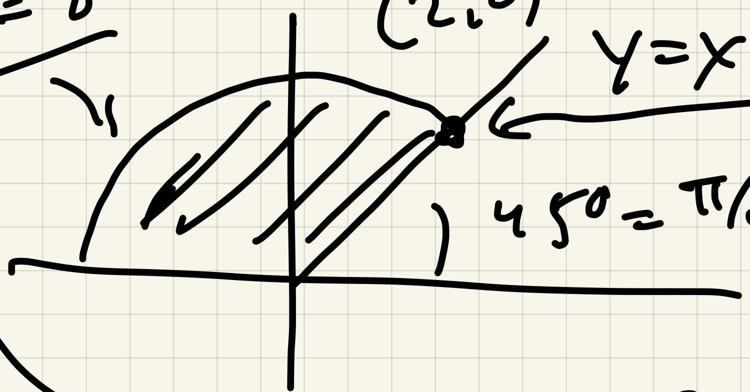
11/6 | Calc3   Quiz 15

$x^2+y^2=8$



$x = y$

$x^2+y^2=8$

$2x^2=8$

$x^2=4$

$x=2$

$x^2+y^2=r^2$

$r=\sqrt{8}$

$\boxed{\#1.}$

$$\int_{\pi/4}^{\pi}\int_{0}^{\sqrt{8}} (r\sin\theta)(r)\,dr\,d\theta$$

$$\iint_R y\,dA$$

$x = r\cos\theta$

$y = r\sin\theta$

conv factr

$\boxed{\#2}$

$$\int_0^{\sqrt{8}} r^2\sin\theta\,dr =$$

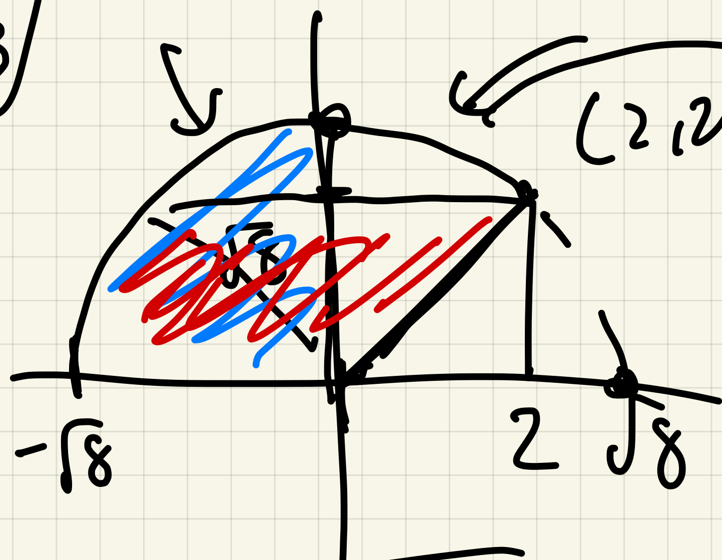
$$\frac{1}{3}r^3\sin\theta\Big|_0^{\sqrt{8}} = \frac{8\sqrt{8}}{3}\sin\theta$$

$$\frac{8\sqrt{8}}{3}\int_{\pi/4}^{\pi} \sin\theta \, d\theta = -\cos\theta \Big|_{\pi/4}^{\pi} =$$

$$-(-1) - -\cos\left(\frac{\pi}{4}\right)$$

$$\frac{8\sqrt{8}}{3}\left(1 + \frac{1}{\sqrt{2}}\right)$$

$\boxed{\#3}$



$x^2+y^2=8$

$y=\sqrt{8-x^2}$

$$\int_{-\sqrt{8}}^{0}\int_{0}^{\sqrt{8-x^2}} y \, dy\, dx \; +$$

left

$$\int_{0}^{2}\int_{x}^{\sqrt{8-x^2}} y \, dy\, dx$$

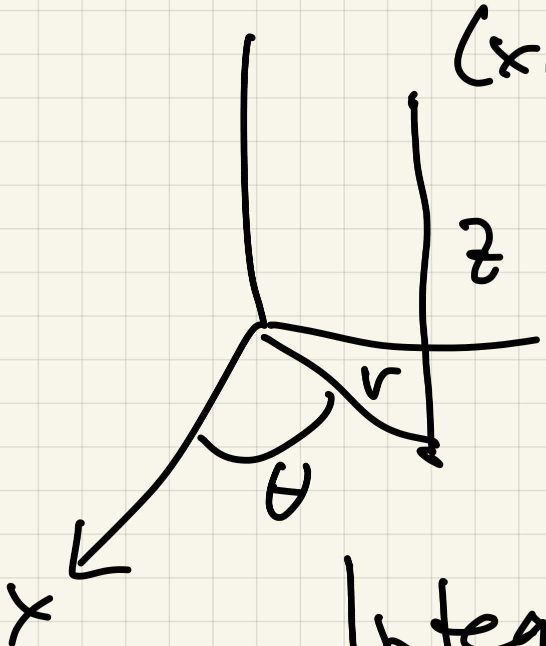
OR

$$\int_0^2 \int_{-\sqrt{8-y^2}}^{y} y\,dx\,dy + \int_2^{\sqrt{8}} \int_{-\sqrt{8-y^2}}^{\sqrt{8-y^2}} y\,dx\,dy$$

y

Last time: **Cylindrical coordinates**

$$(x,y,z) \sim (r,\theta,z)$$

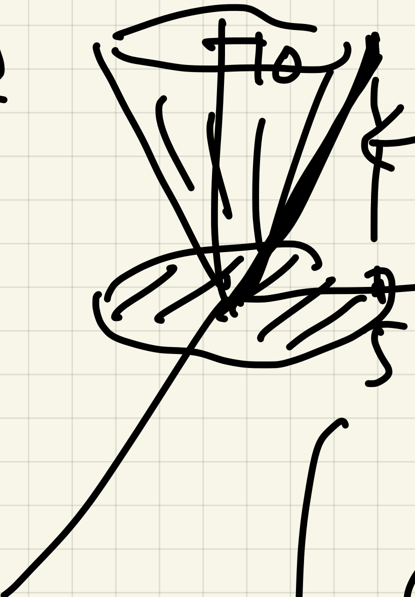


Integrals:

$$\iiint_B f(x,y,z)\,dV = \iiint f(r\cos\theta, r\sin\theta, z)\,r\,dr\,d\theta\,dz$$

end points for B in cylindrical coords

Ex)



$z = 2\boxed{\sqrt{x^2+y^2}}$

$=$

$z = 2r \qquad r = z/2$

$0 \le \theta \le 2\pi$

$0 \le r \le 5$

$\boxed{2r \le z \le 10}$

So $V = \iiint_B 1\, dV = \int_0^{2\pi} \int_0^{5} \int_{2r}^{10} 1\, r\, dz\, dr\, d\theta$

$0 \le \theta \le 2\pi$

$0 \le z \le 10$

$0 \le r \le z/2$

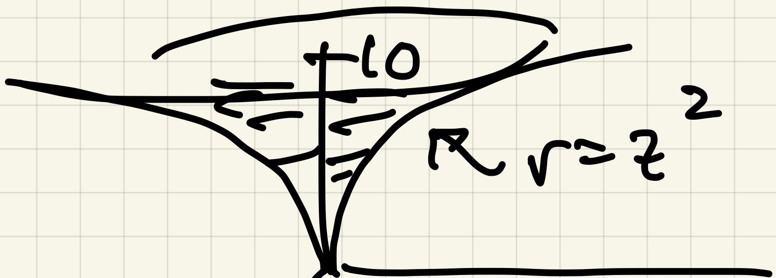
$$\int_0^{2\pi} \int_0^{10} \int_0^{z/2} r\, dr\, dz\, d\theta =$$

$$\frac{1}{2} r^2 \Big|_0^{z/2} = \int_0^{10} \frac{z^2}{8}\, dz =$$

$$\frac{z^3}{24}\Big|_0^{10} = \frac{1000}{24} = \frac{250}{6}$$

$$\int_0^{2\pi} \frac{250}{6}\, d\theta = \frac{250\pi}{3} \checkmark$$

Ex 2



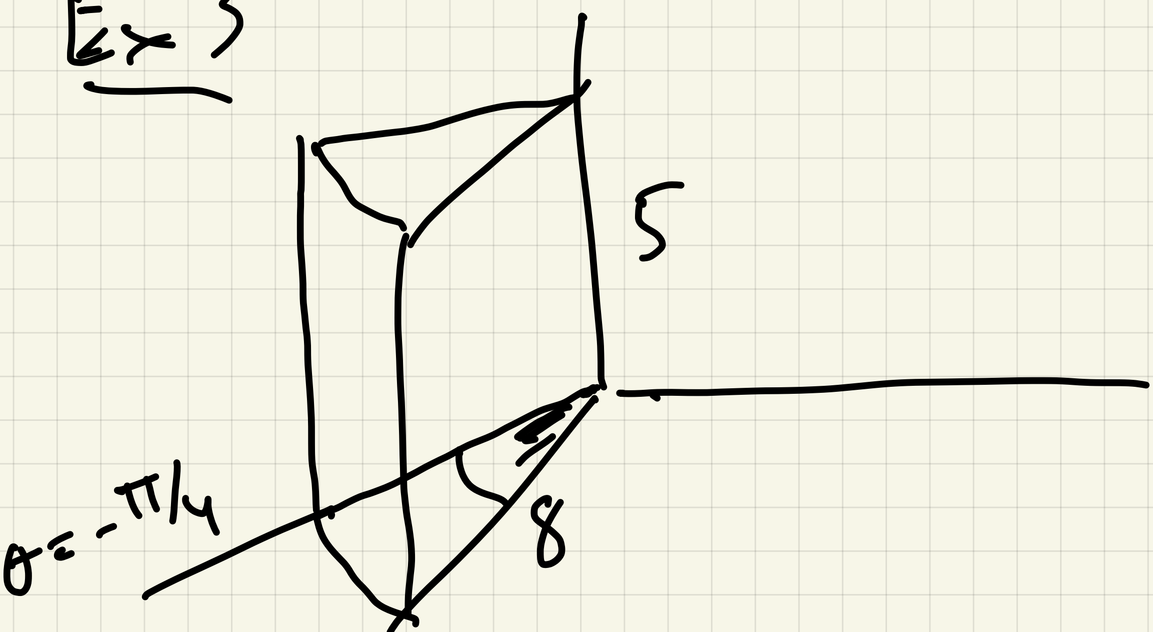
$$\int_0^{2\pi}\int_0^{10}\int_0^{z^2 \checkmark} r\, dr\, dz\, d\theta$$

$$\frac{r^2}{2}\Big|_0^{z^2} = \frac{z^4}{2}$$

$$\int_0^{10} \frac{z^4}{2}\, dz = \frac{z^5}{10}\Big|_0^{10} = 10000$$

$$\int_0^{2\pi} 10000\, d\theta = 20000\pi$$

Ex 3



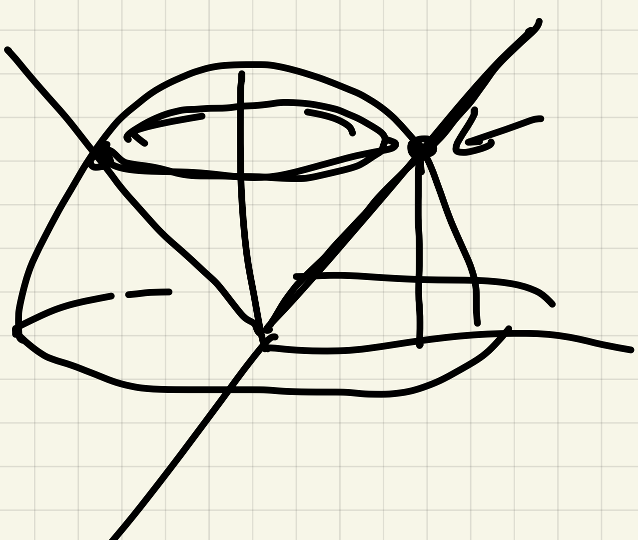
$$\int_{-\pi/4}^{0} \int_{0}^{8} \int_{0}^{5} r \, dz \, dr \, d\theta = 40\pi$$

Ex 4 Find volume of solid

inside sphere $\boxed{x^2+y^2+z^2=100}$

but outside cone $z=\sqrt{x^2+y^2}$,

$z \geq 0$



$$\int_0^{2\pi} \underline{\qquad} \, d\theta$$

$\underbrace{r^2+z^2=100}_{\text{sphere}}$ $z=r$ cone

intersect: $r^2+r^2=100$

$r^2=50$

$r=\sqrt{50}=5\sqrt{2}$

$$\int_0^{2\pi}\int_0^{5\sqrt{2}}\int_0^{r} r\,dz\,dr\,d\theta \;+$$

inside

$$\int_0^{2\pi}\int_{5\sqrt{2}}^{10}\int_0^{\sqrt{100-r^2}} r\,dz\,dr\,d\theta$$

fine, but easier:

$$\int_0^{2\pi}\int_0^{5\sqrt{2}}\underbrace{\int_z^{\sqrt{100-z^2}} r\,dr}\,dz\,d\theta$$

$$\frac{1}{2}r^2\Big|_z^{\sqrt{100-z^2}} =$$

$$\underbrace{\tfrac{1}{2}(100-z^2) - \tfrac{1}{2}z^2}$$

$$\int_0^{5\sqrt{2}} 50 - z^2 \, dz$$

$$50z - z^3/3 \Big|_0^{5\sqrt{2}} =$$

$$250\sqrt{2} - \frac{\boxed{125 \cdot 2}\sqrt{2}}{3} =$$

$$250\sqrt{2} - \tfrac{1}{3}\,250\sqrt{2}$$

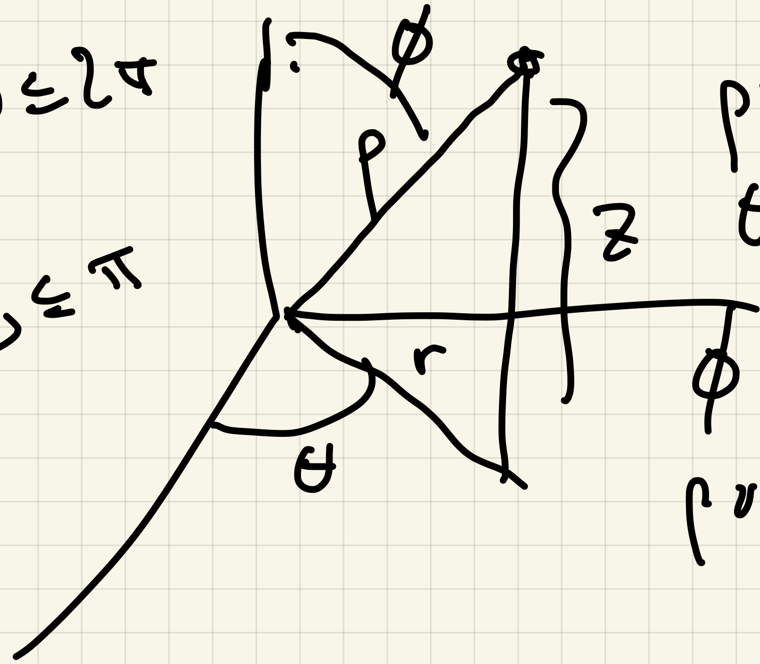
$$\tfrac{2}{3}(250\sqrt{2}) = \frac{500\sqrt{2}}{3}$$

$$\int_0^{2\pi} \frac{500\sqrt{2}}{3} = \boxed{\frac{1000\sqrt{2}\pi}{3}}$$

Spherical Coordinates:

$$(x,y,z) \sim (r,\theta,z)$$

$$\begin{cases} 0 \le \theta \le 2\pi \\ 0 \le \rho \\ 0 \le \phi \le \pi \end{cases}$$



ρ = dist to $(0,0,0)$

θ = polar angle

ϕ = angle from positive z-axis

connections:

$$\boxed{r = \rho \sin\phi} \quad \left(\sin\phi = \frac{r}{\rho} \right)$$

$$\boxed{z = \rho \cos\phi}$$

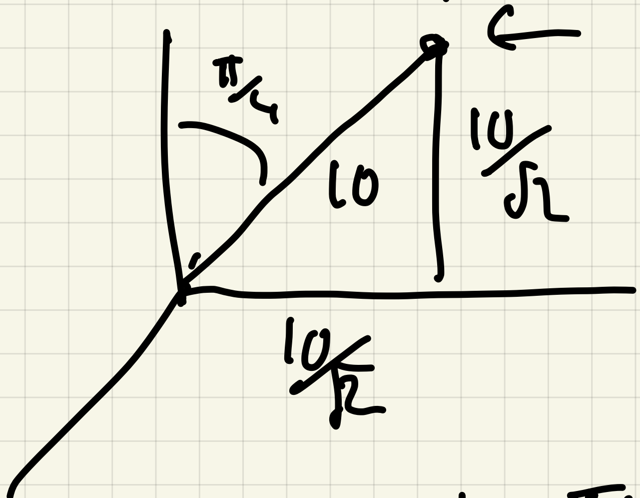
$$\Downarrow$$

$$(*) \begin{cases} x = r\cos\theta = \rho \sin\phi \cos\theta \\ y = r\sin\theta = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}$$

$$\rho = \sqrt{x^2 + y^2 + z^2} \qquad \theta = \arctan \frac{y}{x}$$

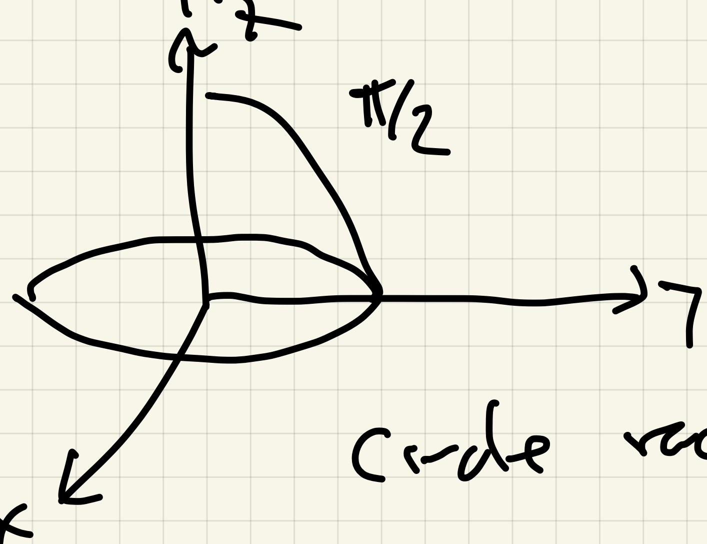
$$\phi = \arccos \frac{z}{\rho}$$

Ex 5 (a) $\rho = 10,\ \theta = \pi/2,\ \phi = \pi/4$



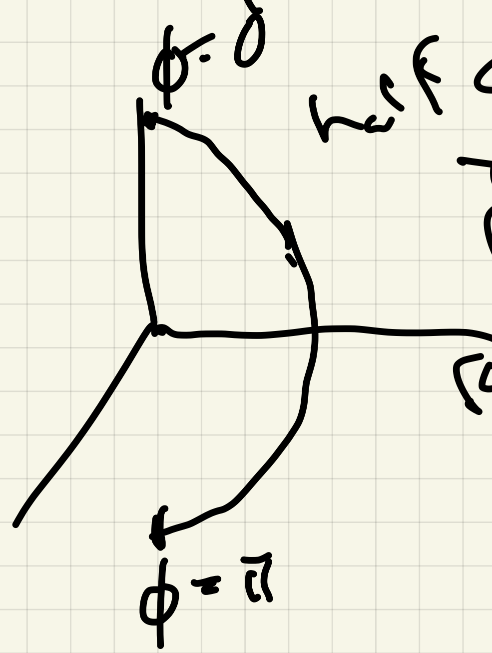
$\leftarrow (0, \frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}})$

(b) $\rho = 5,\ \phi = \pi/2$



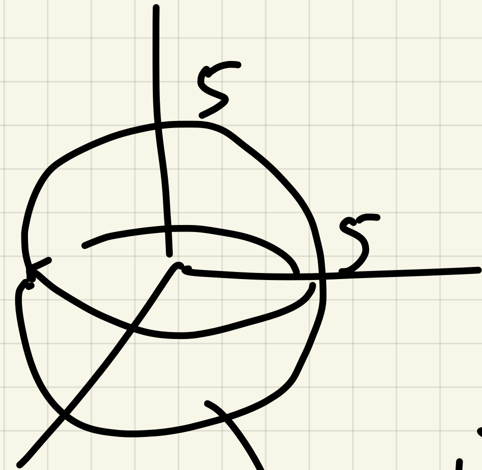
circle radius 5 in xy plane

(c) $\rho = 5$
$\theta = \pi/2$

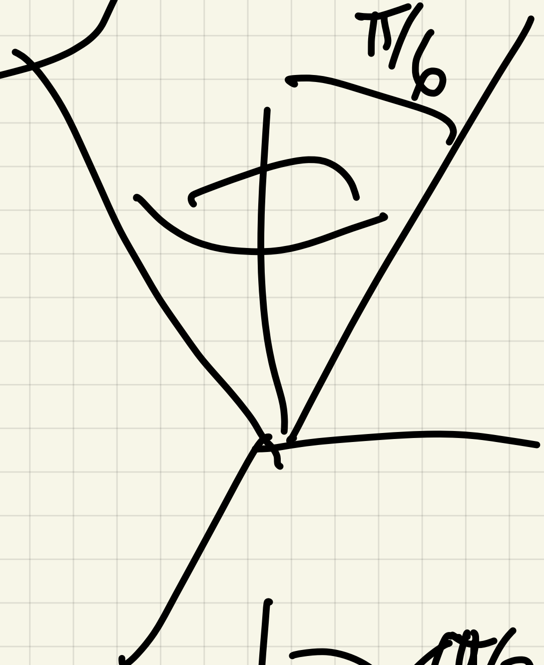


half circle of radius 5

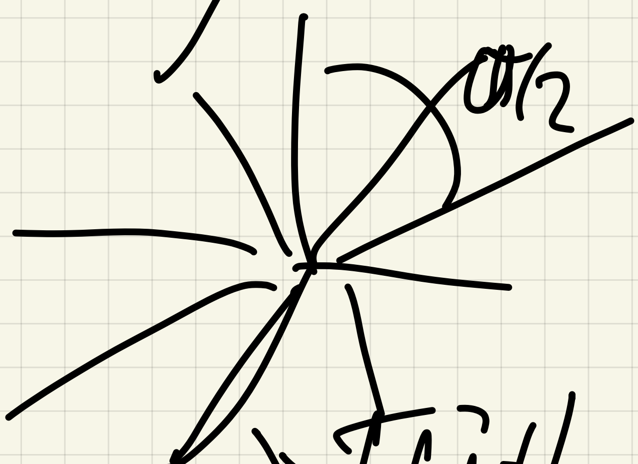
(d) $\rho = 5$



(e) $\phi = \pi/6$



(f) $\phi = \pi/2$

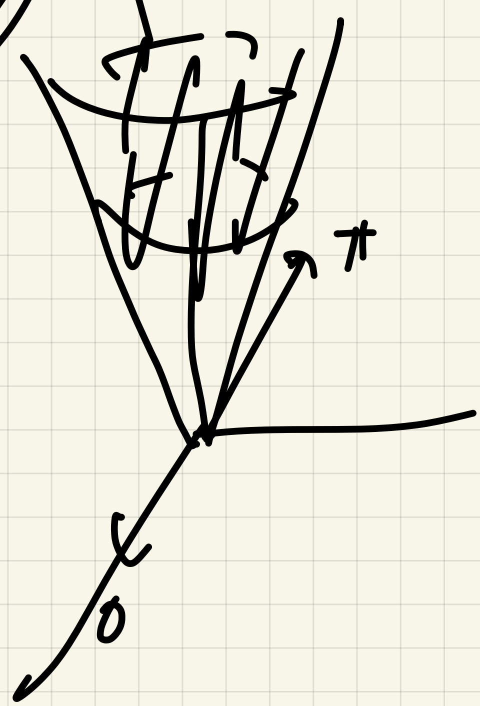


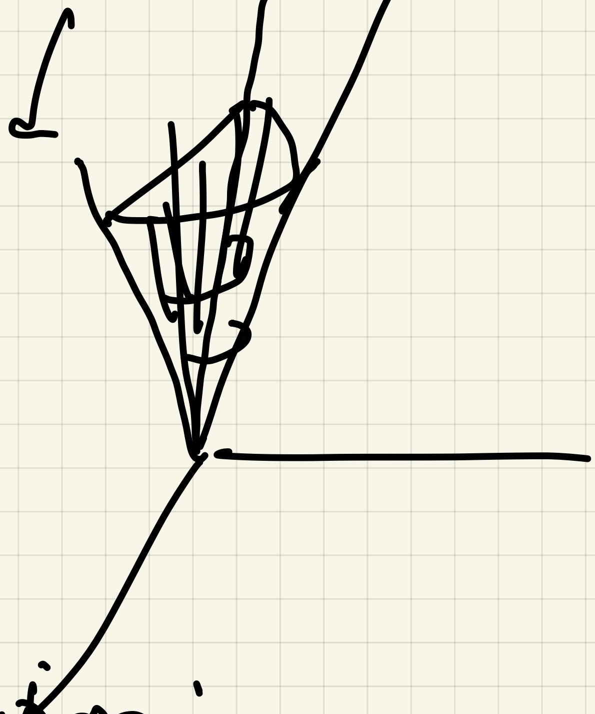
xy plane

(g) $0 \le \rho \le 5$

$0 \le \phi \le \pi/6$

$0 \le \theta \le \pi$



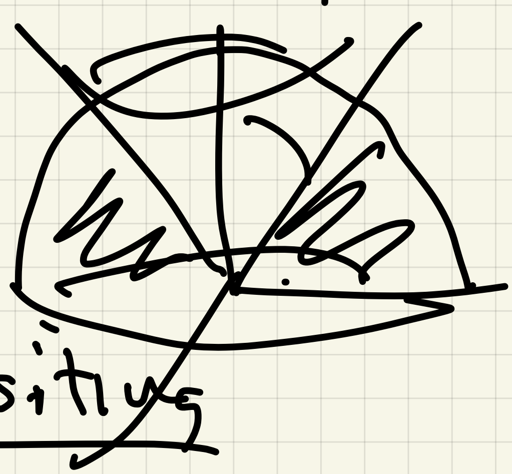


Integration:

$$\iiint_B f(x,y,z)\,dV =$$

$$\iiint f(\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta,\ \rho\cos\phi)\,\rho^2\sin\phi\ d\rho\, d\phi\, d\theta$$

endpoints corresp to B



Ex 4: revisiting

$$V = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \underbrace{\int_0^{10} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}$$

$\theta \quad \phi$

$$\frac{\rho^3}{3} \sin\phi \Big|_0^{10}$$

$$\int_{\pi/4}^{\pi/2} \frac{1000}{3} \sin\phi$$

$$-\frac{1000}{3} \cos\phi \Big|_{\pi/4}^{\pi/2}$$

$$-\left(-\frac{1000}{3} \frac{1}{\sqrt{2}}\right) =$$

$$+\int_0^{2\pi} \frac{1000}{3\sqrt{2}} = \frac{2000\pi}{3\sqrt{2}}$$