11/4/ Calc3

Quiz 15

$$\int_{x=0}^{x=2}\underbrace{\int_{y=x^2}^{y=4} 2x\,dy}\,dx =$$

$$2xy\Big|_{y=x^2}^{y=4} =$$

$$\int_0^2 8x - 2x^3\,dx$$

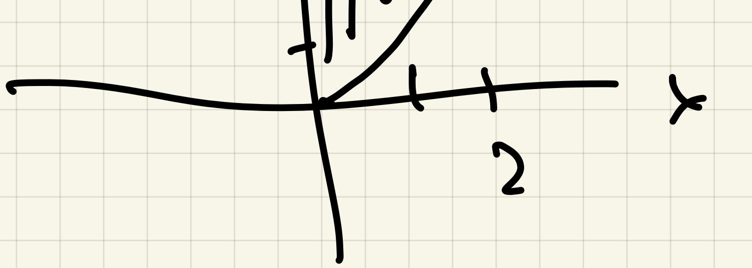
$$4x^2 - \frac{1}{2}x^4\Big|_0^2 =$$

$$16 - 8 = 8$$

$$\underbrace{\int_0^2}_{x}\int_{x^2}^{4}_{y}$$

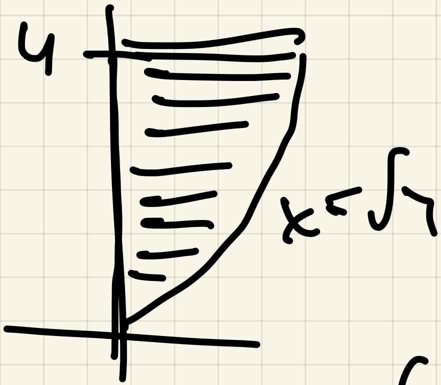
$0 \le x \le 2$

$x^2 \le y \le 4$



$y = x^2$

$x = \sqrt{y}$



$0 \le y \le 4$

$0 \le x \le \sqrt{y}$

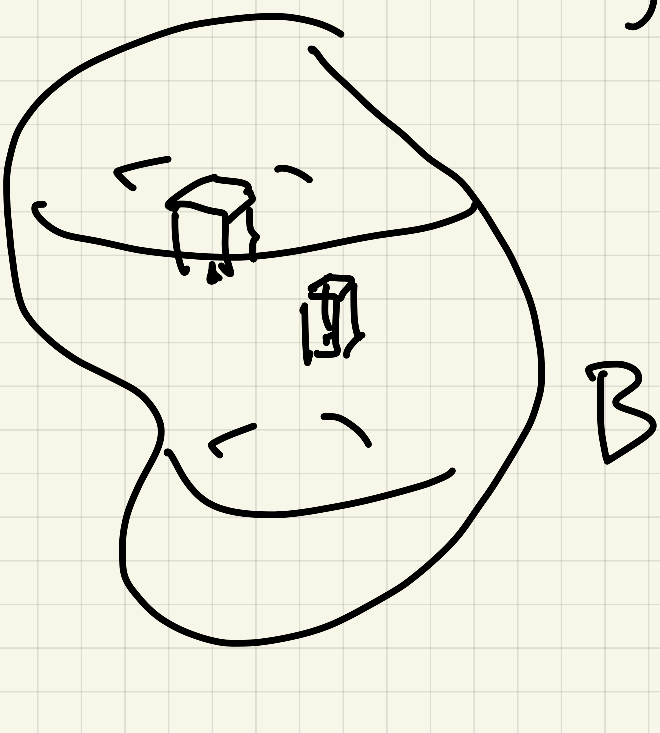
$$\int_0^4 \int_0^{\sqrt{y}} 2x \, dx \, dy$$

Last time

$$\iiint_B f(x,y,z)\, dV$$

$$\parallel$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i, y_i, z_i)\, dV_i$$



n rectangular solids

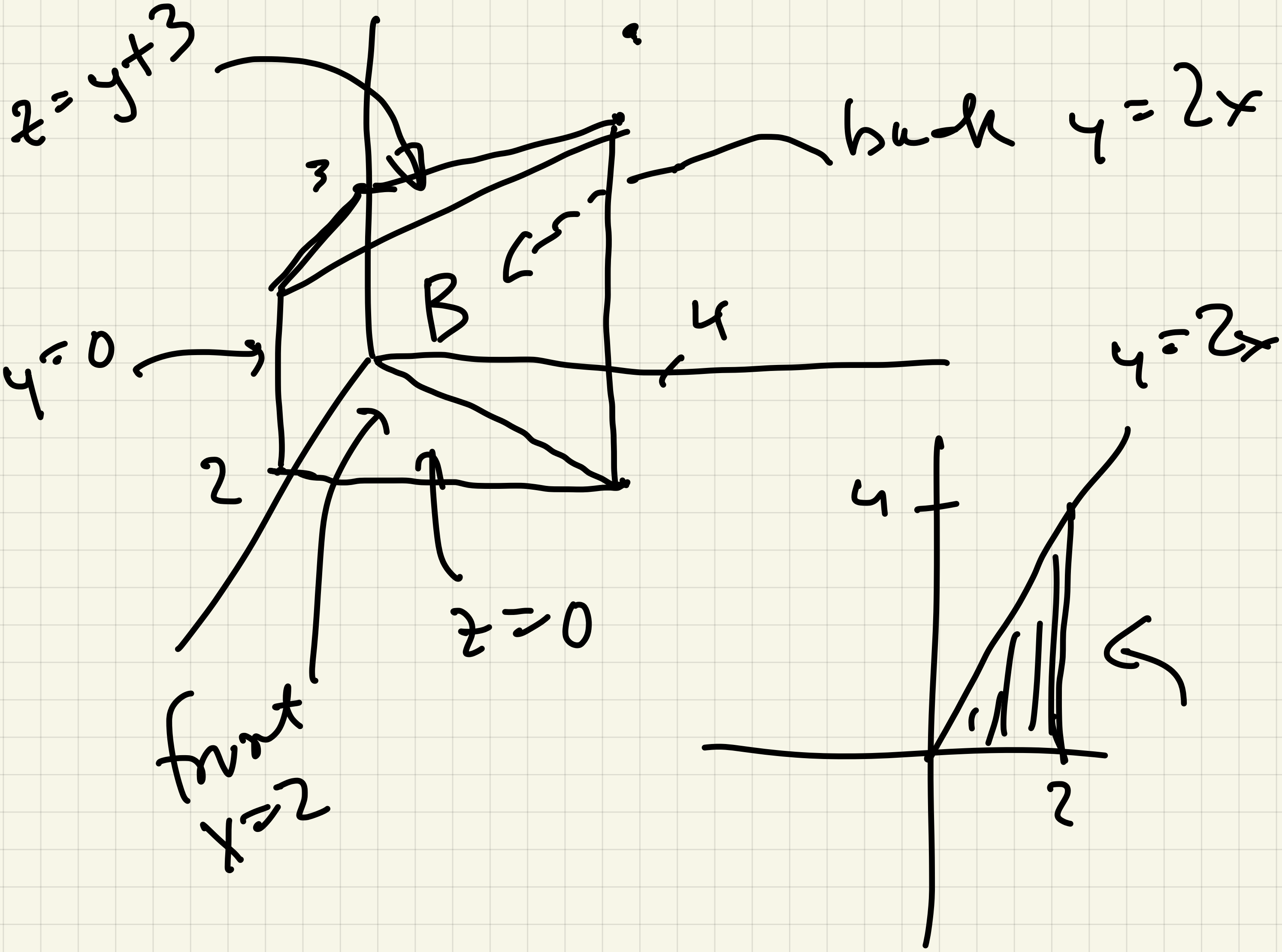
volume of i^{th} rectangle

$$\iiint_B 1 \, dV = \text{Volume of } B$$

$$\iiint_B \rho(x,y,z)\,dV = \text{mass of } B$$

ρ = density at (x,y,z)

find volume of the solid in the sketch



B :

$$0 \le z \le y+3$$
$$0 \le y \le 2x$$
$$0 \le x \le 2$$

$y = 2x$

$x = \frac{y}{2}$

$$\underbrace{\int_0^2}_{x} \underbrace{\int_0^{2x}}_{y} \underbrace{\int_0^{y+3} 1 \, dz}_{z} \, dy \, dx =$$

$$\int_0^2 \int_0^{2x} y+3 \, dy \, dx$$

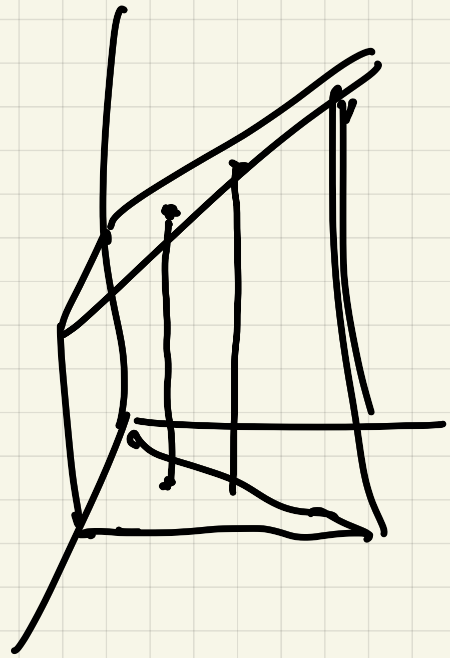
$$\frac{1}{2}y^2 + 3y \Big|_0^{2x}$$

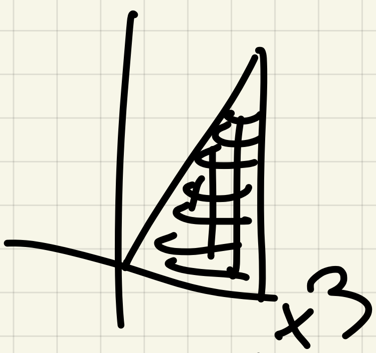
$$\int_0^2 2x^2 + 6x \, dx =$$

$$\frac{4}{3}x^3 + 3x^2 \Big|_0^2 = \frac{16}{3} + 12 = \frac{52}{3}$$

six possible orders of integration:

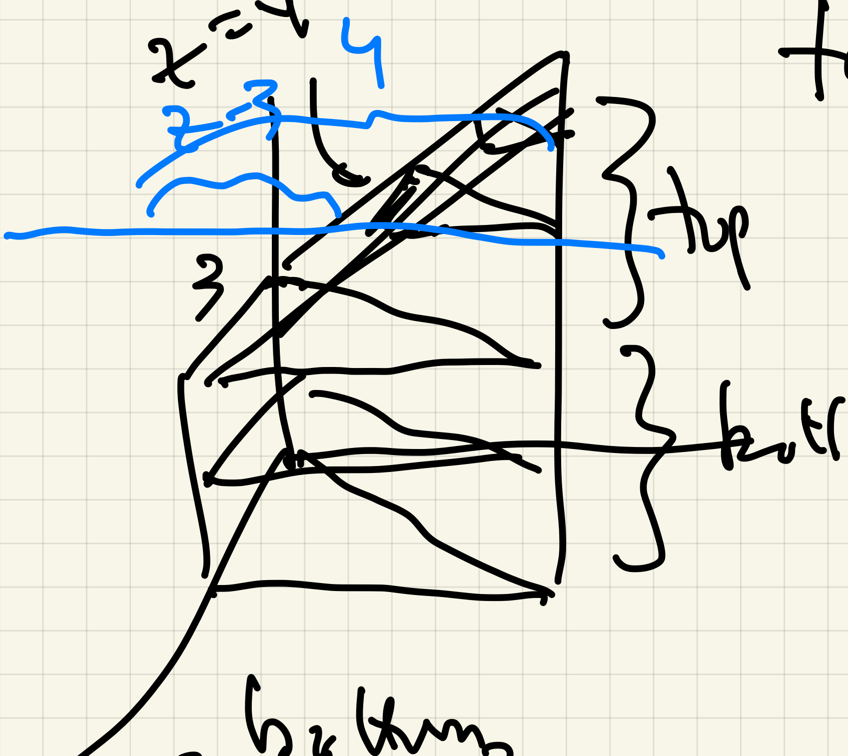
$$\int_0^4 \int_{y/2}^2 \int_0^{y+3} dz\,dx\,dy$$





$0 \le z \le 7$

triangles behave differently for $z \le 3$ / $z \ge 3$



bottom

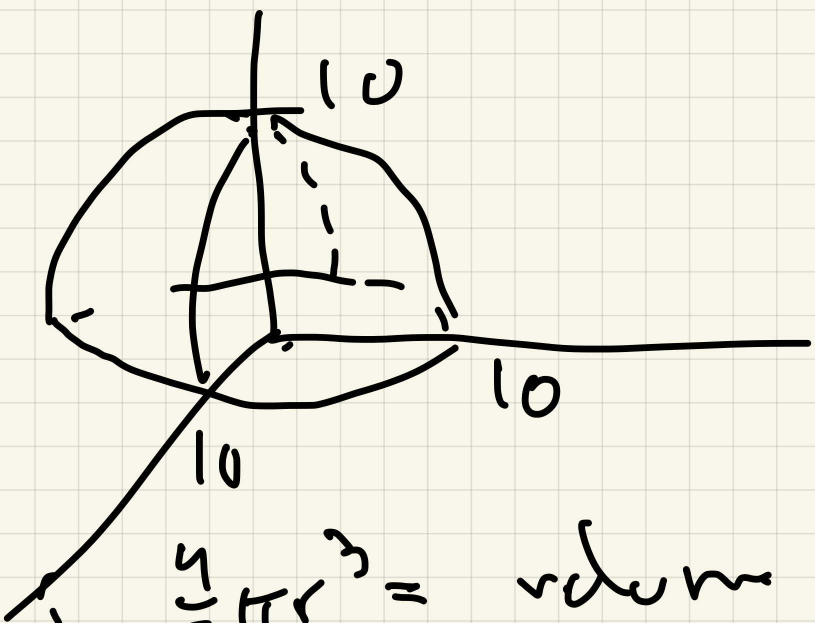
$$\int_0^3 \int_0^4 \int_{y/2}^2 dx\,dy\,dz$$ bottom

$$\int_3^7 \int_{z-3}^4 \int_{y/2}^{2} dx\,dy\,dz \quad \text{top}$$

Ex 3 Find volume of region

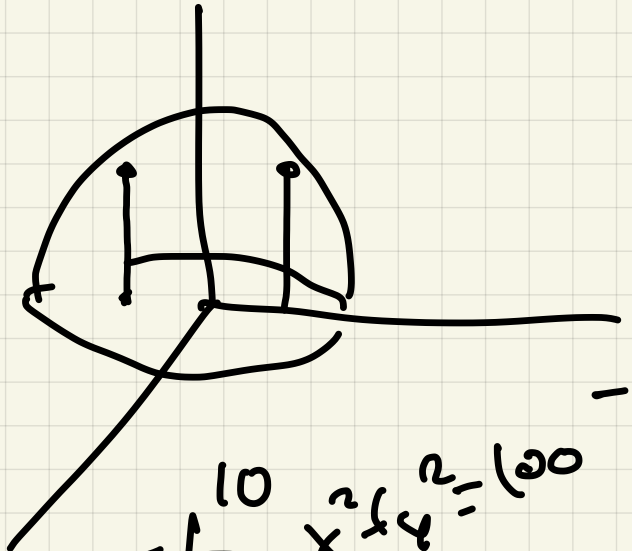
inside sphere $x^2+y^2+z^2=100$

above xy-plane



Ans $\frac{4}{3}\pi r^3 =$ volume of sphere radius r

$$\frac{1}{2}\left(\frac{4}{3}\pi \cdot 10^3\right) = \frac{2000\pi}{3}$$

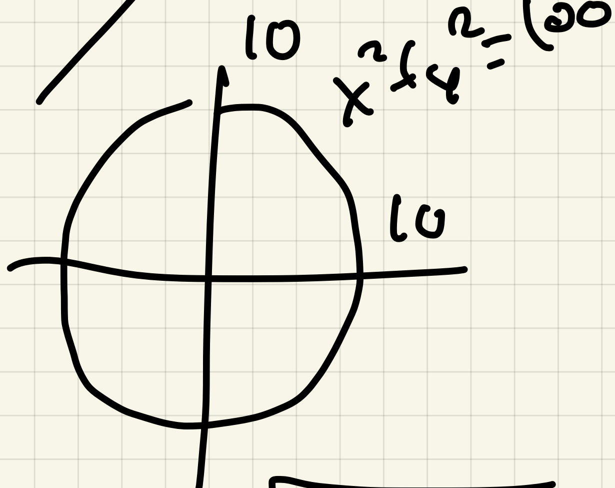


$$x^2+y^2+z^2=100$$

$$0 \le z \le \sqrt{100-x^2-y^2}$$

$$-\sqrt{100-x^2} \le y \le \sqrt{100-x^2}$$

$$-10 \le x \le 10$$



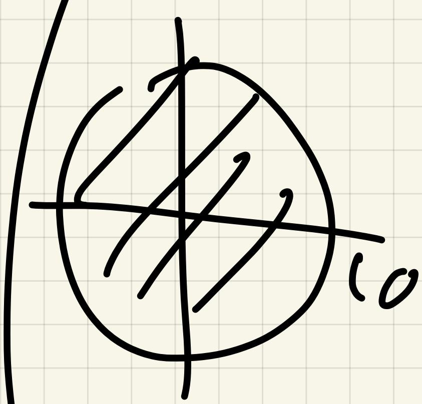
$$y = \pm\sqrt{100-x^2}$$

$$\text{Volume} = \int_{-10}^{10}\int_{-\sqrt{100-x^2}}^{\sqrt{100-x^2}} \boxed{\int_0^{\sqrt{100-x^2-y^2}} 1 \, dz} \, dy \, dx$$

$$\int_{-10}^{10}\int_{-\sqrt{100-x^2}}^{\sqrt{100-x^2}} \sqrt{100-x^2-y^2} \, dy \, dx$$

Go to polar coordinates

$$\int_0^{2\pi}\int_0^{10} \sqrt{100-r^2}\; \underbrace{r}_{\text{conversion factor}}\,dr\,d\theta$$



$u = 100 - r^2$

$du = -2r\,dr$

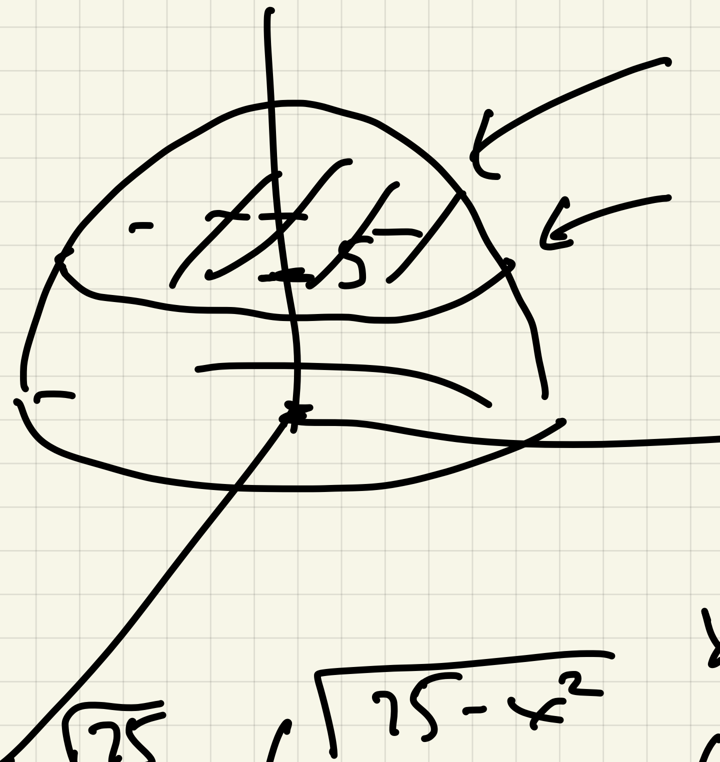
$$\int_{u=100}^{u=0} -\frac{1}{2}\sqrt{u}\,du = \int_0^{100} \frac{1}{2}\sqrt{u}\,du =$$

$$\frac{1}{\cancel{2}}\,\frac{\cancel{2}}{3}\,u^{3/2}\Big|_0^{100} = \frac{1}{3}1000$$

$$\int_0^{2\pi} \frac{1000}{3}\,d\theta = \frac{1000}{3}\,\theta\Big|_0^{2\pi} =$$

$$\frac{2000\pi}{3} \checkmark$$

What about the region $z \geq 5$?



$z = 5$?

$x^2+y^2+z^2 = 100$

$z = 5$

$x^2+y^2+25 = 100$

$x^2+y^2 = 75$

$$\int_{-\sqrt{75}}^{\sqrt{75}} \int_{-\sqrt{75-x^2}}^{\sqrt{75-x^2}} \int_{5}^{\sqrt{100-x^2-y^2}} dz\, dy\, dx$$