u>Rmk</u>: The surface is

$$z^2 = x^2 + y^2 - 4$$

$$0 = x^2 + y^2 - z^2 - 4 = g$$

$$\nabla g = \langle 2x, 2y, -2z \rangle$$

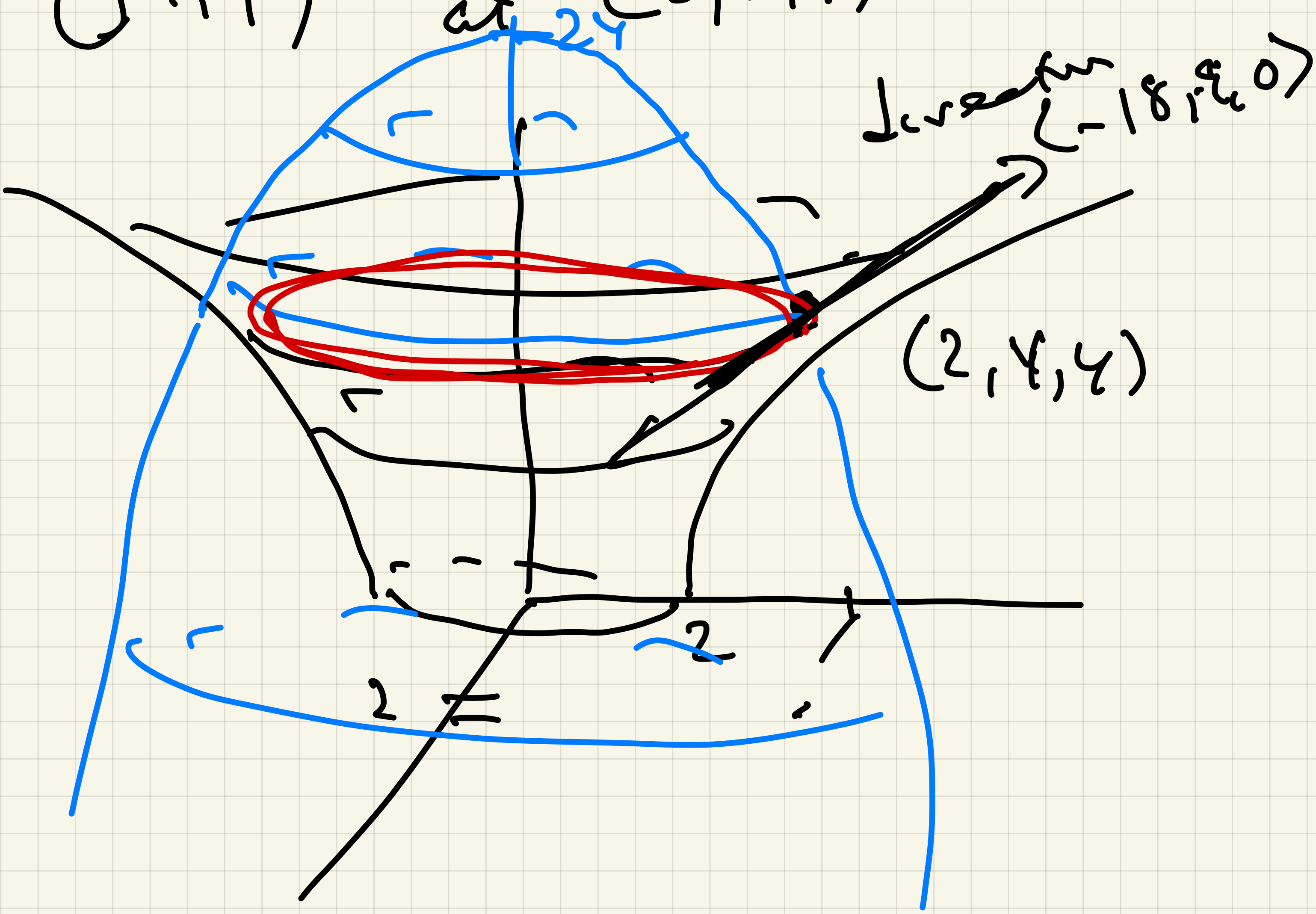
$$\nabla g(2,4,4) = \boxed{\langle 4, 8, -8 \rangle}$$

$$\parallel \text{ to } \left\langle -\frac{1}{2}, -1, 1 \right\rangle$$

<u>Ex 2</u>: Intersect the surface S from Ex 1 with the surface T: $\boxed{z + x^2 + y^2 = 24}$

(a) Find the tangent plane to T at (2, 4, 4)

(b) Find the tangent lines to the intersection of S and T ($S \cap T$) at (2, 4, 4)



(a) $h = z + x^2 + y^2 = 24$

$\nabla h = \langle 2x, 2y, 1 \rangle$

$\nabla h(2,4,4) = \langle 4, 8, 1 \rangle$

$P_0 = (2,4,4)$

<u>Plane</u>: $4(x-2) + 8(y-4) + 1(z-4) = 0$

$4x + 8y + z = 8 + 32 + 4 = 44$

(b) Intersection S & T

is a circle C

Tangent vector to C at (2,4,4)

is $\perp$ normals ∇h

∇f

Use <u>cross product</u>:

$\langle 4, 8, 1 \rangle \qquad \langle -\frac{1}{2}, -1, 1 \rangle$

$\langle -1, -2, 2 \rangle$

$$\begin{vmatrix} i & j & k \\ -1 & -2 & 2 \\ 4 & 8 & 1 \end{vmatrix} = \langle -18, 9, 0 \rangle$$

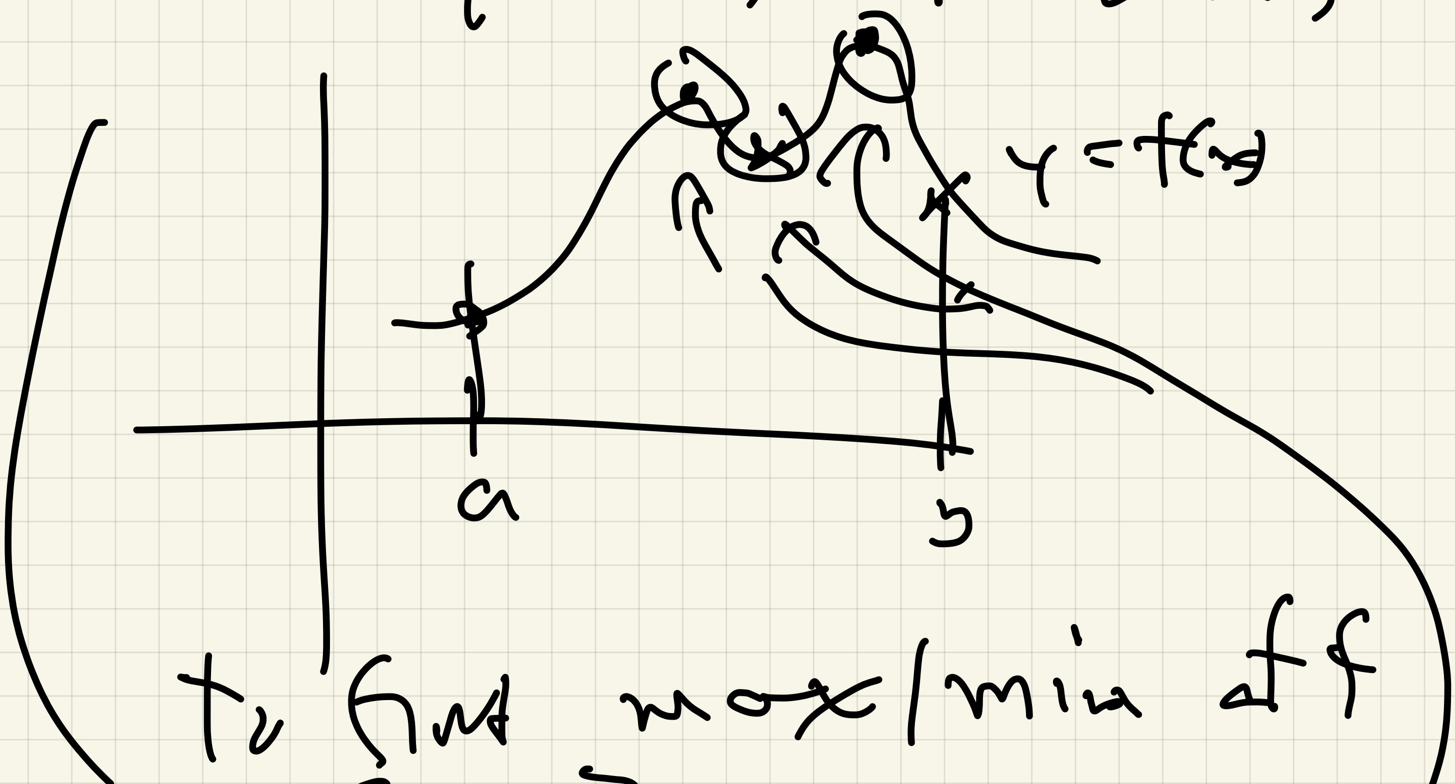
so tangent line is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 - 18t \\ 4 + 9t \\ 4 \end{pmatrix}$$

§13.7 Extrema

(local max/min)

Calc I: $y = f(x)$ on $[a,b]$



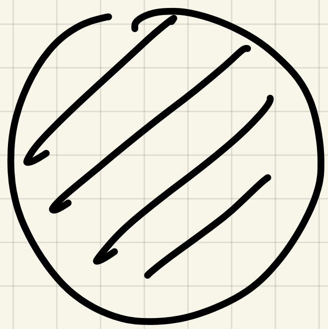
to find max/min of f on $[a,b]$

① Check critical points

($f'(x)=0$) on (a,b)

② Check end points

<u>Calc3</u> $z = f(x,y)$

region more complicated $\longrightarrow$ 

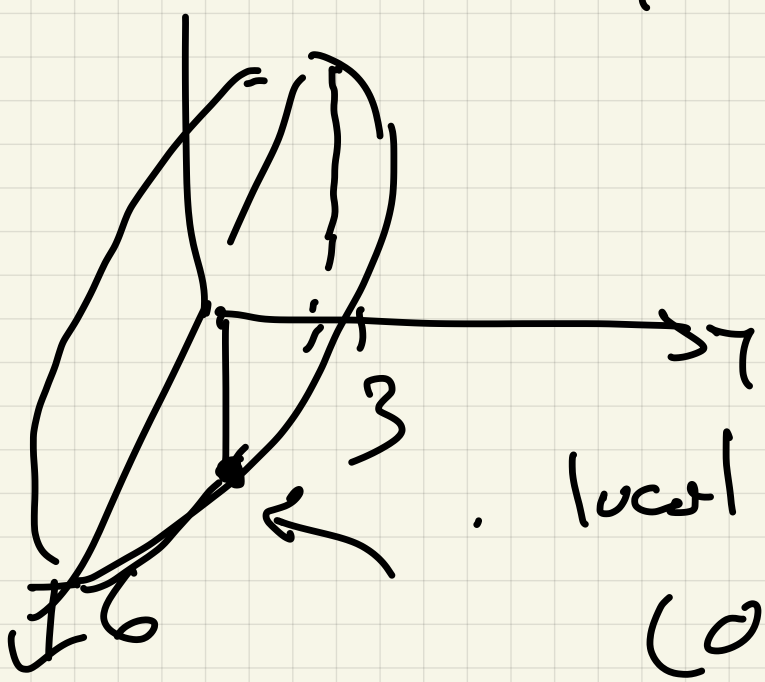
<u>Defn</u>: $z = f(x,y)$ has a <u>(local) maximum</u> (relative maximum) at (a,b) if $f(x,y) \le f(a,b)$ for (x,y) in an open disk about (a,b)

local minimum is similar

<u>Ex1</u>: $z = 25 - x^2 - y^2$

(b)

$$z = -\sqrt{9 - \frac{x^2}{4} - y^2}$$



local min at

$(0, 0, -3)$