1/17/ Calc 3

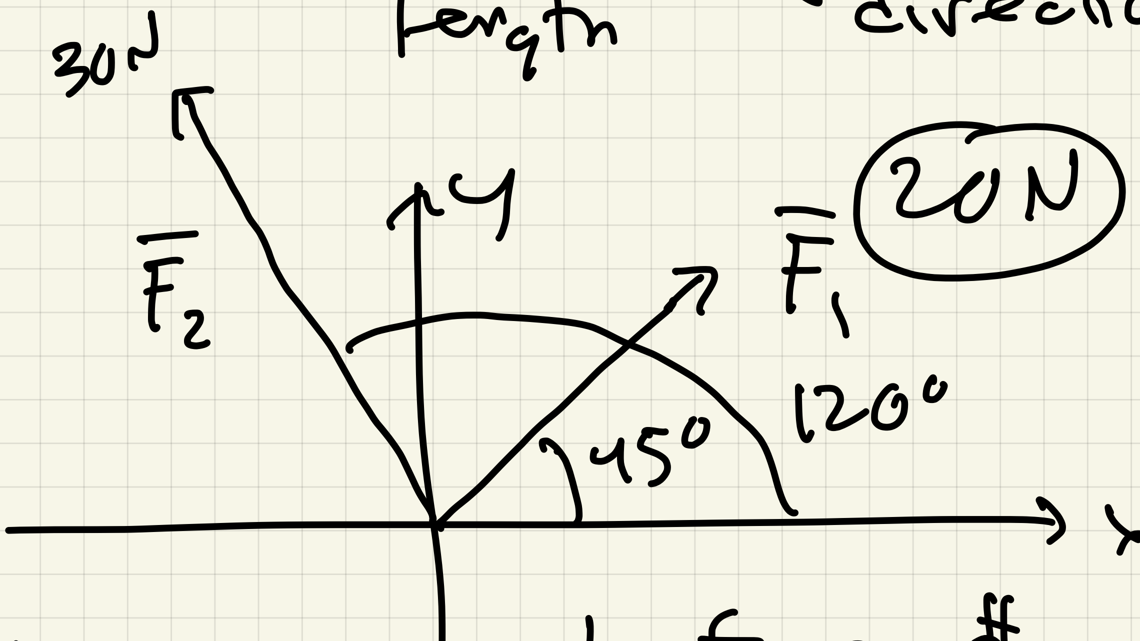
Last time

Vectors
operations
+ / − / scalar mult.
algebra − geometry
unit vectors,

$v \neq 0 \implies v = |v| \cdot \frac{v}{|v|}$

length ↗ ↖ direction.

Ex1



What is the net force of $\bar{F}_1$ and $\bar{F}_2$? i.e, magnitude and direction?

Easy: $\bar{F}_1$ = mag. · unit vector

$= 20 \cdot \langle \cos 45^\circ, \sin 45^\circ \rangle$

$= 20 \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\bar{F}_2 = 30 \langle \cos 120^\circ, \sin 120^\circ \rangle =$

$\cos 120^\circ = -\frac{1}{2}$, $\sin 120^\circ = \frac{\sqrt{3}}{2}$

$= 30 \langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

so $\bar{F}_{net} = \bar{F}_1 + \bar{F}_2 =$

$\langle \frac{20}{\sqrt{2}} - \frac{30}{2}, \frac{20}{\sqrt{2}} + \frac{30\sqrt{3}}{2} \rangle$

$= \langle 10\sqrt{2} - 15, 10\sqrt{2} + 15\sqrt{3} \rangle$

Calculator $\|F_{net}\| = 40.132$

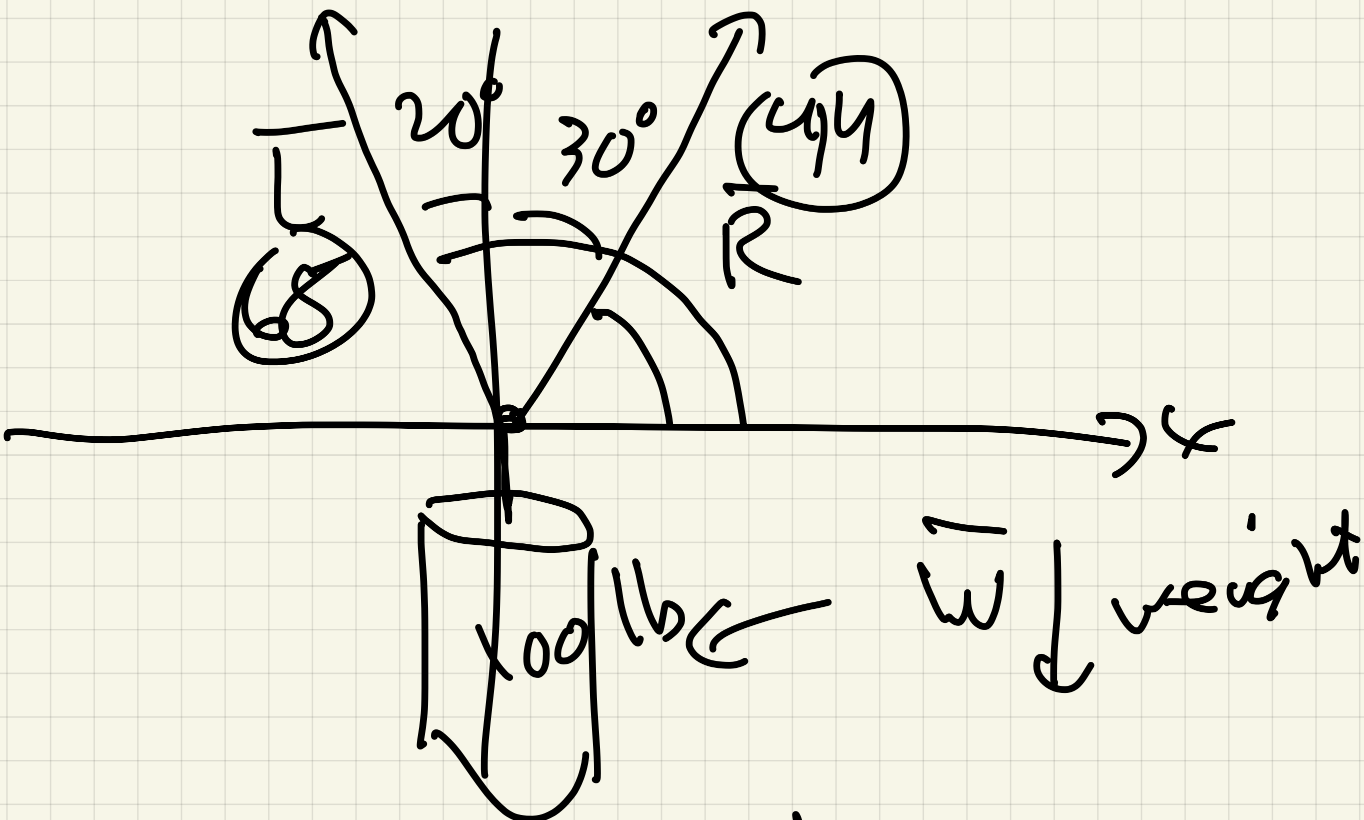
$\theta = \arctan \left(\frac{10\sqrt{2} + 15\sqrt{3}}{10\sqrt{2} - 15} \right) \approx$

-1.54 rad $= -88.78^\circ$ degs

$+180^\circ$

91.22°

Ex2 Two workers hold up a 100 lb weight as shown



How much force does each worker pull?

Key: $\bar{L} + \bar{R} + \bar{W} = 0$

$L = |\bar{L}|$, $R = |\bar{R}|$

$L\langle \cos 110^\circ, \sin 110^\circ\rangle + R\langle \cos 60, \sin 60\rangle$

$\vec{L}$

$+ \langle 0, -100\rangle = \langle 0, 0\rangle$

$\Downarrow$

① x comp: $L\cos 110^\circ + R\cos 60 = 0$

② y comp: $L\sin 110^\circ + R\sin 60^\circ = 100$

① $\Rightarrow$ $L = -\dfrac{R\cos 60}{\cos 110}$

② $-\dfrac{R\cos 60}{\cos 110}\sin 110 + R\sin 60 = 100$

$\Downarrow$

$$R = \frac{100}{\sin 60^\circ - \dfrac{\cos 60 \sin 110^\circ}{\cos 110^\circ}} \approx$$

44.698 lb

$L = 65$ lb

($R + L > 100$ expected)

§11.3 Dot product

Defn: $\bar{u} = \langle u_1, u_2, u_3 \rangle$

$\bar{v} = \langle v_1, v_2, v_3 \rangle$

Dot product is

$$\bar{u} \cdot \bar{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

Ex1 $\langle 1, -3, 2 \rangle \cdot \langle 3, 10, 8 \rangle$

(a) $3 - 30 + 16 = -11$

(b) $\langle 2, 3, 4 \rangle \cdot \langle 2, 3, 4 \rangle =$

$4 + 9 + 16 = 29$

(c) $\langle 3, 7, -4 \rangle \cdot \langle -1, 1, 1 \rangle = 0$

(d) $\langle 5,2\rangle \cdot \langle 8,-4\rangle = 32$

Properties:

① $\bar{u}\cdot\bar{v} = \bar{v}\cdot\bar{u}$

→ ② $\bar{u}\cdot(\bar{v}+\bar{w}) = \bar{u}\cdot\bar{v} + \bar{u}\cdot\bar{w}$

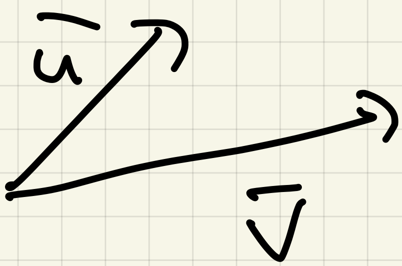
③ $k(\bar{u}\cdot\bar{v}) = (k\bar{u})\cdot\bar{v} =$

④ $\bar{u}\cdot(k\bar{v})$

⑤ $\bar{0}\cdot\bar{v} = \bar{0}$

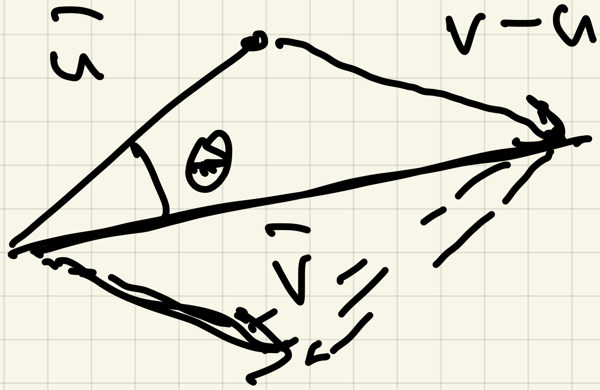
→ ⑥ $\bar{v}\cdot\bar{v} = |v|^2$

Theorem If θ is the angle between vectors $\bar{u}$ and $\bar{v}$,



Then

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$$



Law of cosines

$$|\bar{v}-\bar{u}|^2 = |u|^2 + |v|^2 - 2|u||v|\cos\theta$$

$$(\bar{v}-u)(\bar{v}-\bar{u})$$

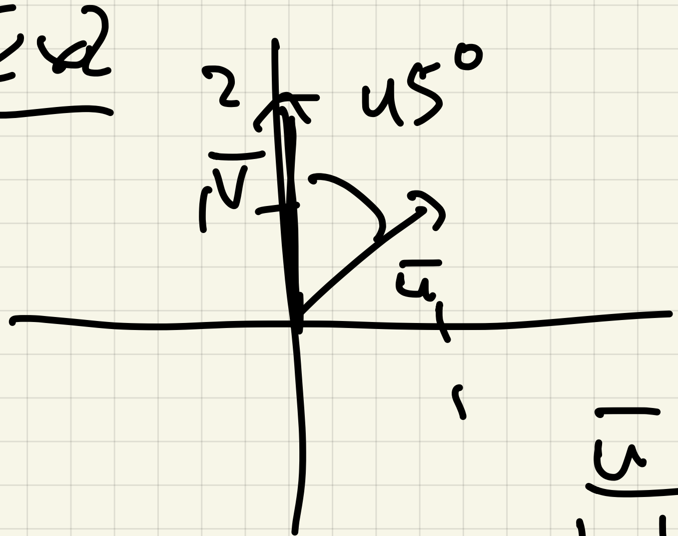
$$\bar{v}\cdot\bar{v} - \bar{v}\cdot\bar{u} - \bar{u}\cdot\bar{v} + \bar{u}\cdot\bar{u}$$

$$u\cdot u + \bar{v}\cdot\bar{v} - 2|u||v|\cos\theta$$

$$-2\bar{u}\cdot\bar{v} = -2|u||v|\cos\theta$$

$$\frac{\bar{u}\cdot\bar{v}}{|u||v|} = \cos\theta$$

Ex 2



$\bar{u} = \langle 1,1 \rangle$

$\bar{v} = \langle 0,2 \rangle$

$$\frac{\bar{u}\cdot\bar{v}}{|u||v|} = \frac{2}{\sqrt{2}\cdot 2} = \frac{1}{\sqrt{2}}$$

$= \cos 45°$