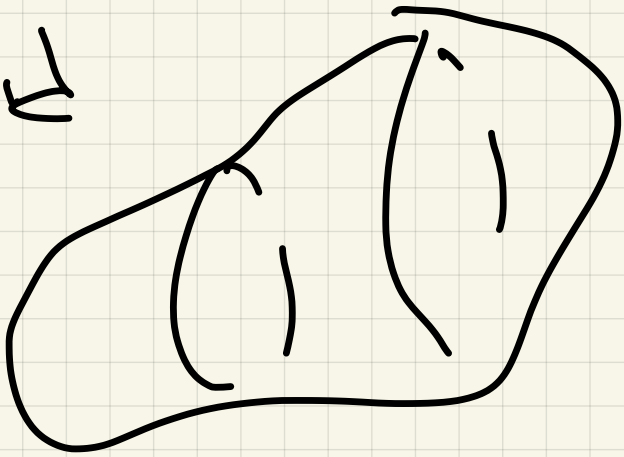
3/28| Calc 3

Definition $\iiint_B f(x,y,z)\,dV$

B → 3D solid



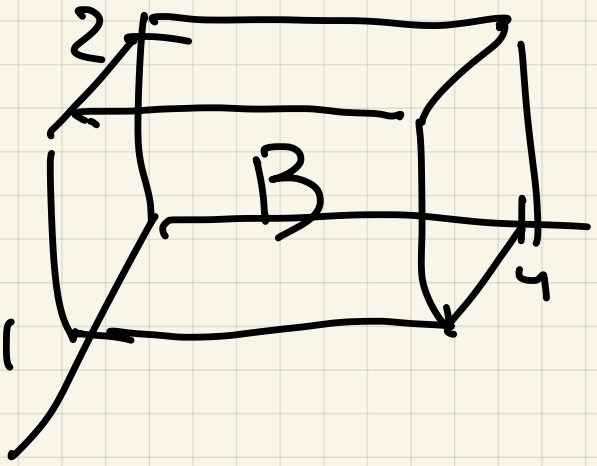
Meaning: If $f(x,y,z) \geq 0$ is the density of solid at position (x,y,z) then

$\iiint_B f(x,y,z)\,dV$ = mass

special case

$\iiint_B 1\,dV$ = Volume of B

Ex 1 : (a)



B = box

density is 5

$$\iiint_B 5 \, dV = 8 \times 5 =$$

volume density

40 mass

(b) What if density is $f(x,y,z) = y + 3$??

Can evaluate with iterated integrals:

B: $0 \le x \le 1$

$0 \le y \le 4$

$0 \le z \le 2$

$f = y + 3$

$$\int_0^2 \int_0^4 \int_0^1 y+3 \, dx \, dy \, dz$$

$$(y+3)x \Big|_0^1$$

$$\int_0^2 \int_0^4 y+3 \, dy \, dz$$

$$\frac{y^2}{2} + 3y \Big|_0^4$$

$$\int_0^2 8+12 \, dz$$

$$\int_0^2 20 \, dz = 20z \Big|_0^2 = 40$$

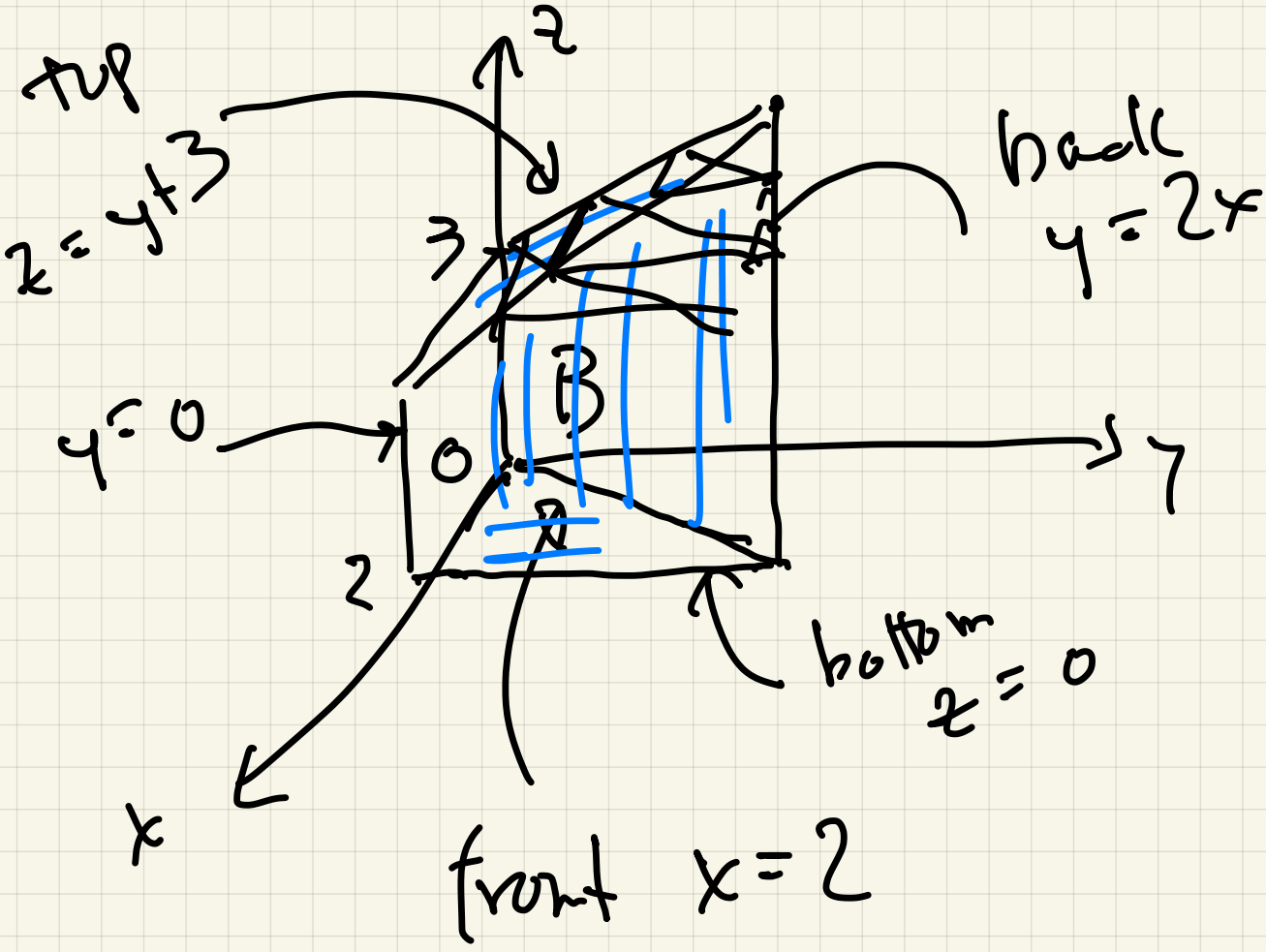
Can switch order of integration if B is still described by new endpoints

for example

$$\int_0^1 \int_0^2 \int_0^4 (y+3)\, dy\, dz\, dx$$

six possible orders,

Find the volume of the solid sketched:

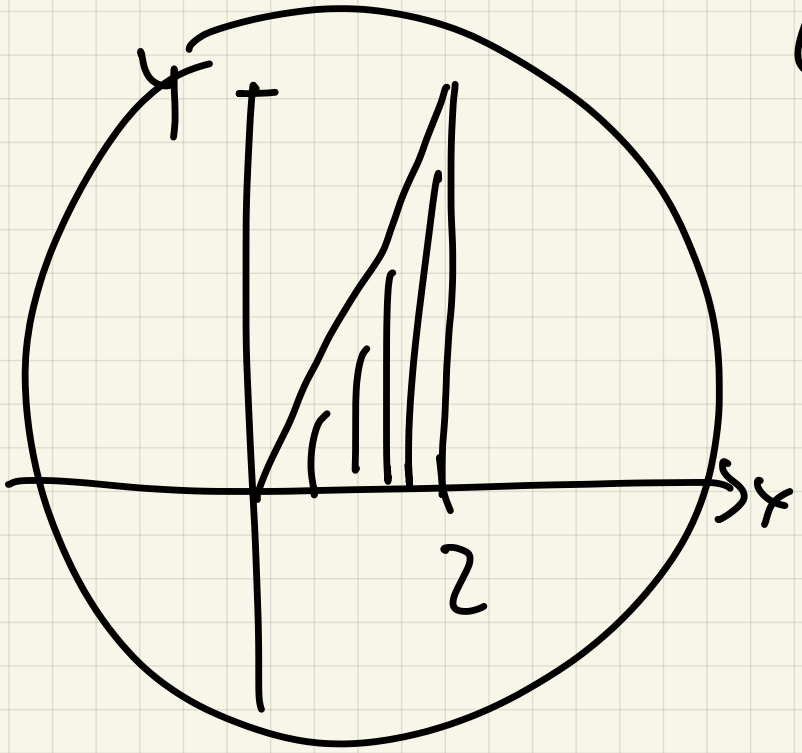


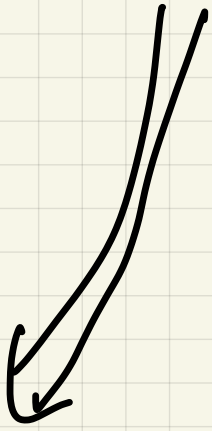
Steps: B:

$0 \le x \le 2$

$0 \le y \le 2x$

$0 \le z \le y+3$





$$V = \int_0^2 \int_0^{2x} \int_0^{y+3} dz\, dy\, dx$$

$$z\Big|_0^{y+3}$$

$$\int_0^{2x} y+3\, dy$$

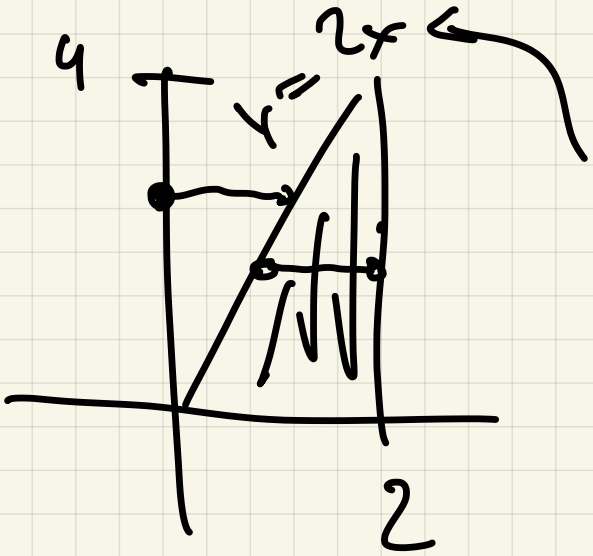
$$\frac{y^2}{2} + 3y\Big|_0^{2x}$$

$$\int_0^2 2x^2 + 6x \, dx$$

$$\frac{2}{3}x^3 + 3x^2 \Big|_0^2 =$$

$$\frac{16}{3} + 12 = \frac{52}{3}$$

six orders of integration possible



$$x = \frac{y}{2}$$

$$0 \le y \le 4$$

$$\frac{y}{2} \le x \le 2$$

$$0 \le z \le y+3$$

$$V = \int_0^4 \int_{y/2}^2 \int_0^{y+3} dz\, dx\, dy$$

less convenient order:

$dx\,dy\,dz$

$0 \le z \le 7$

but the range on y/x depends on z

$z \le 3$ or $7 \ge z \ge 3$

$$\int_0^3 \int_0^4 \int_{y/2}^{2} dx\,dy\,dz$$

bottom

z y x

$z = y + 3$

$y = z - 3$

$$\int_3^7 \int_{z-3}^{4} \int_{y/2}^{2} dx\,dy\,dz$$

top