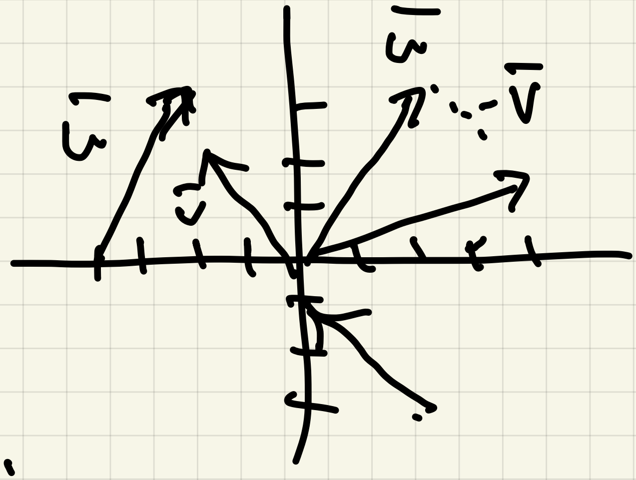
1/24/ Calc 3 Quiz 2 avg 91 med 95

1.



$\bar{u} = \langle 2, 3 \rangle$

$\bar{v} = \langle -2, 2 \rangle$

$\bar{u} - \bar{v} = \langle 4, 1 \rangle$

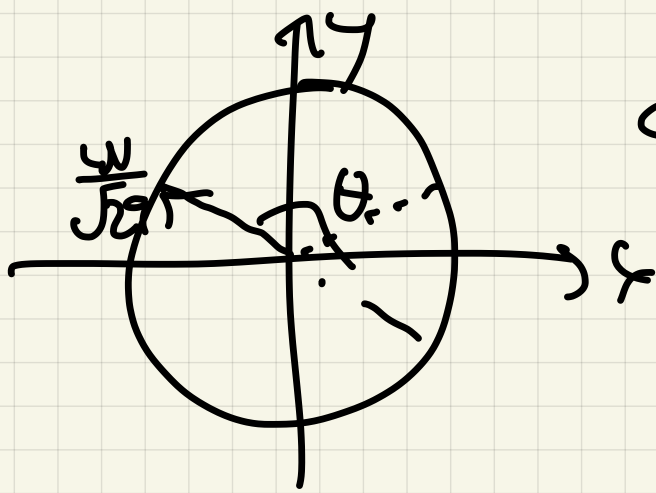
$\bar{u} + 2\bar{v} = \langle -2, 7 \rangle$

2. $\bar{w} = \langle -5, 2 \rangle$

(a) $|\bar{w}| = \sqrt{25+4} = \sqrt{29}$

(b) $\dfrac{\bar{w}}{\sqrt{29}} = \langle -5/\sqrt{29}, 2/\sqrt{29} \rangle$

$\cos\theta$ $\sin\theta$



(c) $\theta = \arccos -5/\sqrt{29}$

$\theta = \arctan \frac{2}{-5} + \pi$

$\theta = \pi - \arctan \frac{2}{5}$

$\theta = \pi - \arcsin \frac{2}{\sqrt{29}}$

Last time Determinants

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + \dots$$

cross product of $\bar{u}, \bar{v}$ is

$$\bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

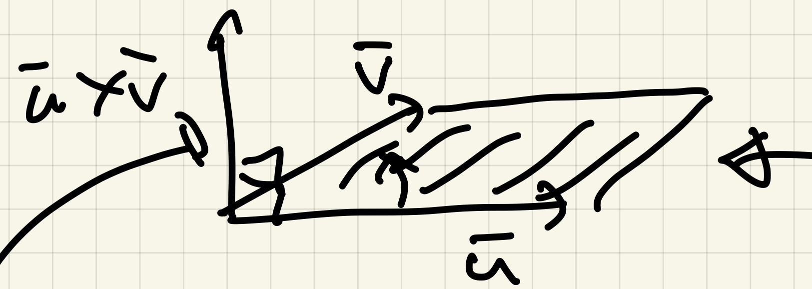
algebraic properties

Geometric properties

(1) $\bar{u} \times \bar{v} = 0 \iff \bar{u} \parallel \bar{v}$

(2) $\bar{u}, \bar{v} \perp \bar{u} \times \bar{v}$

follows the right hand rule



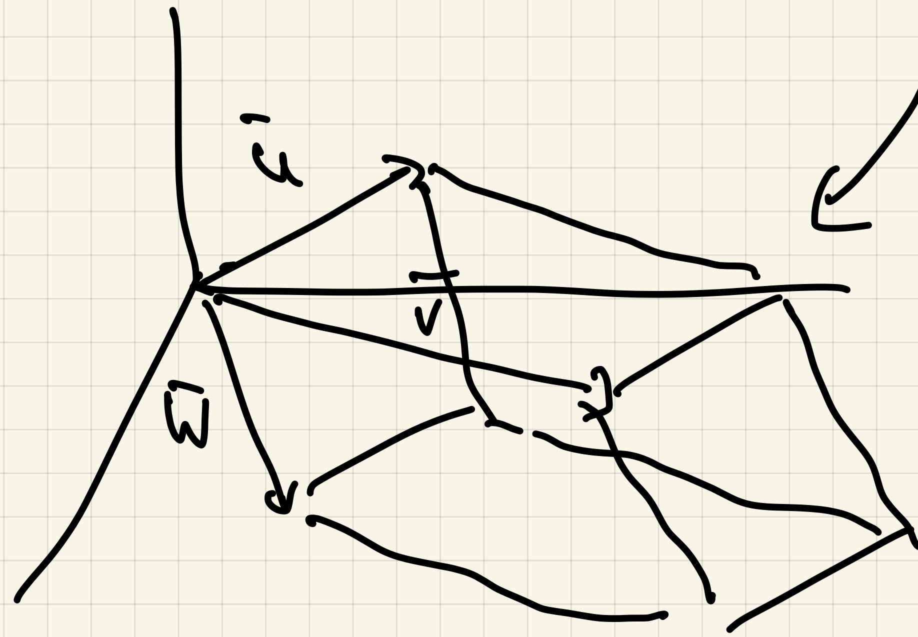
length $|\bar{u} \times \bar{v}|$ = area of parallelogram

Triple scalar product

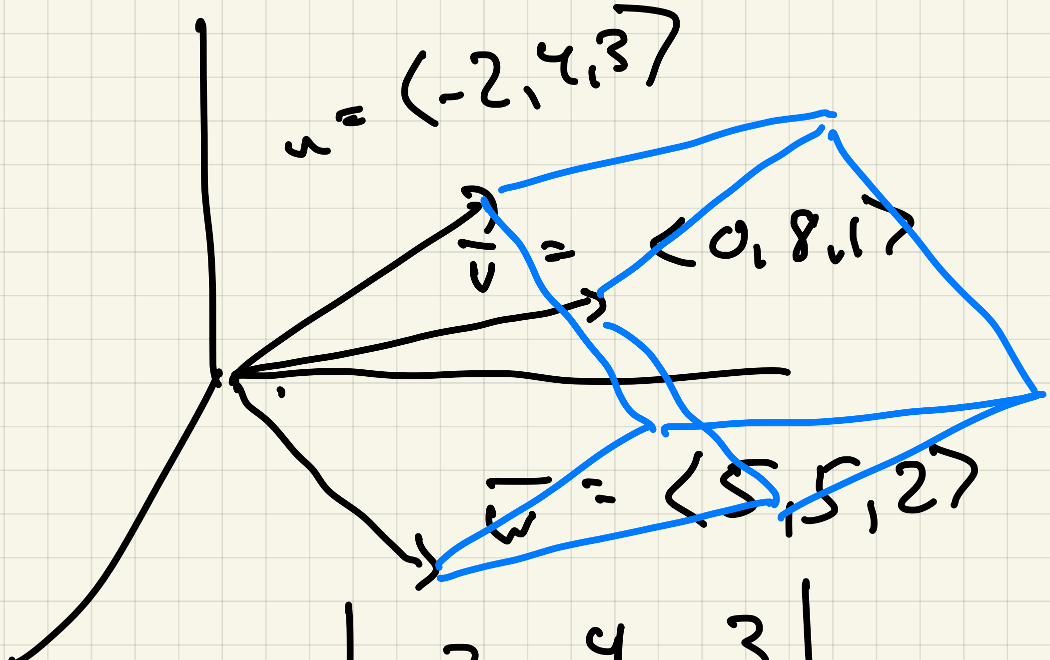
$$(\bar{u} \times \bar{v}) \cdot \bar{w} = \bar{u} \cdot (\bar{v} \times \bar{w}) =$$

$$\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Absolute value is volume of the parallelapiped



Ex



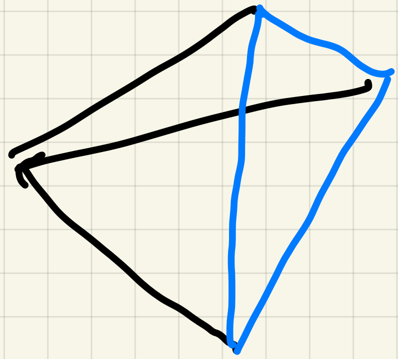
$$\text{Volume} = \left| \underbrace{\begin{vmatrix} -2 & 4 & 3 \\ 0 & 8 & 1 \\ 5 & 5 & 2 \end{vmatrix}}_{} \right|$$

$$-2 \underbrace{\begin{vmatrix} 8 & 1 \\ 5 & 2 \end{vmatrix}}_{11} - 4 \underbrace{\begin{vmatrix} 0 & 1 \\ 5 & 2 \end{vmatrix}}_{-5} + 3 \underbrace{\begin{vmatrix} 0 & 8 \\ 5 & 5 \end{vmatrix}}_{-40}$$

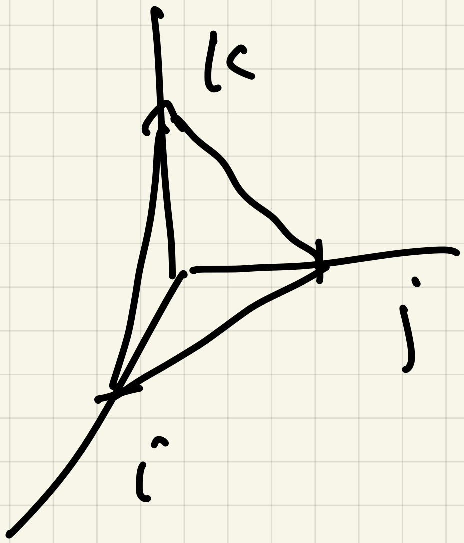
$-22 + 20 - 120 = -122$

so volume $= 122$

Rmk



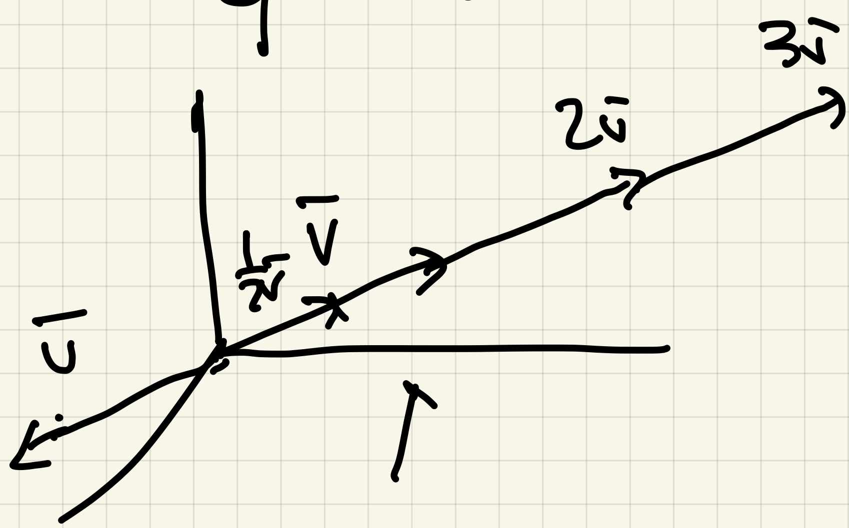
$\leftarrow$ Tetrahedron has volume $122/6$



§11.5 lines & planes

In $\mathbb{R}^2$: $y = mx + b$

In $\mathbb{R}^3$



Notice the line through $\bar{O}$ with direction $\bar{v}$ is

$$t\bar{v} \ , \quad t \in \mathbb{R}$$

Ex 1 $\bar{v} = \langle -1, 4, 2 \rangle$

line $\begin{cases} x = -t \\ y = 4t \\ z = 2t \end{cases} , \quad t \in \mathbb{R}$

Parametric equations for a line:

If line L is parallel to $\bar{v} = \langle a, b, c \rangle$ and passes through point $P_0 = (x_1, y_1, z_1)$

Then L is given by

$$(x, y, z) = \langle x_1 + at, y_1 + bt, z_1 + ct \rangle$$

$$= \bar{P}_0 + \bar{v}t$$

$$\begin{cases} x = x_1 + at \\ y = y_1 + bt \\ z = z_1 + ct \end{cases}$$

Remarks

① t is a parameter

② Many descriptions of same line

Ex 2

$$x = 1 + 8t$$
$$y = 6 - 48t$$
$$z = 1 - 8t$$

$(9, -42, -7)$

$$x = 9 + t$$
$$y = -42 - 6t$$
$$z = -7 - t$$

same!