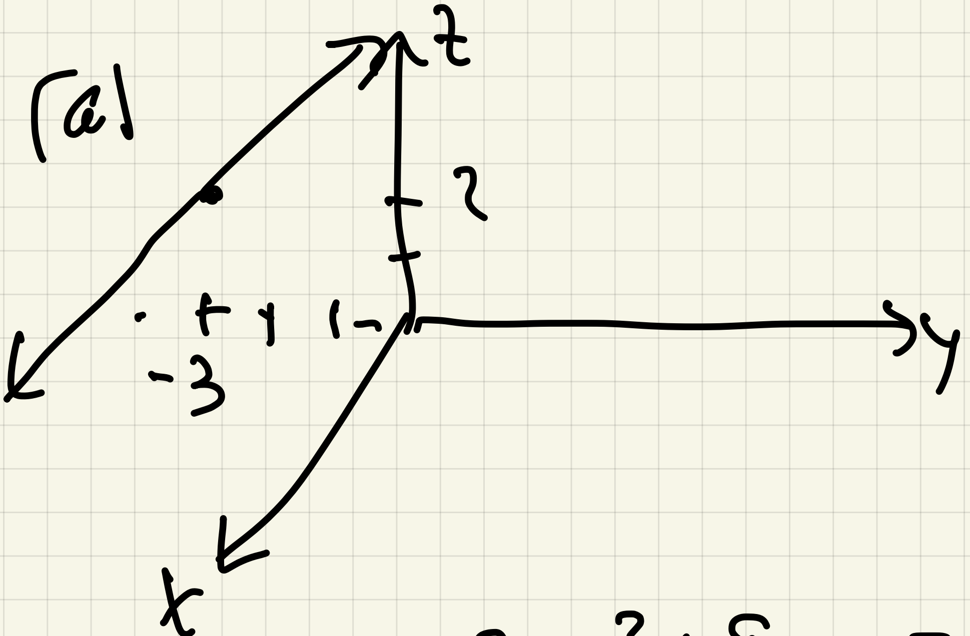
1/21/Calc3

Quiz 1

avg 95%
med 100%

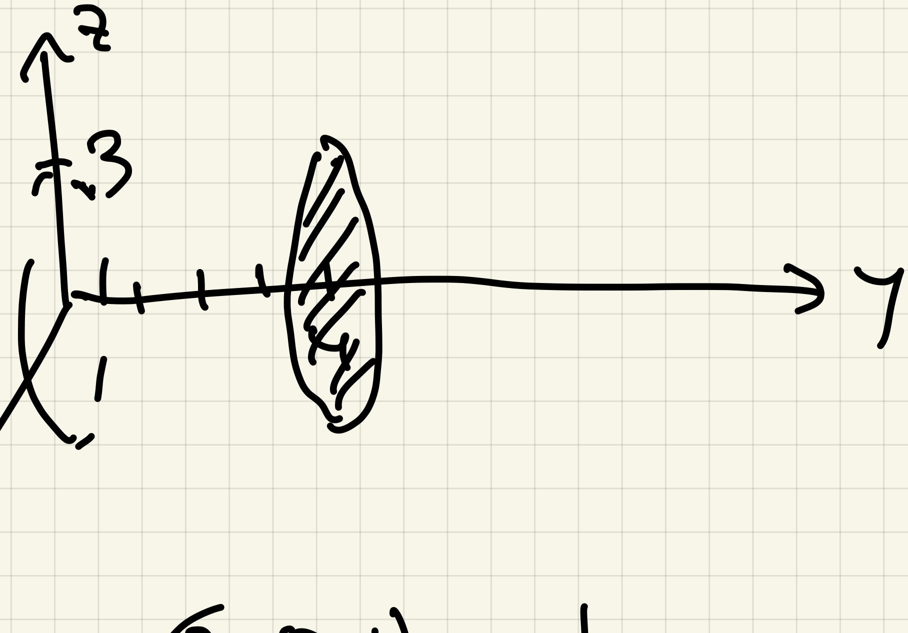
(a)



$y = -3$

$z = +2$

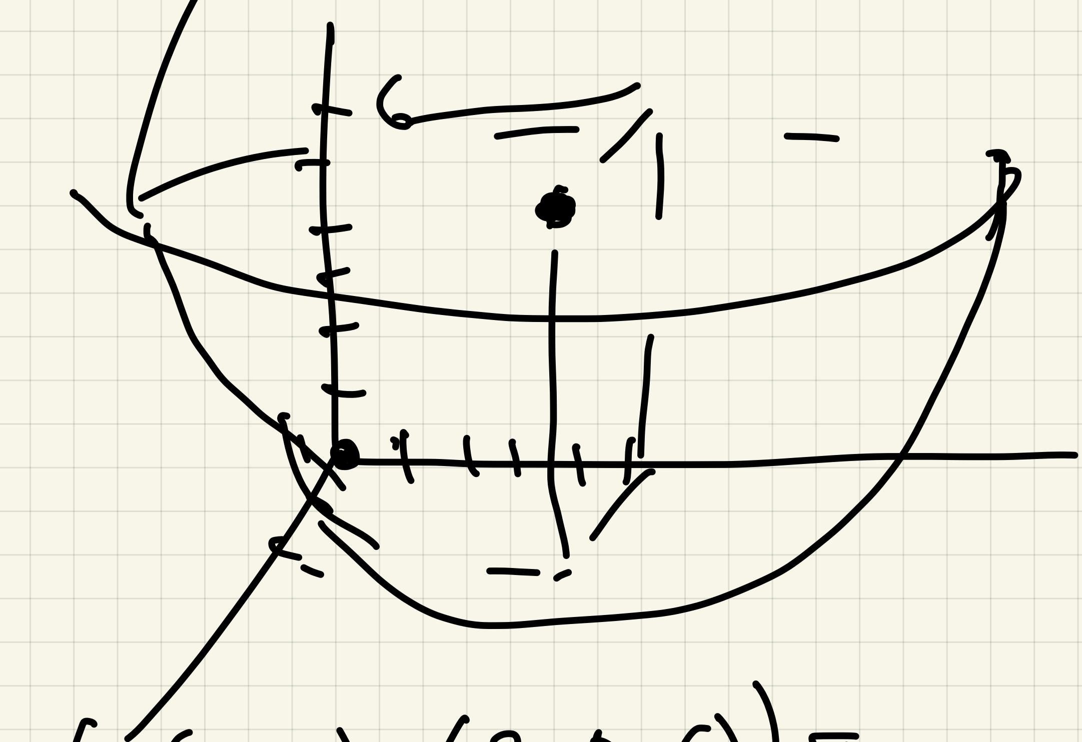
(b) $x^2 + z^2 \le 9, \; y = 4.$



2. (a) $(2, 5, 6)$ ctr $r = 8$

$$(x-2)^2 + (y-5)^2 + (z-6)^2 = 64$$

(b)



$$\text{dist}((0,0,0),(2,5,6)) =$$

$$\sqrt{2^2+5^2+6^2} =$$

$$\sqrt{4+25+36} = \sqrt{65} > 8$$

outside

11.3 Dot product

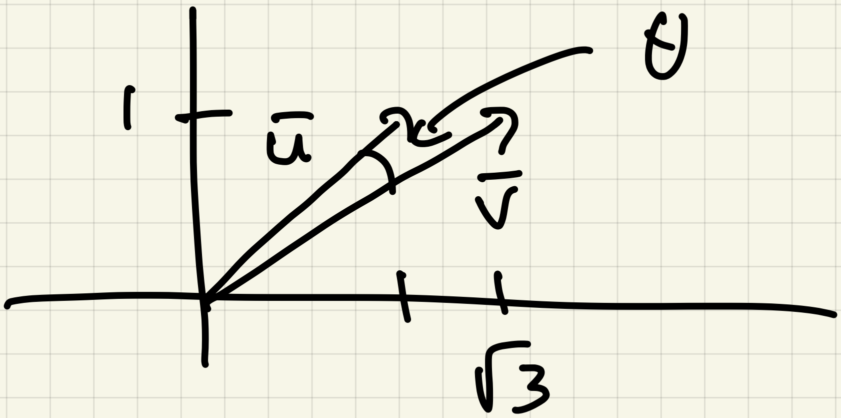
$\bar{u}\cdot\bar{v}$ = scalar

$$\implies \cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$$

Rmk $|\bar{w}| = \|\bar{u}\|$

Ex)

(a)



$\bar{u} = \langle 1, 1 \rangle$

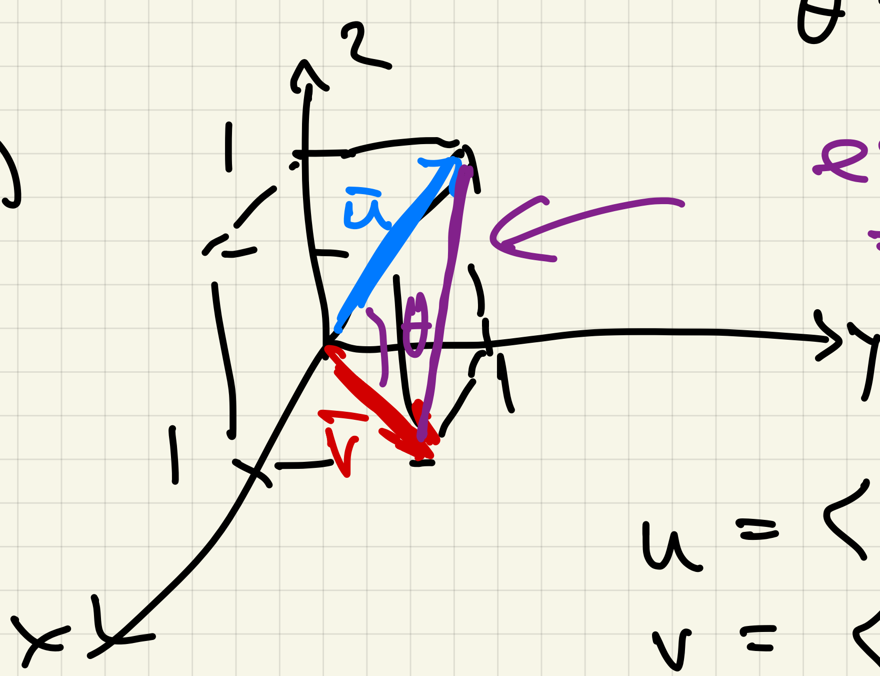
$\bar{v} = \langle 1, \sqrt{3} \rangle$

$$\frac{\bar{u} \cdot \bar{v}}{|\bar{u}||v|} = \frac{1+\sqrt{3}}{\sqrt{2 \cdot 2}} = \cos\theta$$

$\Downarrow$

$\theta = 15^\circ$

(b)



equilateral $\triangle$
$\Rightarrow$ all $\angle$'s $= 60^\circ$

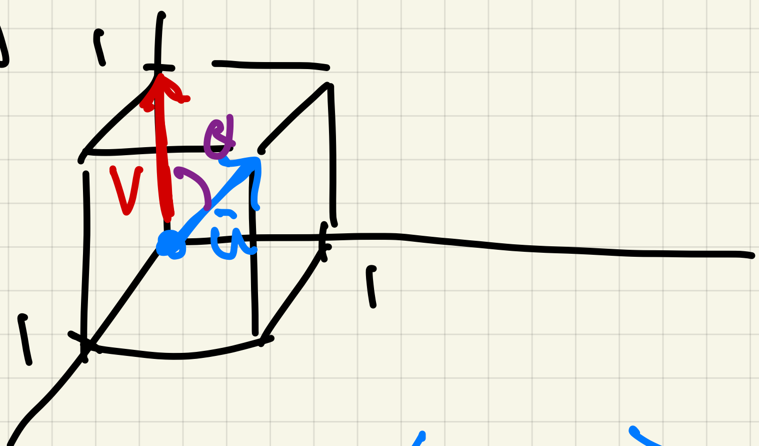
$u = \langle 0, 1, 1 \rangle$

$v = \langle 1, 1, 0 \rangle$

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$$\theta = 60^\circ = \pi/3 \text{ rad}$$

(c)



$\bar{u} = \langle 1, 1, 1\rangle$

$\bar{v} = \langle 0, 0, 1\rangle$

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|} = \frac{1}{\sqrt{3}\cdot 1} = \frac{1}{\sqrt{3}}$$

$$\left(\theta = \arccos\frac{1}{\sqrt{3}} \approx 54.74^\circ\right)$$

<u>Special Case:</u>

$\bar{u} \perp \bar{v} \iff \theta = 90° = \pi/2$ rad

perpendicular
normal
orthogonal

$$\Updownarrow$$

$$\cos\theta = 0$$

$$\Updownarrow$$

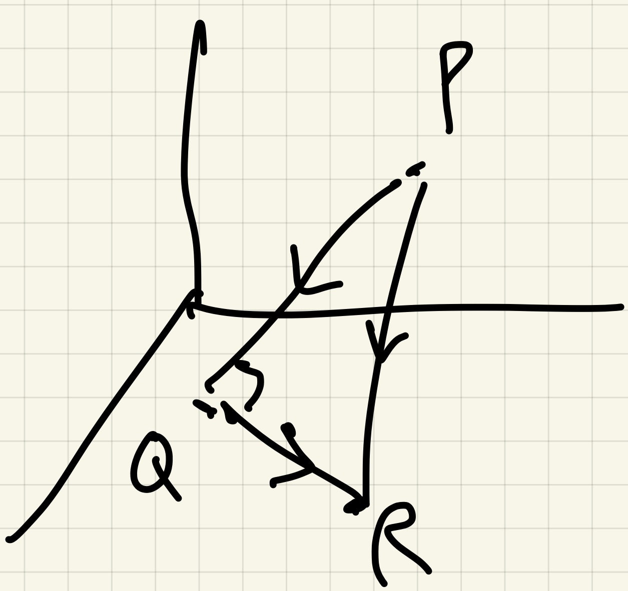
$$\bar{u} \cdot \bar{v} = 0$$

<u>Ex 2</u> Is ΔPQR a right Δ?
Is acute? obtuse?

Cal $\quad P = (5, 3, 4)$

$Q = (7, 1, 3)$

$R = (8, 0, 7)$



$\overrightarrow{PQ} = \langle 2, -2, -1 \rangle$

$\overrightarrow{QR} = \langle 1, -1, 4 \rangle$

$\overrightarrow{PR} = \langle 3, -3, 3 \rangle$

Right: $\vec{PQ} \cdot \vec{QR} = 0$

double check: $|\vec{PQ}| = \sqrt{9} = 3$

$|\vec{QR}| = \sqrt{18}$

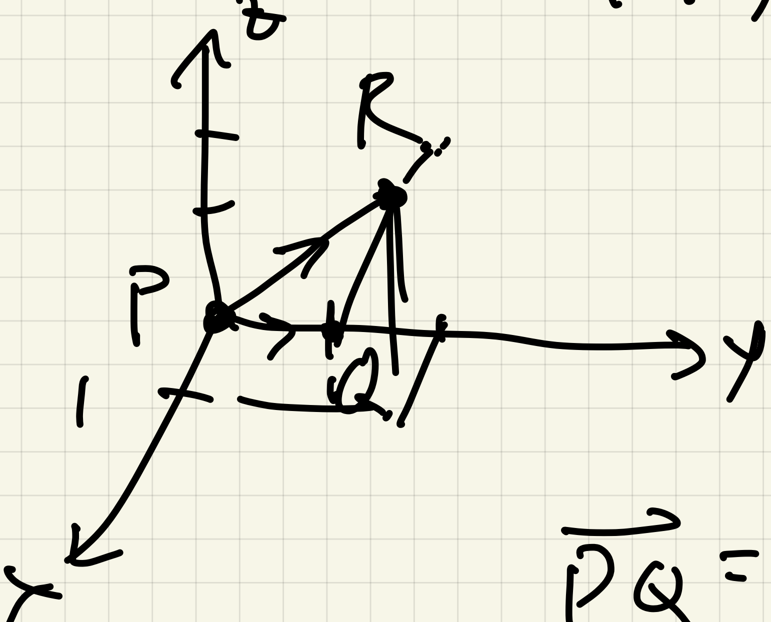
$|\vec{PR}| = \sqrt{27}$

$$|\vec{PR}|^2 = |\vec{PQ}|^2 + |\vec{QR}|^2$$

(b) $P = (0,0,0)$

$Q = (0,1,0)$

$R = (1,2,2)$



$\vec{PQ} = \langle 0, 1, 0 \rangle$

$\vec{PR} = \langle 1,2,2 \rangle$

$\vec{QR} = \langle 1,1,2 \rangle$

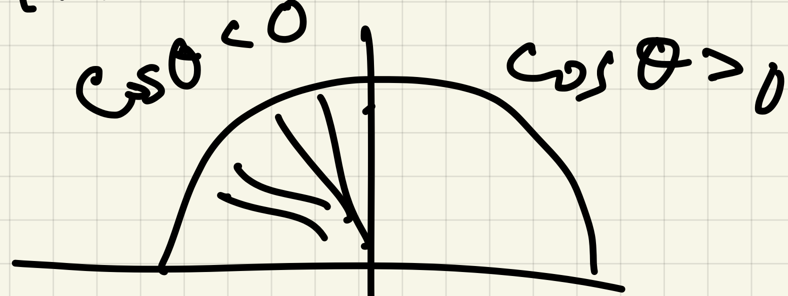
all dot prods > 0 $\Rightarrow$

not right Δ.

<u>But</u>: $\vec{PQ}\cdot\vec{QR} > 0 \Rightarrow$

$\vec{QP}\cdot\vec{QR} < 0 \Rightarrow$

$$\cos\theta = \frac{\vec{QP}\cdot\vec{QR}}{|\overline{QP}||\overline{QR}|} < 0$$



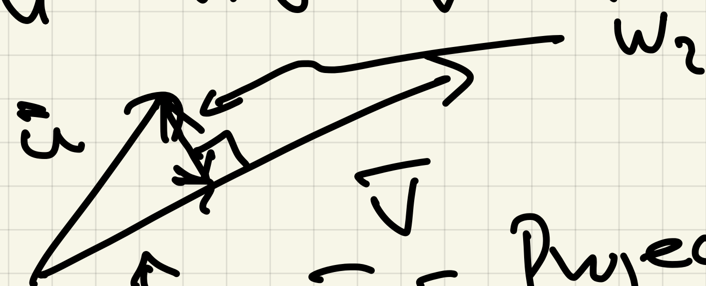
So $\angle PQR > 90^\circ \Rightarrow$ obtuse

<u>Projections</u> Given vectors

$\bar{u}$ and $\bar{v}$

<u>component of $\bar{u}$ along $\bar{v}$</u>

"



$\bar{w}_1$ = <u>Projection of $\bar{u}$ onto $\bar{v}$</u>

Want to decompose $\bar{u}$ as

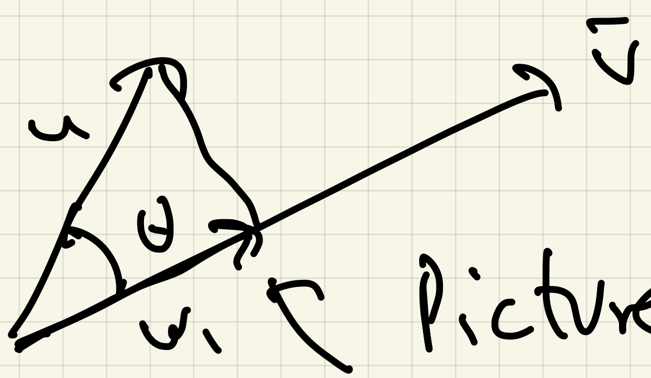
$$\bar{u} = \bar{w}_1 + \bar{w}_2 \quad \text{with}$$

$$\bar{w}_1 \parallel \bar{v}, \quad \bar{w}_2 \perp \bar{v}$$

$$\bar{w}_1 = \mathrm{Proj}_{\bar{v}} \bar{u}$$

(<u>note</u>: $w_2 = \underset{\text{vector component of } \bar{u} \text{ orthogonal to } \bar{v}}{\underset{\shortparallel}{\bar{u}}} - \bar{w}_1$)

How to compute:



Picture $\Rightarrow$

$$\bar{w}_1 \text{ has } \begin{cases} \text{direction } \bar{v} \\ \text{length } |u|\cos\theta \end{cases}$$

$$\Downarrow$$

$$\bar{w}_1 = |u|\cos\theta \, \frac{\bar{v}}{|v|} =$$

$\cancel{|u|}\,\frac{\bar u\cdot\bar v}{\cancel{|u|}|v|}\,\frac{\bar v}{|v|} =$

$$= \frac{\bar u\cdot\bar v}{|v|^2}\,\bar v$$

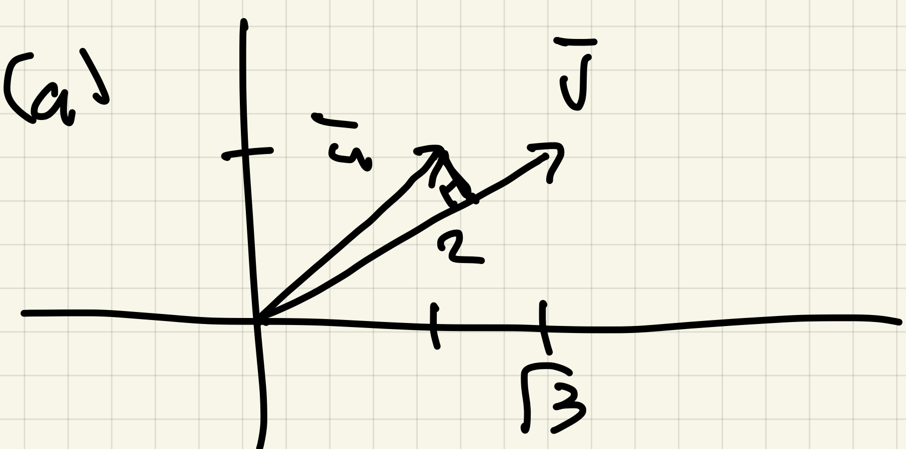
$$\boxed{\underline{\text{Thm}}\quad Proj_{\bar v}\bar u = \frac{\bar u\cdot\bar v}{|v|^2}\cdot\bar v}$$

Book.

Scalar component is $|\bar u|\cos\theta = \frac{\bar u\cdot\bar v}{|v|}$

Ex 3 Find vector components of $\bar u$ along $\bar v$, also the component orthogonal to $\bar v$ and scalar component

(a)



(a) $\bar{u} = \langle 1, 1 \rangle$

$\bar{v} = \langle \sqrt{3}, 1 \rangle$

$$\underline{w_1} = \operatorname{proj}_{\bar{v}} \bar{u} = \frac{\bar{u} \cdot \bar{v}}{|v|^2} \bar{v}$$

along v

$$= \frac{\sqrt{3}+1}{2^2} \langle \sqrt{3}, 1 \rangle =$$

$$\left\langle \frac{3+\sqrt{3}}{4}, \frac{\sqrt{3}+1}{4} \right\rangle$$

$$\bar{w}_2 = \bar{u} - \bar{w}_1 = \langle 1, 1 \rangle - \left\langle \frac{3+\sqrt{3}}{4}, \frac{\sqrt{3}+1}{4} \right\rangle$$

orthog to v

$$= \left\langle \frac{1-\sqrt{3}}{4}, \frac{-\sqrt{3}+3}{4} \right\rangle$$

$\left(\underline{\text{scalar component}} \quad \frac{\sqrt{3}+1}{2} \right)$

(b) $\bar{u} = \langle 3, -2, 7 \rangle$

$\bar{v} = i = e_1 = \langle 1, 0, 0 \rangle$

(i) $\bar{w}_1 = \frac{\bar{u} \cdot \bar{v}}{|v|^2} \bar{v} = \frac{3}{1} \langle 1, 0, 0 \rangle =$

$$\begin{cases} \overline{w_1} = \langle 3, 0, 0 \rangle \\ w_2 = \overline{u} - \langle 3, 0, 0 \rangle = \langle 0, -2, 7 \rangle \\ \text{scalar comp} = 3 \end{cases}$$

(ii) : $v = j = \langle 0, 1, 0 \rangle$

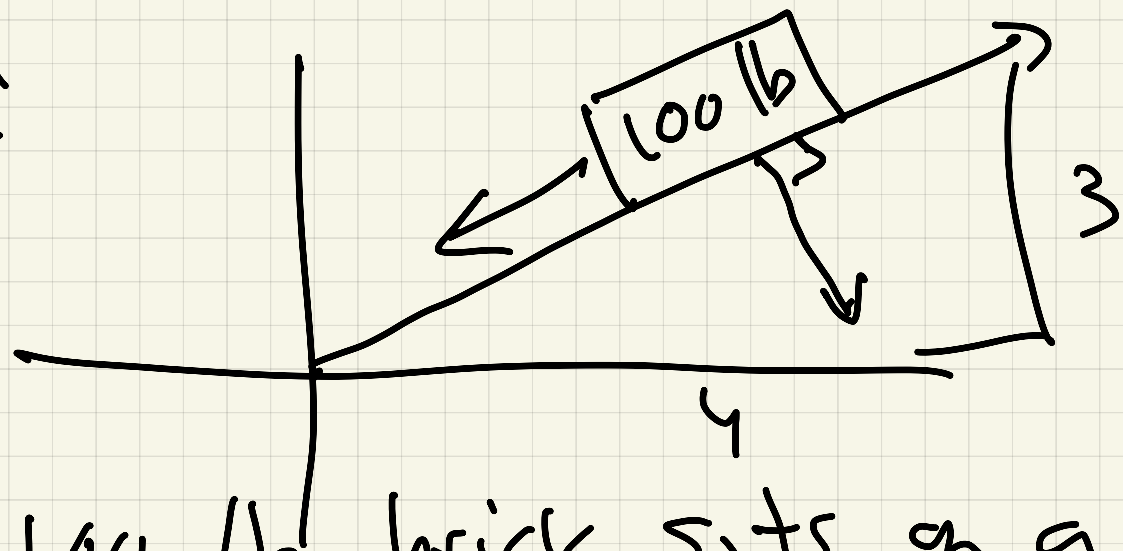
$$\overline{w_1} = -2j$$

$$\overline{w_2} = \langle 3, 0, 7 \rangle$$

negative OK

$$\text{scalar comp} = -2$$

Ex 4



A 100 lb brick sits on a ramp of slope 3/4

Find components of gravity force

with along and orthogonal
to ramp.

$\bar{v} = \langle 4, 3 \rangle$ $\quad \left(\langle 4/5, 3/5 \rangle \text{ ok} \right)$

~~Along $\bar{v}$~~:

Force of gravity
is $\bar{g} = \langle 0, -100 \rangle$

$\bar{g}$ Along $\bar{v}$ $\quad \text{Proj}_{\bar{g}} \bar{v} =$

$$\frac{g \cdot \bar{v}}{|v|^2} \cdot \bar{v} = \frac{-300}{25} \langle 4, 3 \rangle$$

$$= -12 \langle 4, 3 \rangle = \langle -48, -36 \rangle$$

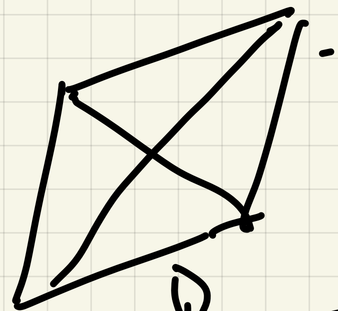
$\bar{g}$ orthog to v

$$\langle 0, -100 \rangle - \langle -48, -36 \rangle$$

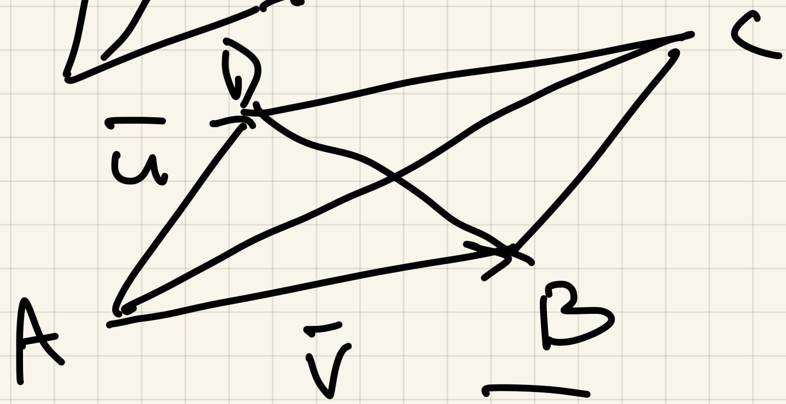
$$= \langle 48, -64 \rangle$$

Ex5 (geometry)

Show A parallelogram is a rectangle $\iff$ diagonals have same length



plausible:



Let $\bar{u} = \overline{AD}$

$\bar{v} = \overline{AB}$

$ABCD$ is rectangle $\iff$

$\angle DAB = 90^\circ \iff$

$\bar{u} \perp \bar{v} \iff \bar{u} \cdot \bar{v} = 0$

Diagonals

$\text{diags} = \iff |u+v| = |u-v| \iff$

$|u+v|^2 = |u-v|^2 \iff$

$(u+v)\cdot(u+v) = (u-v)\cdot(u-v)$

$\cancel{\bar{u}\cdot\bar{u}} + 2\bar{u}\cdot v + \cancel{v\cdot v} = \cancel{\bar{u}\cdot\bar{u}} - 2u\cdot v + \cancel{v\cdot v}$

$\iff 2u\cdot v = -2\bar{u}\cdot v$

$\iff 4u\cdot v = 0$

$u\cdot v = 0 \quad \perp$