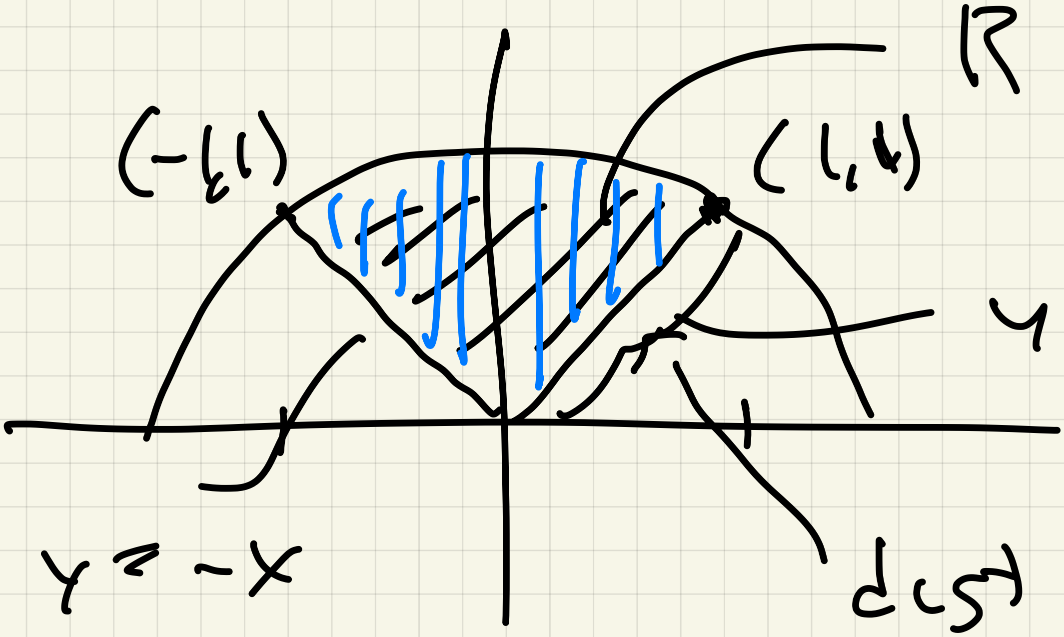
11/21 Calc3

Quiz 16



$r = \sqrt{2}$

1. $$\int_{-1}^{1}\int_{|x|}^{\sqrt{2-x^2}} (x+2)\,dy\,dx$$

$$\boxed{x^2+y^2=2}$$

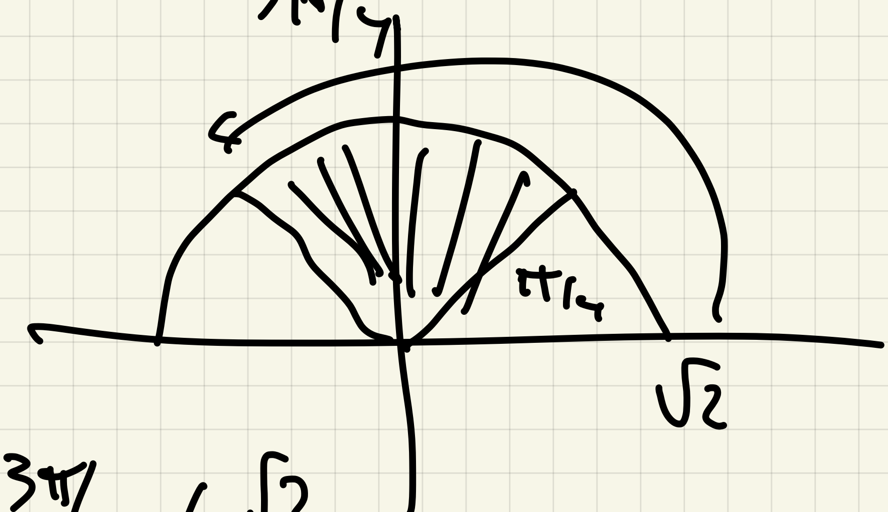
x

$$= \int_{-1}^{0}\int_{-x}^{\sqrt{2-x^2}} dx\,dy + \int_{0}^{1}\int_{x}^{\sqrt{2-x^2}} dy\,dx$$

dy dx

$$\int_{\substack{0\\y}}^{1} \int_{-y}^{y} (x+2)\,dy\,dx + \int_{\substack{1\\y}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} (x+2)\,dy\,dx$$

2.



$$\int_{\pi/4}^{3\pi/4} \int_{0}^{\sqrt{2}} (r\cos\theta + 2)\,r\,dr\,d\theta$$

$$= \frac{1}{3} r^3 \cos\theta + r^2 \Big|_0^{\sqrt{2}} =$$

$$\int_{\pi/4}^{3\pi/4} \frac{2\sqrt{2}}{3}\cos\theta + 2\,d\theta$$

$$\cancelto{0}{\frac{2\sqrt{2}}{3}\sin\theta} + 2\theta \Big|_{\pi/4}^{3\pi/4}$$

$$2\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) = \pi,$$

Last time

Rectangular (x, y, z)

r ~ Cylindrical (r, θ, z)

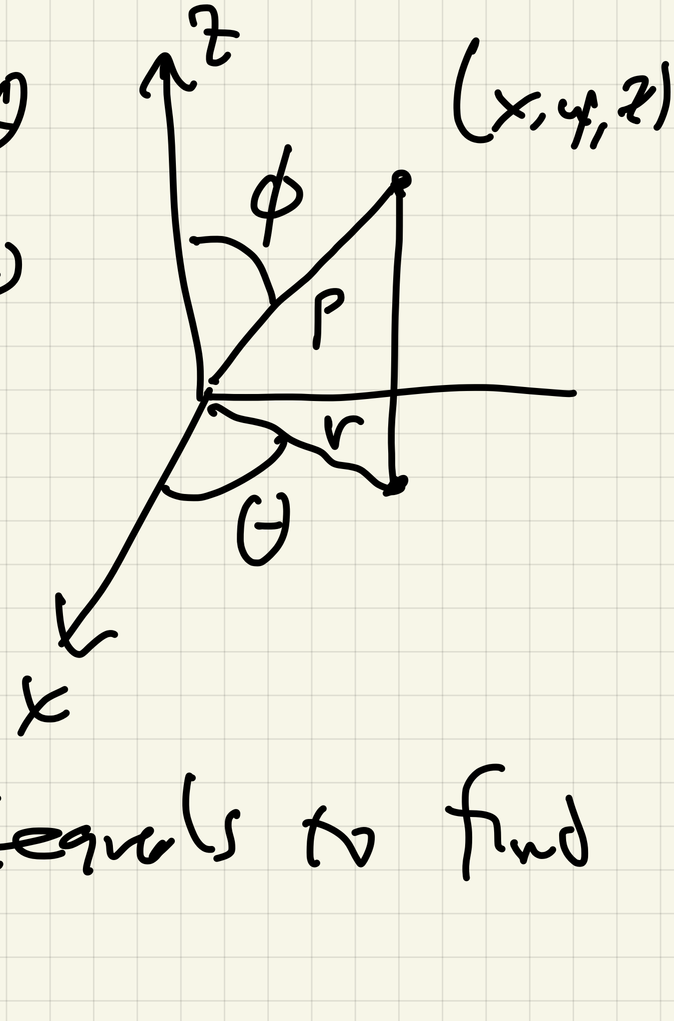
$\rho^2 \sin\phi$ ~ Spherical (ρ, ϕ, θ)

$x = r\cos\theta = \rho \sin\phi \cos\theta$

$y = r\sin\theta = \rho \sin\phi \sin\theta$

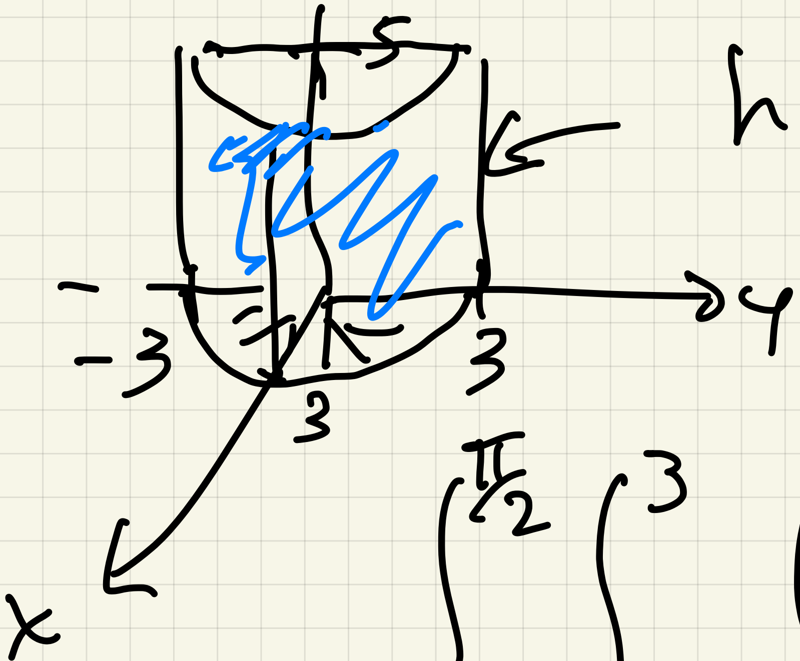
$z = \rho\cos\phi$

$(z = z)$



Ex) Set up integrals to find volume

Cal $\quad \uparrow^{z}$

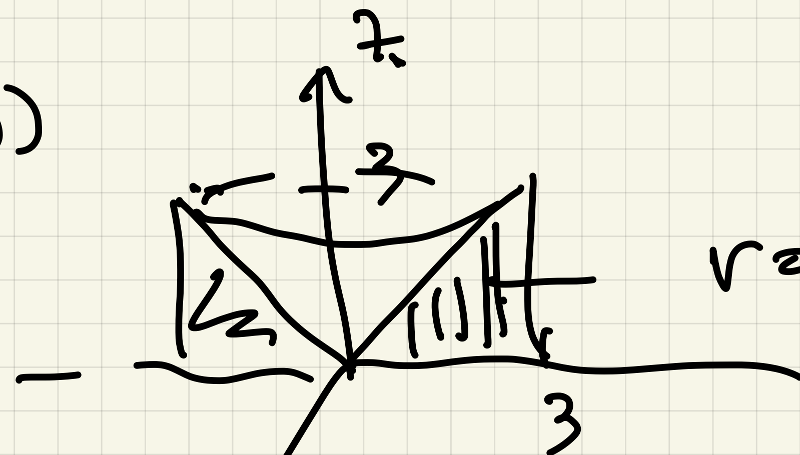


half solid cylinder

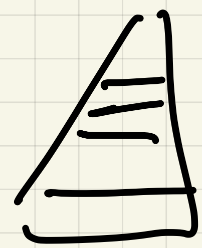
$$\int_{-\pi/2}^{\pi/2}\int_{0}^{3}\int_{0}^{5} r\,dz\,dr\,d\theta = \frac{45\pi}{2}$$

$\theta \quad r \quad z$

(b)



$r = z$



$$V = \int_{0}^{2\pi}\int_{0}^{3}\int_{0}^{r} r\,dz\,dr\,d\theta$$

$\theta \quad r$

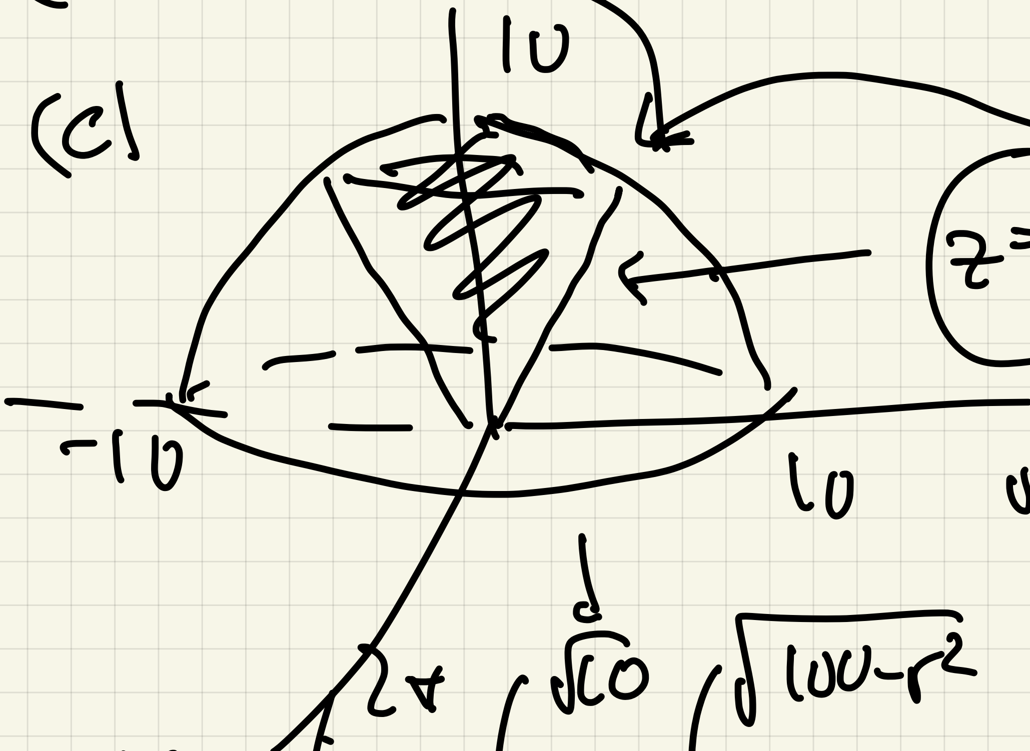
$$\int_{0}^{2\pi}\int_{0}^{3}\int_{z}^{3} r\,dr\,dz\,d\theta$$

$(\sqrt{10}, 3\sqrt{10})$ θ z r

4

18π

(c)



$\underbrace{x^2+y^2}_{r^2}+z^2=100$

$z=3r$

$r^2+z^2=100$

$z=\sqrt{100-r^2}$

$$V=\int_0^{2\pi}\int_0^{\sqrt{10}}\int_{3r}^{\sqrt{100-r^2}} r\,dz\,dr\,d\theta$$

$r^2+z^2=100$

$z=3r$

$\rightarrow \quad r^2+\underbrace{9r^2}_{z^2}=100$

$10r^2=100$

$r^2=10$

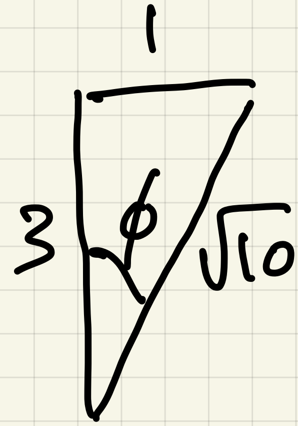
$r=\sqrt{10}$

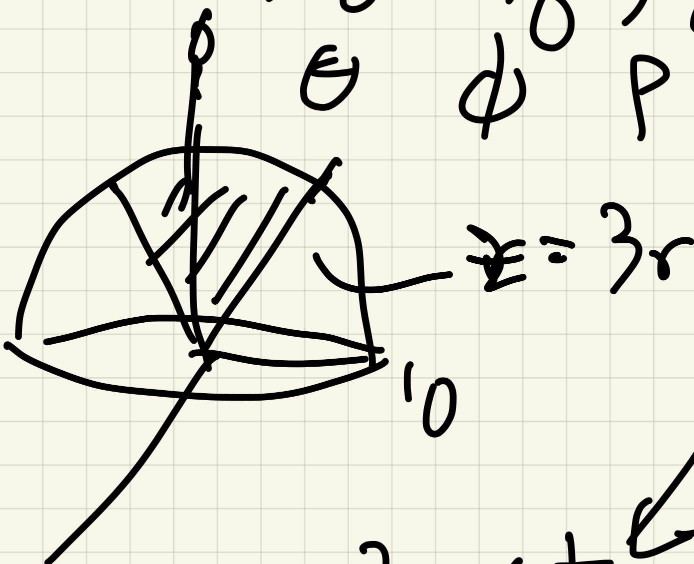
(c) continued:

spherical coordinates:

$\tan^{-1} 1/3$

$$V = \int_0^{2\pi}\int_0^{}\int_0^{10}$$
$\theta \quad \phi \quad \rho$





$\phi = \arctan \frac{1}{3}$

so $$V = \int_0^{2\pi}\int_0^{\tan^{-1}\frac{1}{3}}\underbrace{\int_0^{10} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta}_{\rho}$$
$\theta \quad \phi$

$$\frac{\rho^3}{3}\Bigg|_0^{10} \qquad \int_0^{\tan^{-1}1/3} \frac{1000}{3}\sin\phi\, d\phi =$$

$$-\frac{1000}{3}\cos\phi\,\Bigg|_0^{\tan^{-1}1/3} =$$

$$-\frac{1000}{3}\,\frac{3}{\sqrt{10}} - -\frac{1000}{3} =$$

$$\int_0^{2\pi} \frac{1000}{3}\left(1 - \frac{3}{\sqrt{10}}\right) d\theta =$$

$$\frac{2000\pi}{3}\left(1 - \frac{3}{\sqrt{10}}\right)$$

(d) (#59)

find volume region below $z = \sqrt{x^2+y^2}$

above sphere $\rightarrow$ $\boxed{\rho = 2\cos\phi}$ (spherical coords)

<u>Aside</u>:

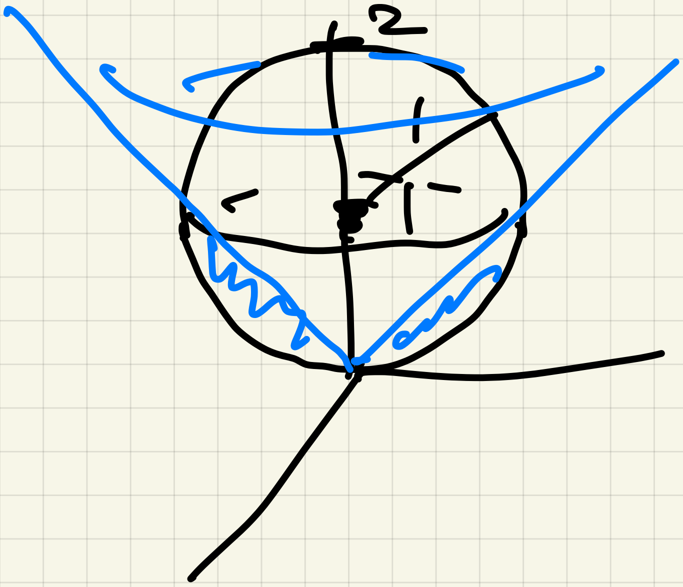
$$\rho = 2\cos\phi$$

$$\rho^2 = 2\rho\cos\phi$$

$$x^2+y^2+z^2 = 2z$$

$$x^2+y^2+z^2-2z+1 = 1$$

$$x^2+y^2+(z-1)^2 = 1$$



$$V = \int_{\theta=0}^{2\pi} \int_{\phi=\pi/4}^{\pi/2} \underbrace{\int_{\rho=0}^{2\cos\phi} \rho^2 \sin\phi \, d\rho}\, d\phi \, d\theta$$

$$\frac{1}{3}\rho^3 \sin\phi \Big|_{0}^{\rho = 2\cos\phi}$$

$$\int_{\pi/4}^{\pi/2} \frac{8}{3}\cos^3\phi \sin\phi \qquad -u^3 \; du$$

$u = \cos\phi$

$du = -\sin\phi \, d\phi$

$$\|$$

$$-\frac{8}{3}\frac{1}{4}\cos^4\phi \Big|_{\pi/4}^{\pi/2} =$$

$$0 - \left(-\frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)^4\right)$$

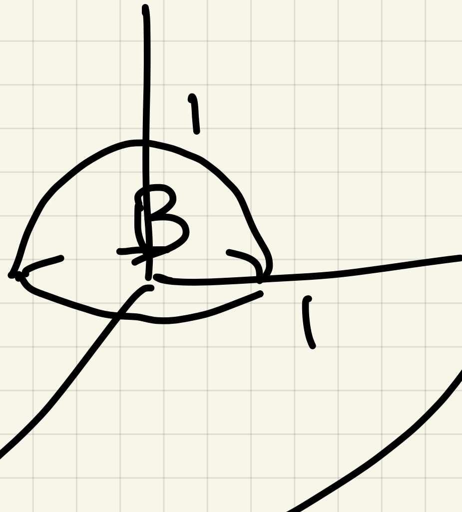
$$-\left(-\frac{1}{6}\right) = \int_0^{2\pi} \frac{1}{6}\, d\theta =$$

$$\frac{2\pi}{6} = \frac{\pi}{3}$$

__Ex2__ If B = Upper solid hemisphere of radius 1 compute

$$\bar{z} = \frac{\iiint_B z\, dV}{\iiint_B 1\, dV}$$

(§ 14.6 z coordinate of centroid)



$$\iiint_B dV =$$

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^1 \rho^2 \sin\phi =$$

$$\int_0^{2\pi}\int_0^{\pi/2} \frac{1}{3}\sin\phi$$

$$-\frac{1}{3}\cos\phi \Big|_0^{\pi/2}$$

$$0 - \left(-\frac{1}{3}\right) =$$

$$\int_0^{2\pi} \frac{1}{3}\, d\theta = \frac{2\pi}{3}$$

$$\iiint_B z\, dV =$$

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^1 \overbrace{\rho\cos\phi}^{z}\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

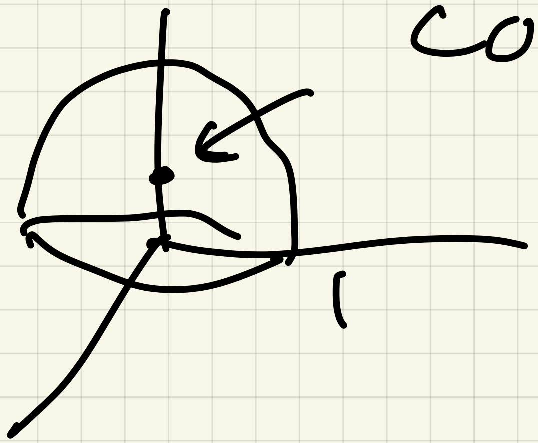
$$\int\int\underbrace{\int_0^1 \rho^3\cos\phi\sin\phi\,d\rho}_{\frac{1}{4}\rho^4\cos\phi\sin\phi\Big|_0^1}$$

$$\int_0^{\pi/2}\frac{1}{4}\underline{\cos\phi\sin\phi} =$$

$$\frac{1}{4}\,\frac{1}{2}\sin^2\phi\Big|_0^{\pi/2} = \frac{1}{8}$$

$$\int_0^{2\pi}\frac{1}{8}\,d\theta = \frac{2\pi}{8} = \frac{\pi}{4}$$

so $\quad \bar{z} = \dfrac{\pi/4}{2\pi/3} = \dfrac{3}{8}$



COM $= (0, 0, 3/8)$

Ch 15

§15.1 line integrals of scalar functions

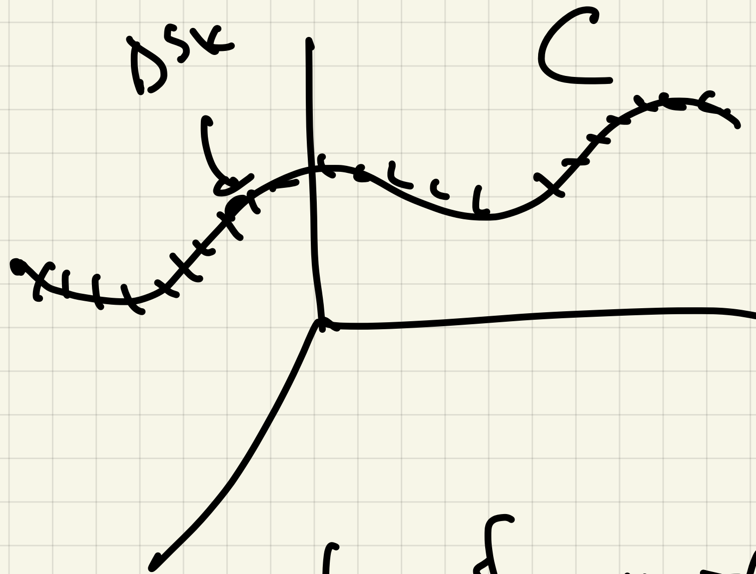
Calc 1 $\quad \int_a^b f(x)\,dx \quad$ 1-D

Ch 14 $\quad \iint_R f(x,y)\,dA \quad$ 2-D

$\iiint_B f(x,y,z)\,dV \quad$ 3-D

line integral:

Idea:



C = curve

$f(x,y,z)$

Want to integrate f over C

Concept: Break C into n small segments of length Δs_k $\quad 1 \le k \le n$

Choose (x_k, y_k, z_k) in segment k

~~Th~~ The line integral of f over C is

$$\int_C f(x,y,z)\,ds$$

||

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k, y_k, z_k) \cdot \Delta s_k$$

How calculate it ??

If C is curve

$\bar{r}(t)$: $a \le t \le b$

Then

$$\int_C f\,ds = \int_{t=a}^{t=b} f(\bar{r}(t))\,|v(t)|\,dt$$

?

speed

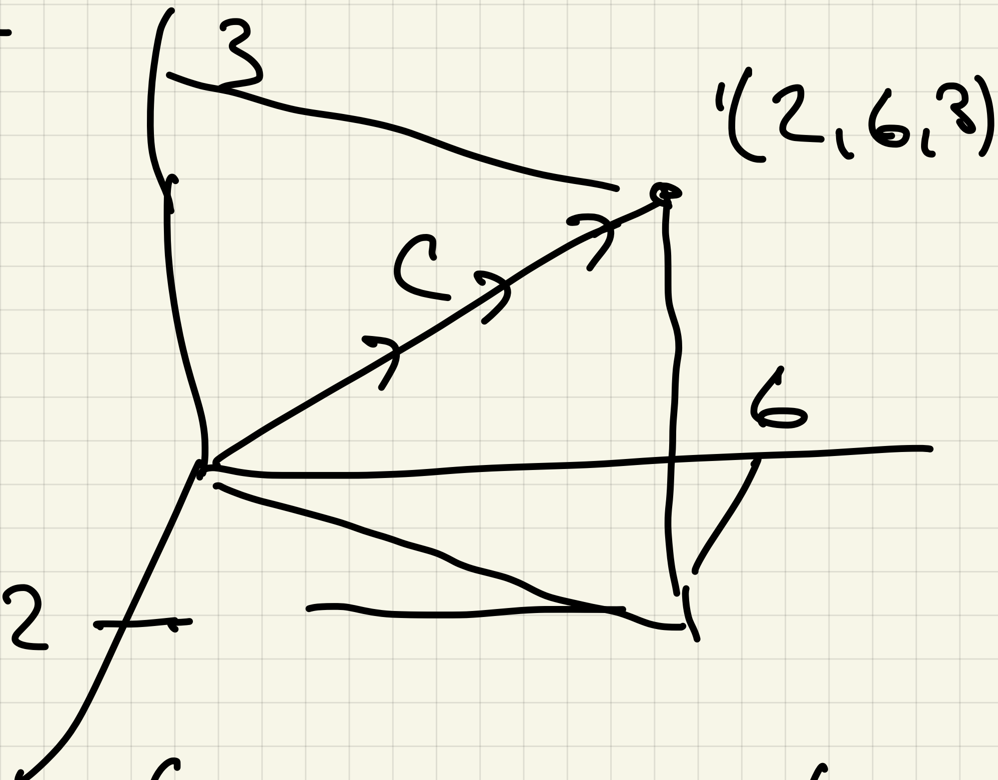
Physical interpretation !

$\int 1\,ds$ = arc length

$\int_C$ If f is density of C at (x, y, z) then

$$\int_C f\,ds = \text{mass of } C$$

Ex



Find $\int_C 1\,ds$ and $\int_C z\,ds$

Parametrize C:

$\bar{r}(t) = \langle 2t, 6t, 3t \rangle,\ 0 \le t \le 1$

$v(t) = \langle 2, 6, 3 \rangle$

$|v(t)| = |\langle 2, 6, 3 \rangle| =$

$\sqrt{4+36+9} = \sqrt{49} = 7$

So $\int_0^1 1 \cdot 7 \, dt = 7t \Big|_0^1 = 7$

$$\int_C 1 \, ds$$