11/11/Calc3

Quiz 16

$$\int_{-3}^{3}\int_{0}^{x+3}\int_{0}^{y} 2\,dz\,dy\,dx$$

$$2z\Big|_0^y = 2y$$

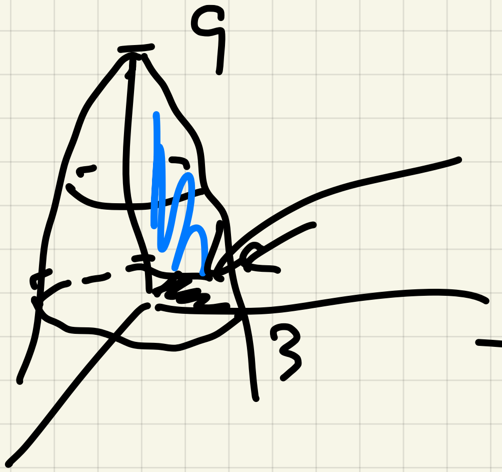
$$\int_0^{x+3} 2y\,dy = y^2\Big|_0^{x+3} =$$

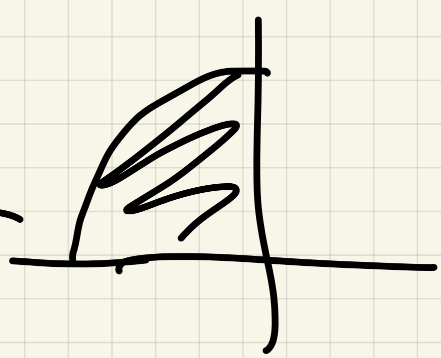
$u = x+3$

$$\int_{-3}^{3}(x+3)^2\,dx = \int_0^6 u^2\,du =$$

$$\frac{u^3}{3}\Big|_0^6 = \frac{6^3}{3} = 72$$

B: $0 \le x \le 9 - x^2 - y^2$, $\boxed{\begin{matrix} x \le 0 \\ y \ge 0 \end{matrix}}$



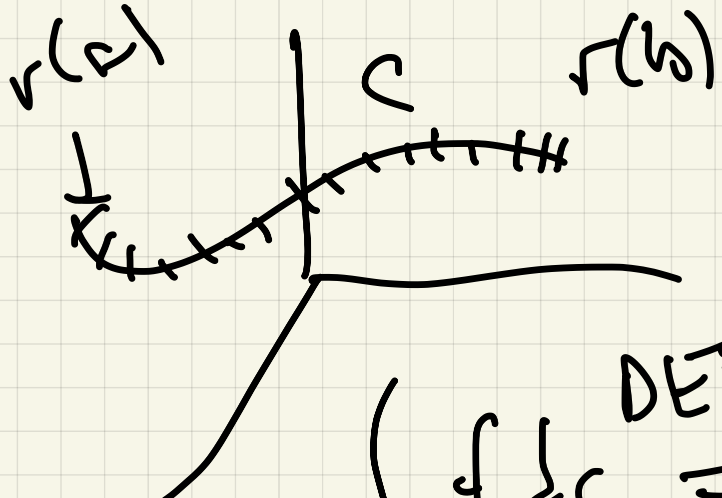


$$\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} 5 \, dz\, dy\, dx$$

Last time

Line integrals of <u>scalar</u> <u>functions</u>

along a curve C



$$\int_C f\, ds \overset{\text{DEFN}}{=} \lim_{n\to\infty} \sum f(x_k, y_k, z_k)\Delta s_k$$

<u>Calculate</u>:

If C is given by

$\bar{r}(t) \qquad a \le t \le b$,

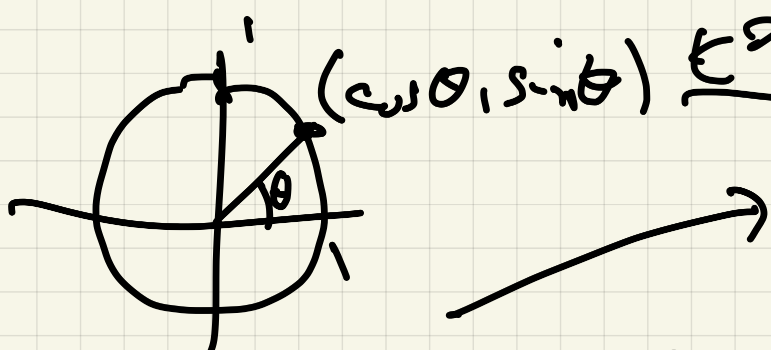
$$\int_C f\,ds = \int_{t=a}^{t=b} f(r(t))\,\underbrace{|r'(t)|\,dt}_{ds}$$

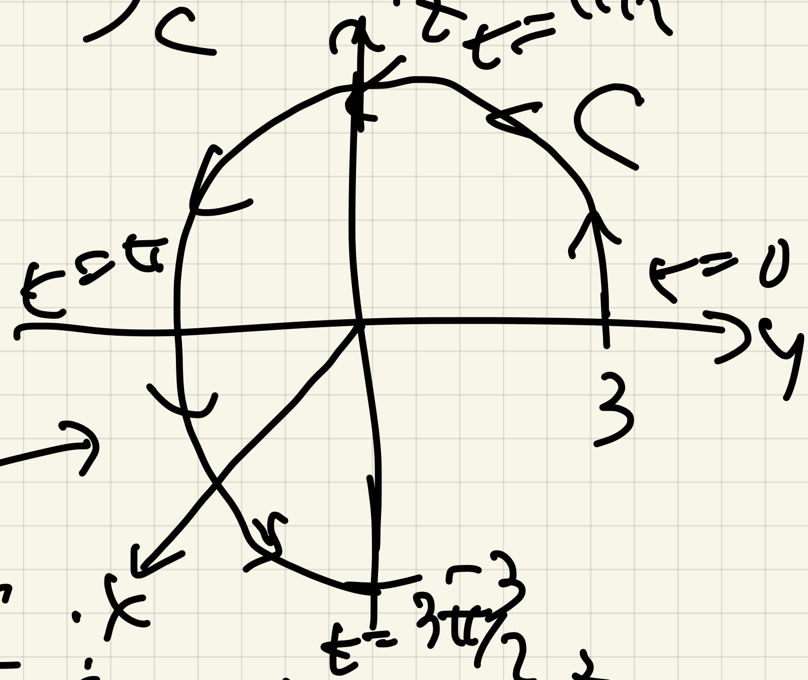
$|r'(t)|$: speed

Note: $f=1$, $\int_C f\,ds =$ arc length

Ex) Find $\int_C z^3 + x\,ds$

(a) C :





parametrize C :

$$\vec{r}(t) = \langle \overset{x}{0}, \overset{y}{3\cos t}, \overset{z}{3\sin t} \rangle$$

$$0 \le t \le 3\pi/2$$

$$\vec{r}'(t) = \langle 0, -3\sin t, 3\cos t \rangle$$

$$|r'(t)| = 3$$

$$\int_C f\,ds = \int_0^{3\pi/2} \underline{f(\bar{r}(t))}\,|r'(t)|\,dt$$

$$\parallel$$

$$\int_0^{3\pi/2} \left((3\sin t)^3 + \cancel{0}^{\,0}\right) \cdot \underline{3} \cdot dt$$

$z^3 + x$

$$= 81 \int_0^{3\pi/2} \underline{\sin^3 t}\, dt =$$

$$81 \int_{t=0}^{3\pi/2} \underline{(1-\cos^2 t)\, \boxed{\sin t\, dt}}$$

$u = \cos t$

$du = -\sin t\, dt$

$$\parallel$$

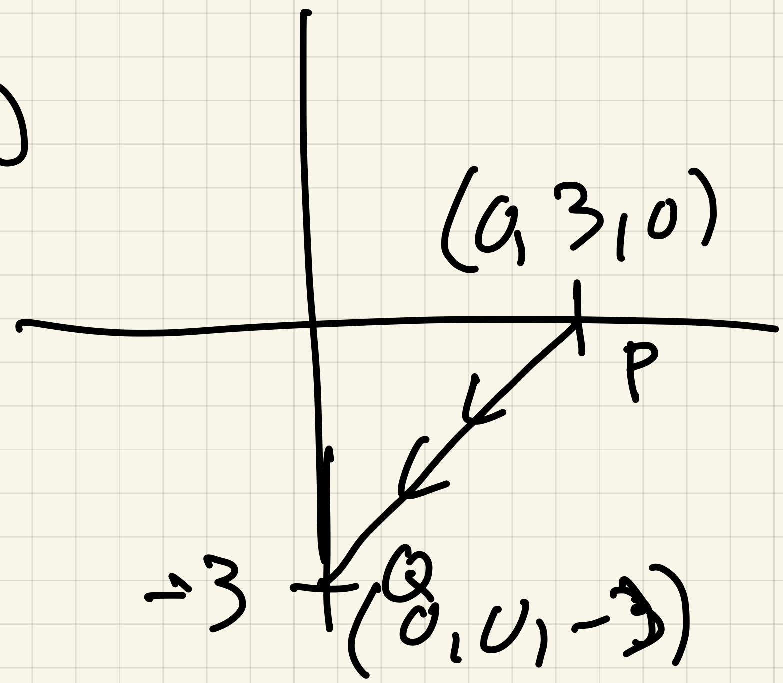
$$-81 \int_{u=1}^{u=0} (1-u^2)\, du =$$

$$81\int_0^1 (1-u^2)\,du =$$

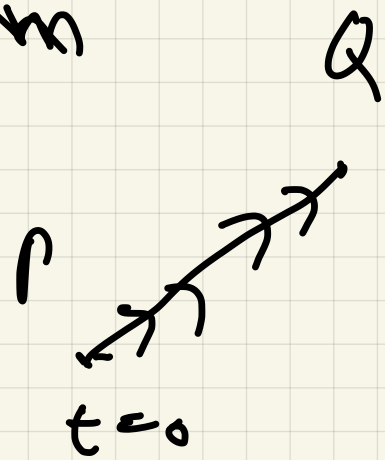
$$81 \left. \underline{u - u^3/3} \right|_0^1 = 81\left(\tfrac{2}{3}\right)$$

$$= 54$$

(b)



<u>Aside</u> to parametrize the line segment from P to Q



direction $\vec{v} = \overrightarrow{PQ}$

$\vec{r}(t) = \bar{P} + \bar{v}\,t \quad , \quad 0 \le t \le 1$

$$\vec{PQ} = \langle 0, -3, -3 \rangle = \vec{v}$$

$$(0, 0, -3) - (0, 3, 0)$$

$$\vec{r}(t) = P + vt$$

$$= \begin{array}{c} x \\ y \\ z \end{array} \begin{pmatrix} 0 + 0 \\ 3 - 3t \\ 0 - 3t \end{pmatrix} \quad 0 \le t \le 1$$

$$\begin{array}{ccc} x & y & z \end{array}$$

$$\vec{r}(t) = \langle 0, 3 - 3t, -3t \rangle$$

$$\vec{r}'(t) = \langle 0, -3, -3 \rangle$$

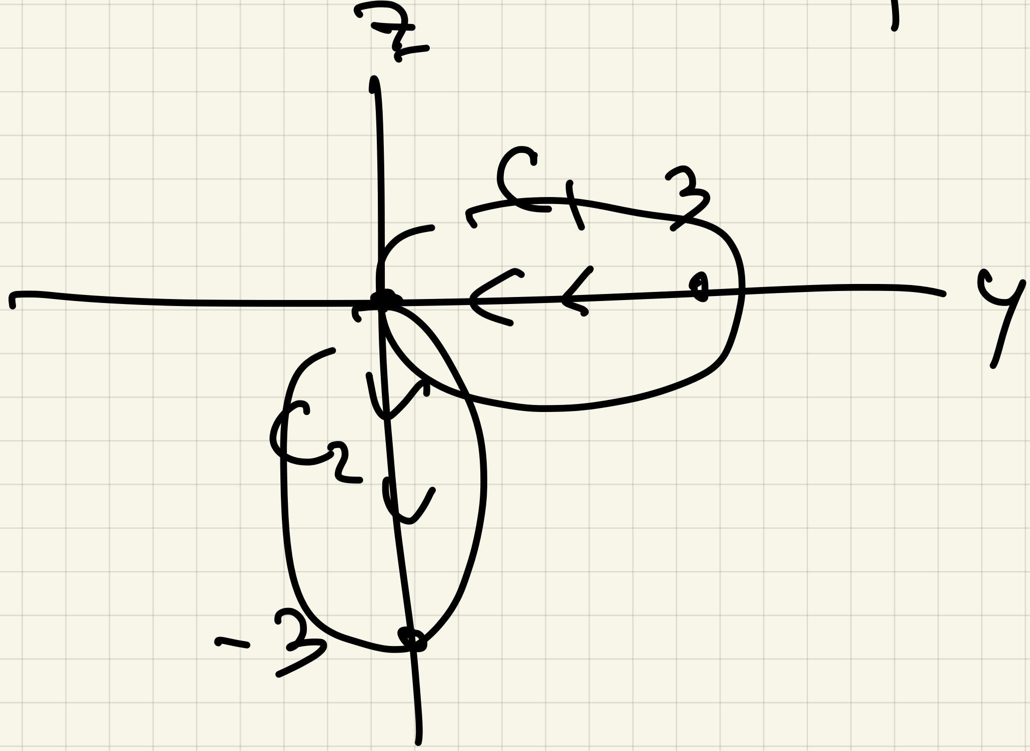
$$|\vec{r}'(t)| = 3\sqrt{2}$$

$$\int_C f\, ds = \int_0^1 \Big(\underbrace{(-3t)^3}_{-27t^3} + 0 \Big) 3\sqrt{2}\, dt$$

$$-\int_0^1 27 t^3 \cdot 3\sqrt{2}\, dt =$$

$$-81\sqrt{2}\ \frac{t^4}{4}\Big|_0^1 = \frac{-81\sqrt{2}}{4}$$

$$\int_C f\,ds$$



$$\| \quad \int_{C_1} f\,ds + \int_{C_2} f\,ds$$

$0 \le t \le 1$

$C_1 \quad \bar{r}(t) = \langle \overset{x}{0}, 3-3t, \overset{z}{0} \rangle$

$\bar{r}'(t) = \langle 0, -3, 0 \rangle$

$|r'| = 3$

$$\int t^3 + x = \int 0 \, 3\,dt = 0$$

C_2 $\quad \vec{r}(t) = \langle 0, 0, -3t \rangle$

$0 \le t \le 1$

$|\vec{r}'| = 3$

$$\int_{C_2} f\,ds = \int_0^1 (-3t)^3 \cdot 3\,dt$$

$$= -81 \int_0^1 t^3\,dt =$$

$$-81 \left. \frac{t^4}{4} \right|_0^1 = = -\frac{81}{4}.$$

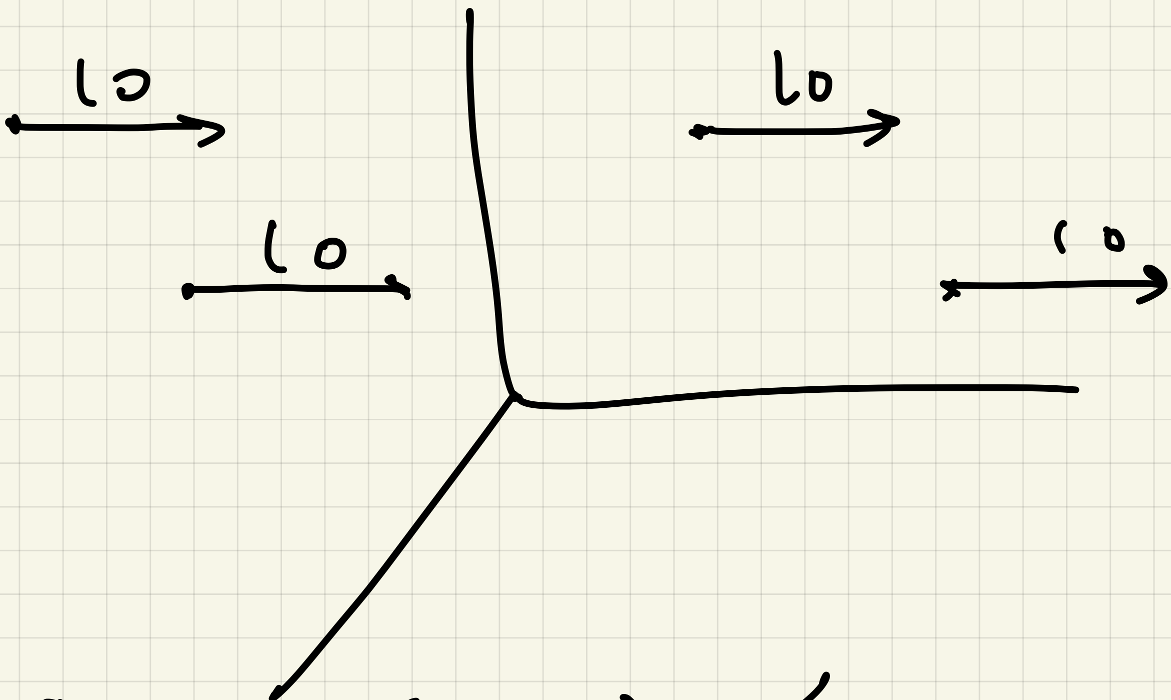
§15.2

Line integrals over vector fields

Vector field: on $\mathbb{R}^3$: is

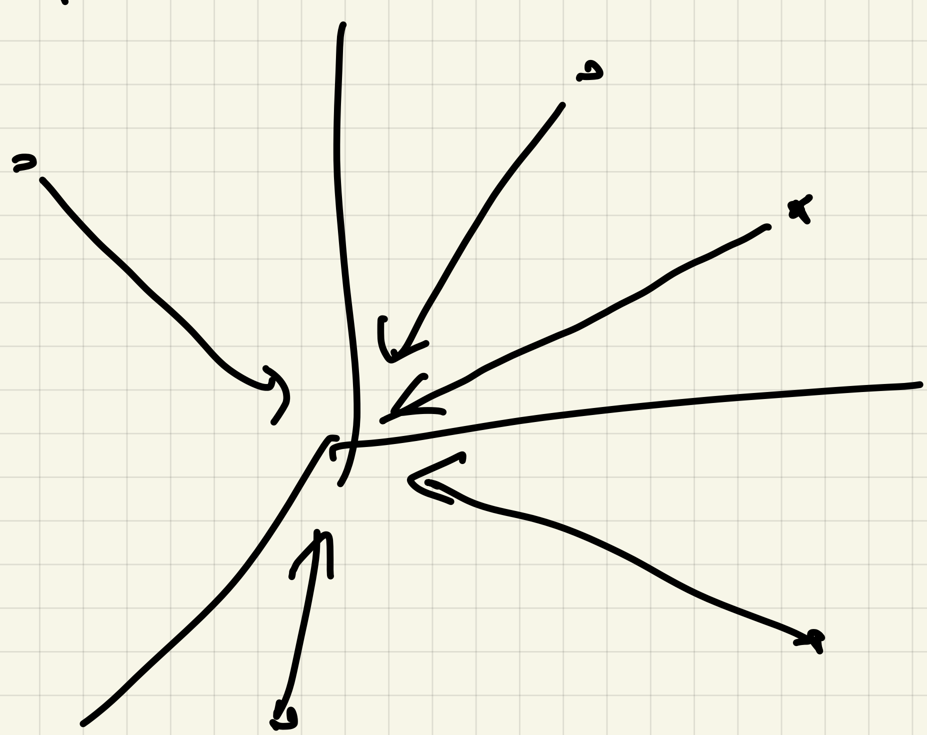
a function $F: \mathbb{R}^3 \to \mathbb{R}^3$

Ex1 $\quad F(x,y,z) = \langle 0, 10, 0 \rangle$



Ex2 $\quad F(x,y,z) = \langle -x, -y, -z \rangle$

(a)



looks like gravity; but length wrong

(y) $\vec{F}(x,y,z) = \dfrac{\langle -x,-y,-z\rangle}{|\langle -x,-y,-z\rangle|^2}$