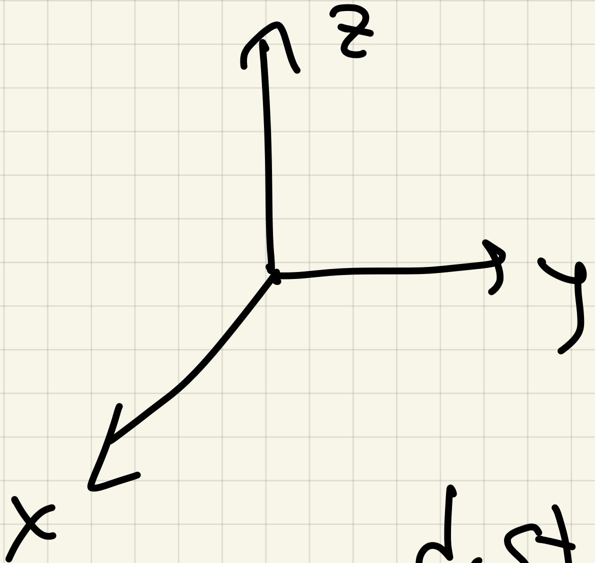
1/16 | Calc3

Tomorrow $\begin{cases} \text{Hw } 1 \\ \text{Q } 1 \end{cases}$

syllabus

Last time



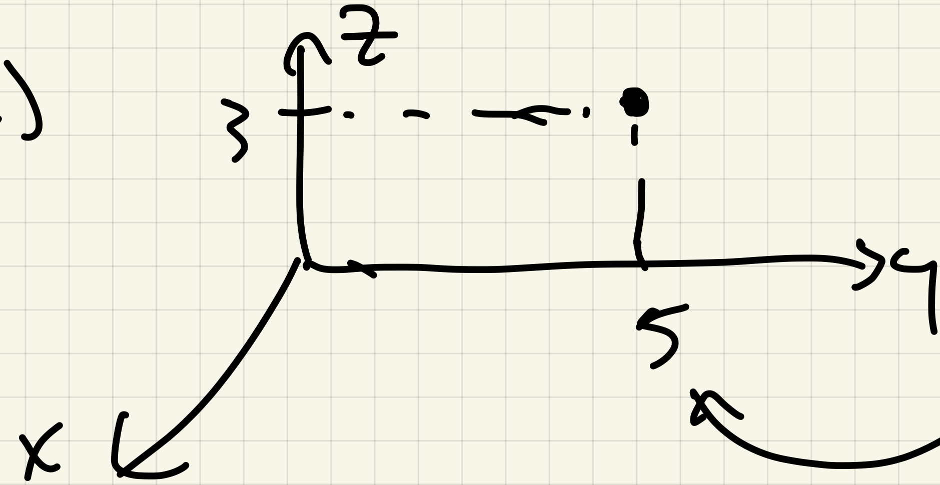
§11.1 understand 3D coords x/y/z

distance formula

Equation of sphere

Ex1 Give equations to describe the following

(a)



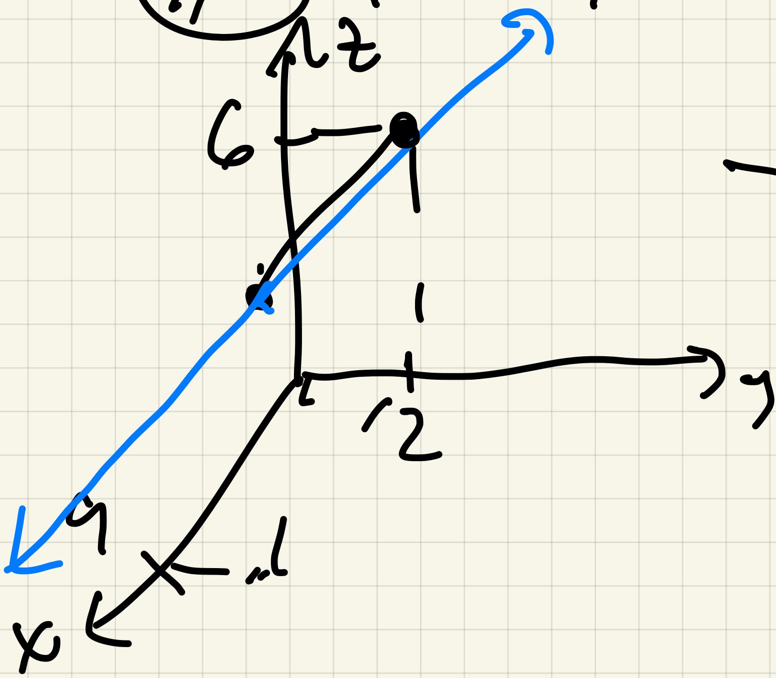
$x = 0$

$y = 5$

$z = 3$

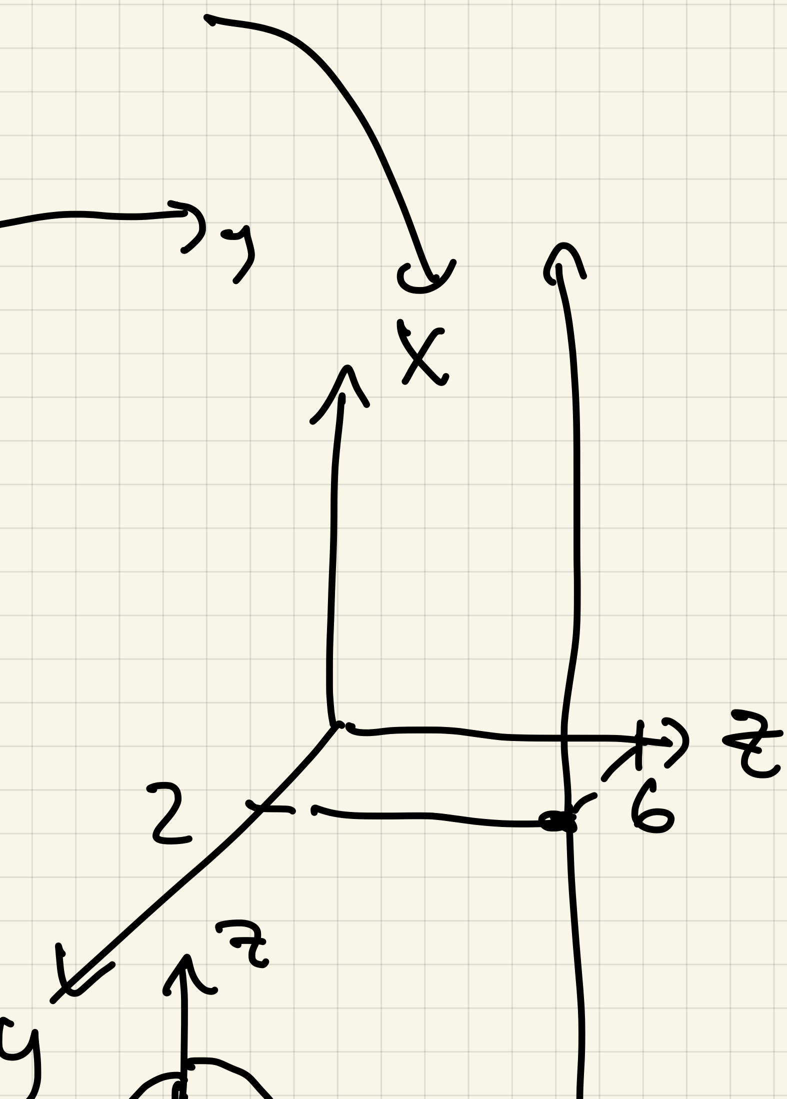
(b) The line through (9,2,6)

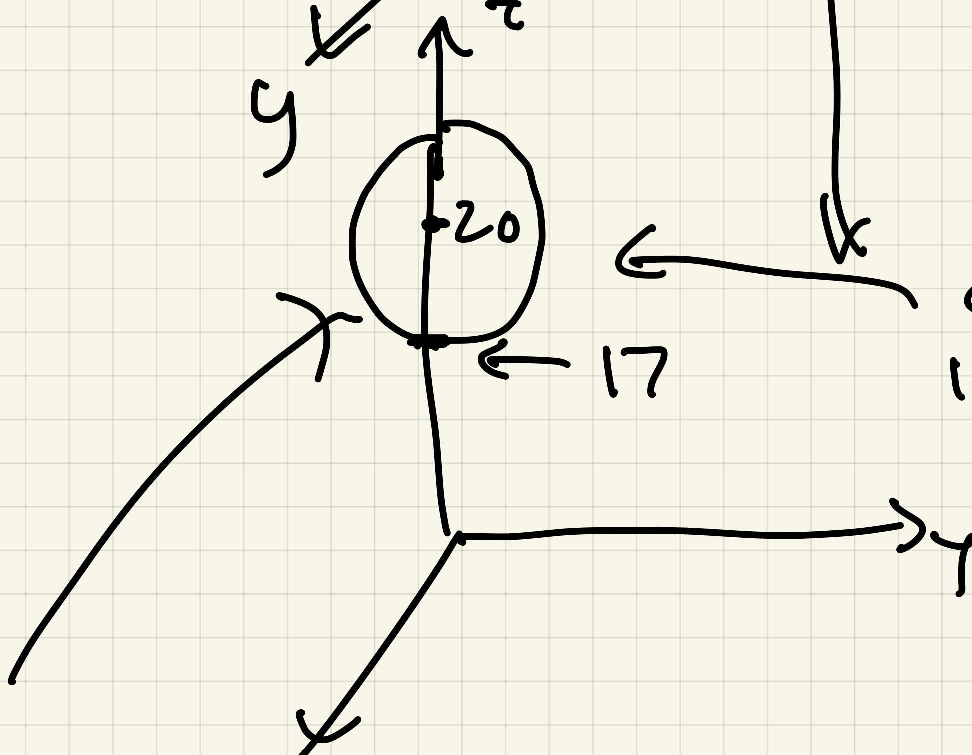
// to x-axis



$y = 2$

$z = 6$



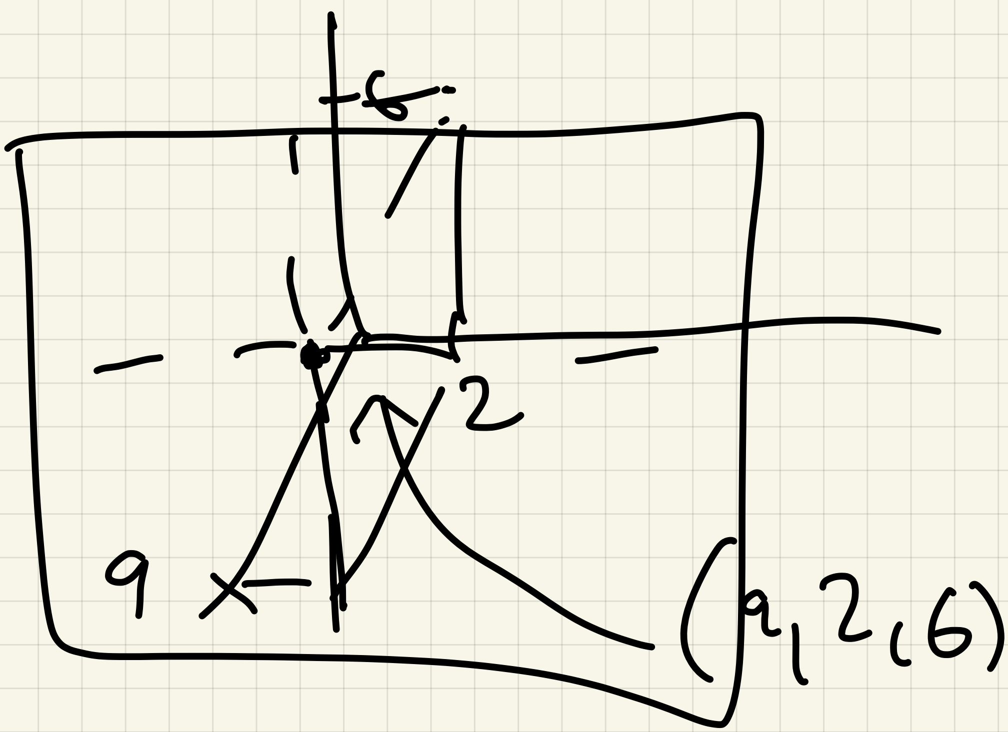


circle in yz-plane

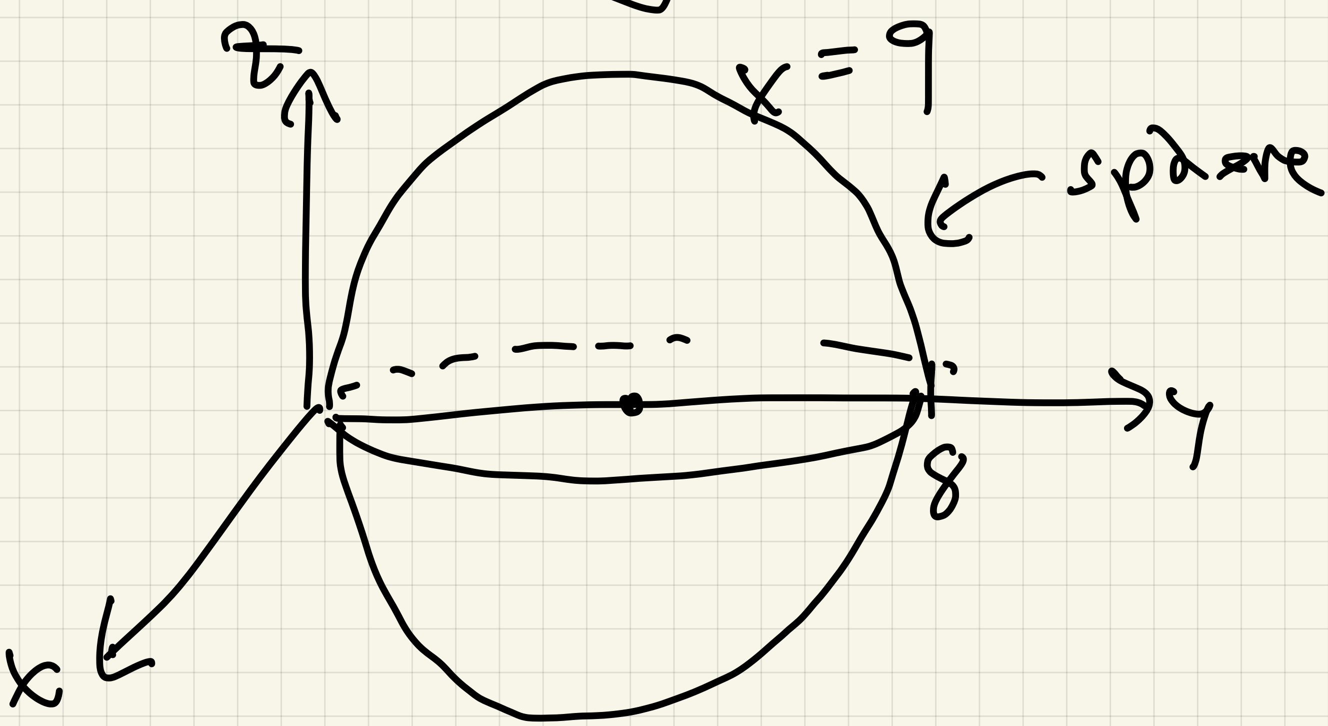
(c)

$$z=20, \; \overset{x}{y}=0 \;\rightarrow\; \begin{cases} y^2+(z-20)^2=9 \\ x=0 \end{cases}$$

(d) Plane $\perp$ to x-axis
contains the point $(9,2,6)$



(e)



center $(0,4,0)$

radius $= 4$

$$x^2+(y-4)^2+z^2=16$$

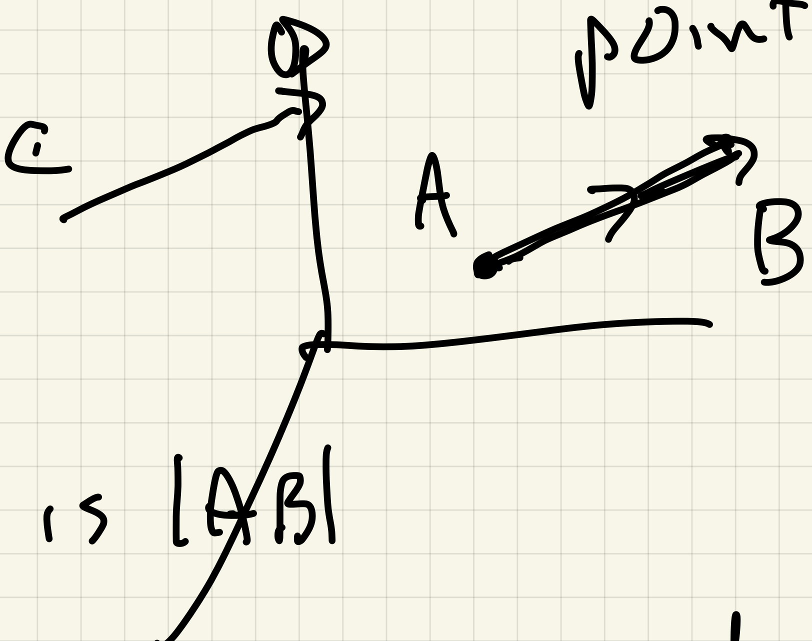
§11.2 Vectors:

A <u>vector</u> $\bar{v}$ is a directed line segment from

A to B

initial point

terminal point

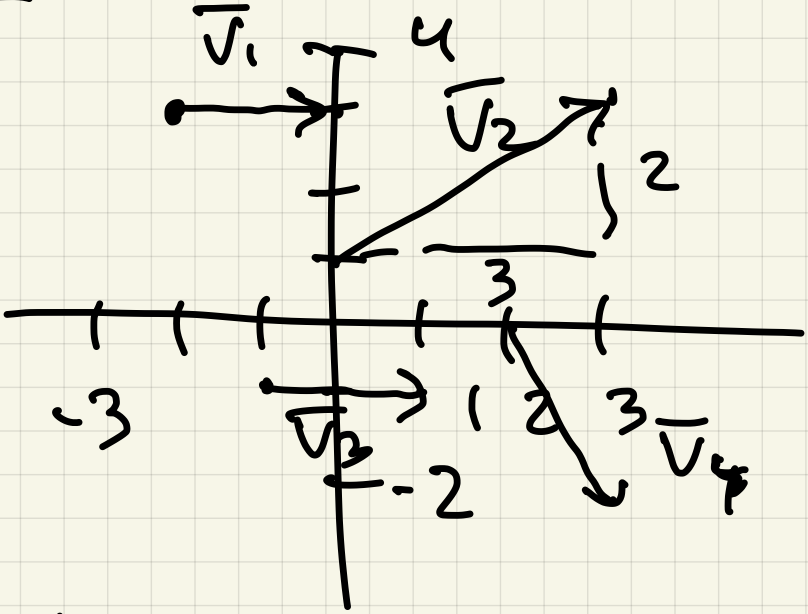


length is $|AB|$

Two vectors are <u>equal</u> if they have same length and direction

Notation: $\bar{v} = \overrightarrow{AB}$

Ex (a) in $\mathbb{R}^2$



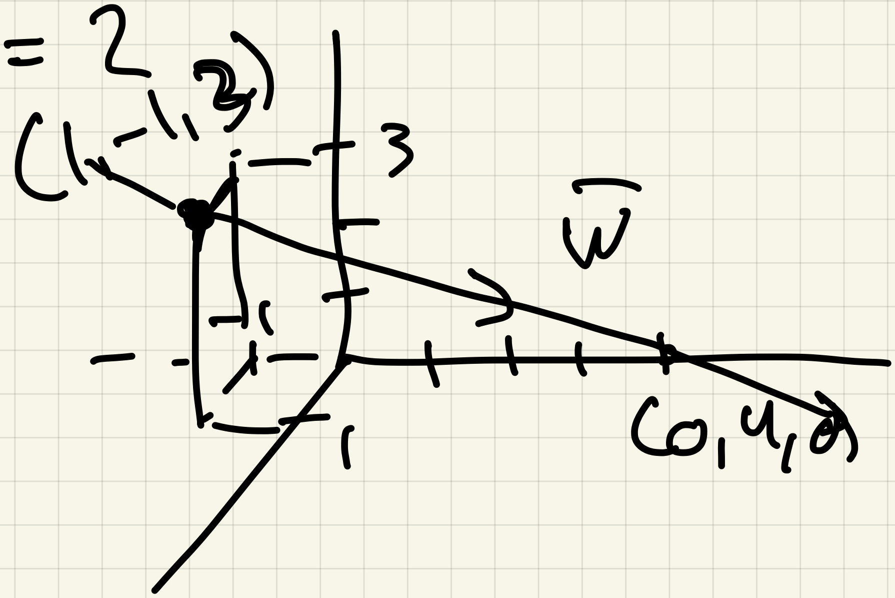
$|\bar{v}_1| = 2$ $\qquad$ $\bar{v}_1 = \bar{v}_3$

$|v_2| = \sqrt{13}$

$|v_4| = \sqrt{5}$

$|v_3| = 2$

(b)



Component form: Given vector $\bar{v}_1$, can find an equal vector with initial point the origin, the terminal point gives the components form

$$\bar{v}_1 = \langle a, b \rangle \quad (\mathbb{R}^2)$$
$$\bar{v}_1 = \langle a, b, c \rangle \quad (\mathbb{R}^3)$$

In Ex)

(a)

$$v_1 = \langle 2, 0 \rangle$$
$$v_2 = \langle 3, 2 \rangle$$
$$v_3 = \langle 2, 0 \rangle$$
$$v_4 = \langle 1, -2 \rangle$$

(b) $\bar{w} = \langle -1, 5, -3 \rangle$

In general: subtract coords for A from coords for B

Vector operations : $\mathbb{R}^2$

If $\bar{u} = \langle u_1, u_2 \rangle$, $\bar{v} = \langle v_1, v_2 \rangle$,
$k \in \mathbb{R}$

1. sum $\bar{u} + \bar{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$
2. difference $\bar{u} - \bar{v} = \langle u_1 - v_1, u_2 - v_2 \rangle$
3. scalar multiplication:

$$k \cdot \bar{u} = \langle k u_1, k u_2 \rangle$$

(Analogous in $\mathbb{R}^3$)

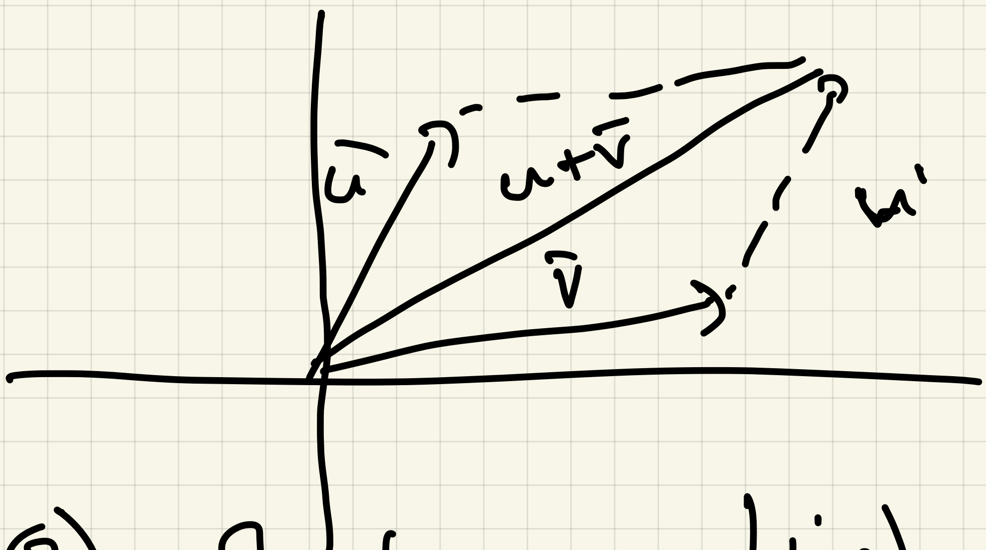
Ex 2

$$u = \langle 2, 3 \rangle$$
$$v = \langle 4, -7 \rangle$$
$$\bar{u} + \bar{v} = \langle 6, -4 \rangle$$
$$\bar{u} - \bar{v} = \langle -2, 10 \rangle$$
$$7\bar{u} = \langle 14, 21 \rangle$$
$$-8\bar{v} = \langle -32, 56 \rangle$$

Geometry :

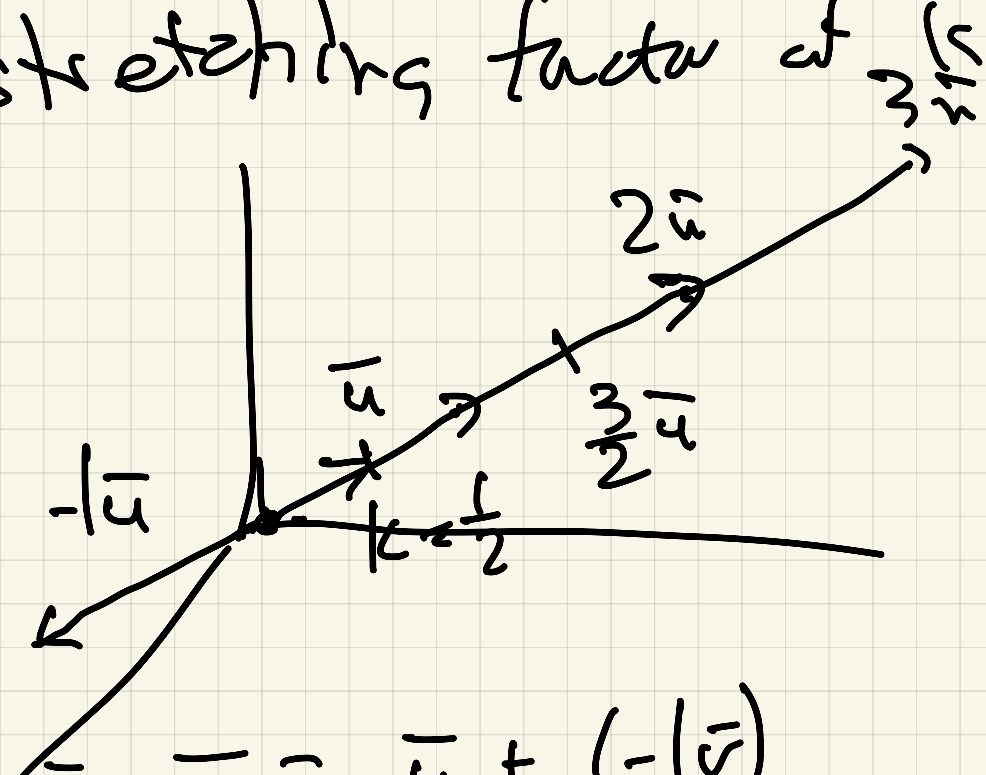
Visual interpretation:

① Addition "tip to tail"

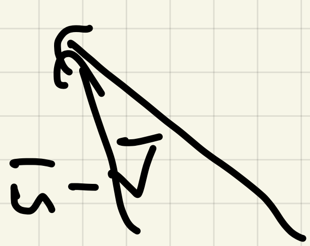


② Scalar multipl.: $k\bar u$

stretching factor of k

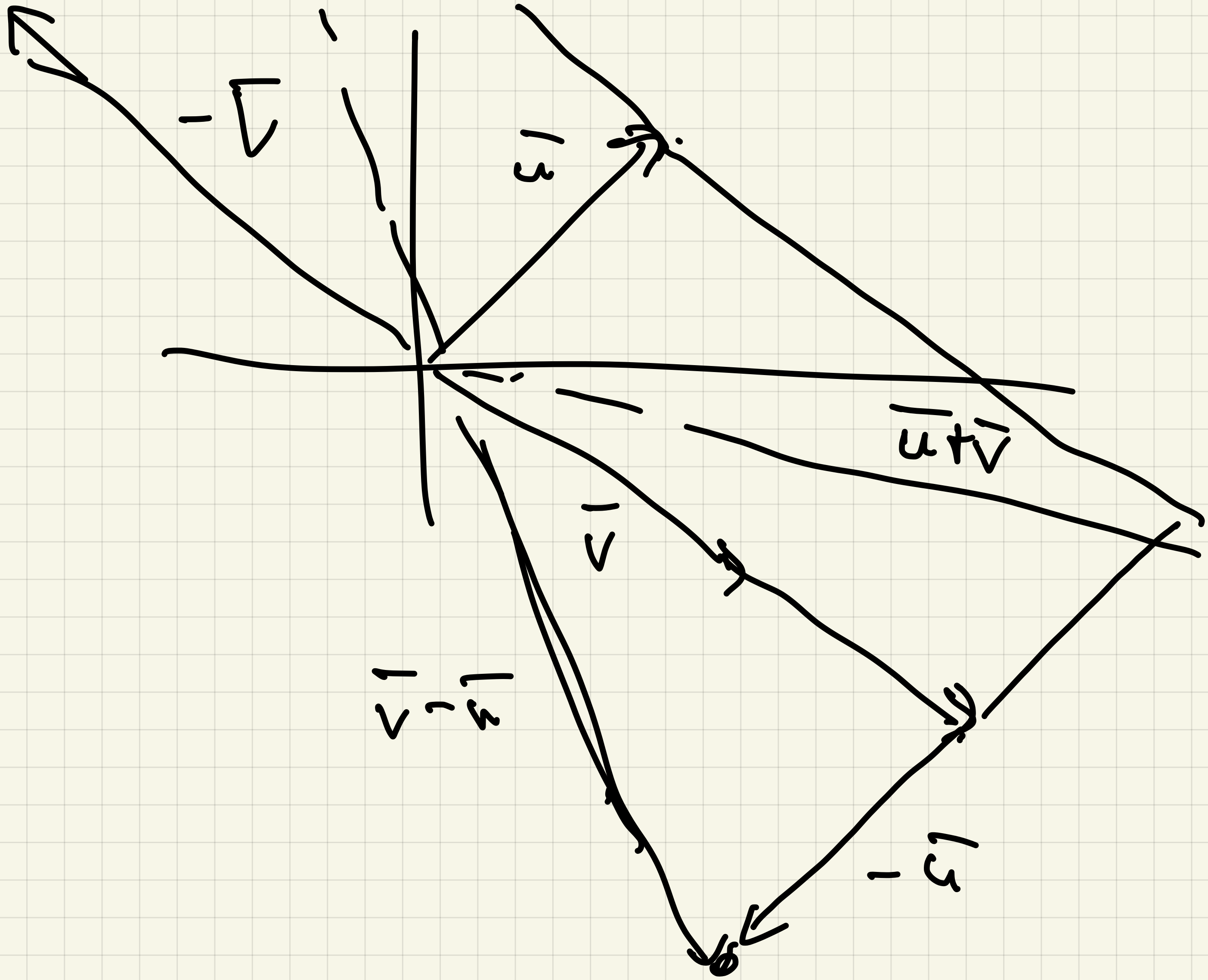


③ $\bar u - \bar v = \bar u + (-1\bar v)$



$\bar v - \bar u =$

$-(\bar u - \bar v)$



<u>Ex 3</u> For $\bar{u}, \bar{v}$ below:

a) Find components

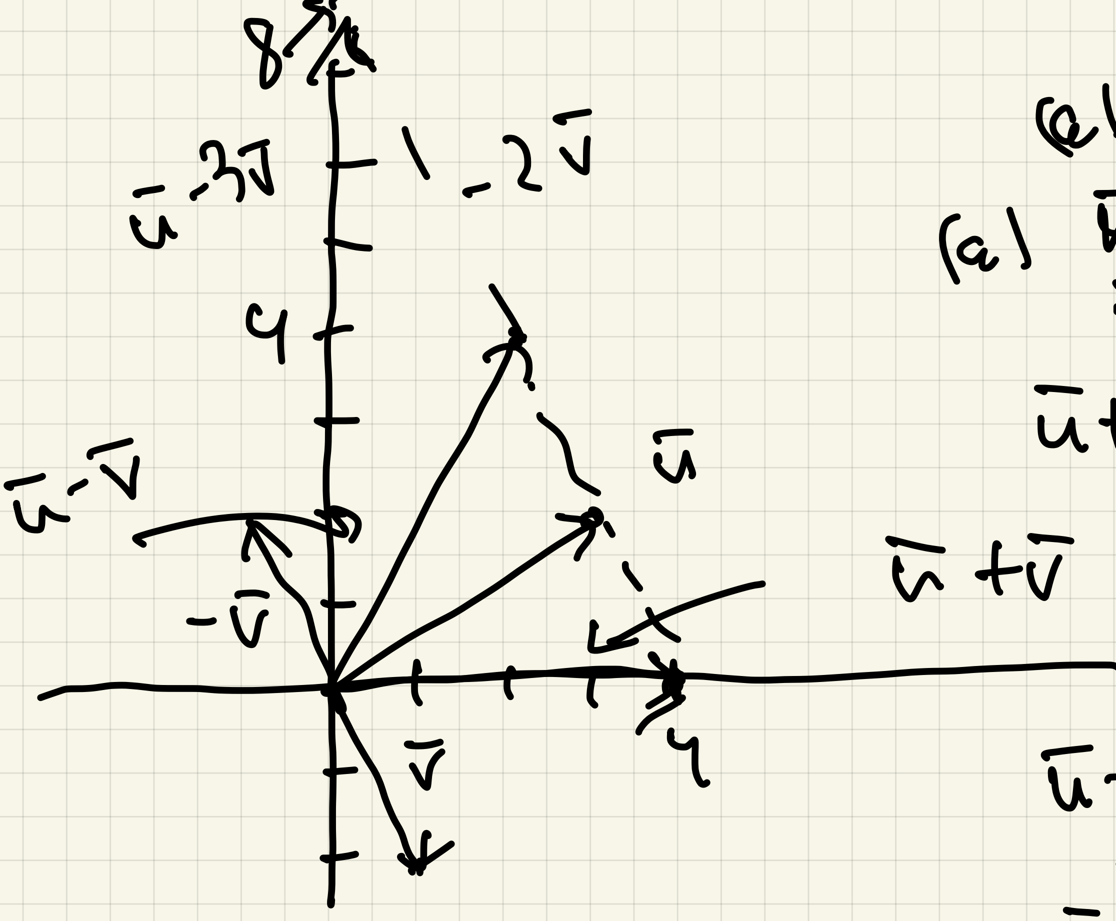
b) $\bar{u}+\bar{v}$

c) $\bar{u}-\bar{v}$

d) $-\bar{v}$

e) $\bar{u}-3\bar{v}$

f) sketch each



(a)

(a) $\bar{u} = \langle 3,2\rangle$
$\bar{v} = \langle 1,-2\rangle$

$\bar{u}+\bar{v} = \langle 4,0\rangle$

$\bar{u}-\bar{v} = \langle 2,4\rangle$

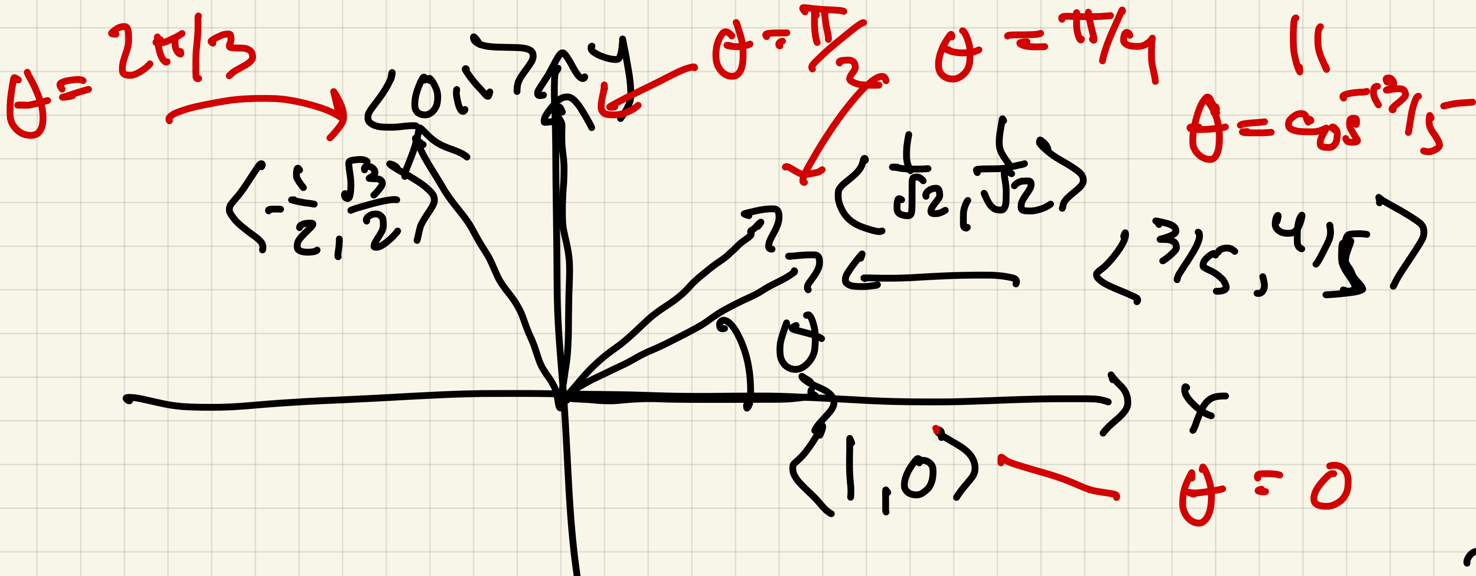
$-\bar{v} = \langle -1,2\rangle$

$\bar{u} - 3\bar{v} =$
$\langle 3,2\rangle - \langle 3,-6\rangle$
$= \langle 0,8\rangle$

Definition A unit vector has length 1.

Ex 4 In $\mathbb{R}^2$

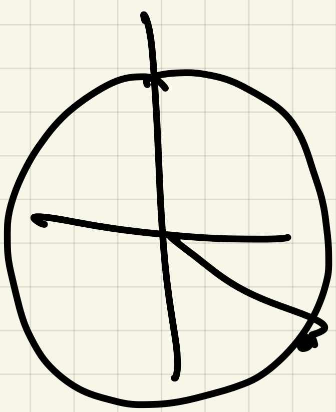
arctan $4/3$
$=$
arccos $3/5$
$=$



<u>Observe</u> a unit vector in $\mathbb{R}^2$ can be written

$$\bar{u} = \langle \cos\theta, \sin\theta\rangle,$$

$$0 \le \theta < 2\pi$$



$\leftarrow$ points on unit circle

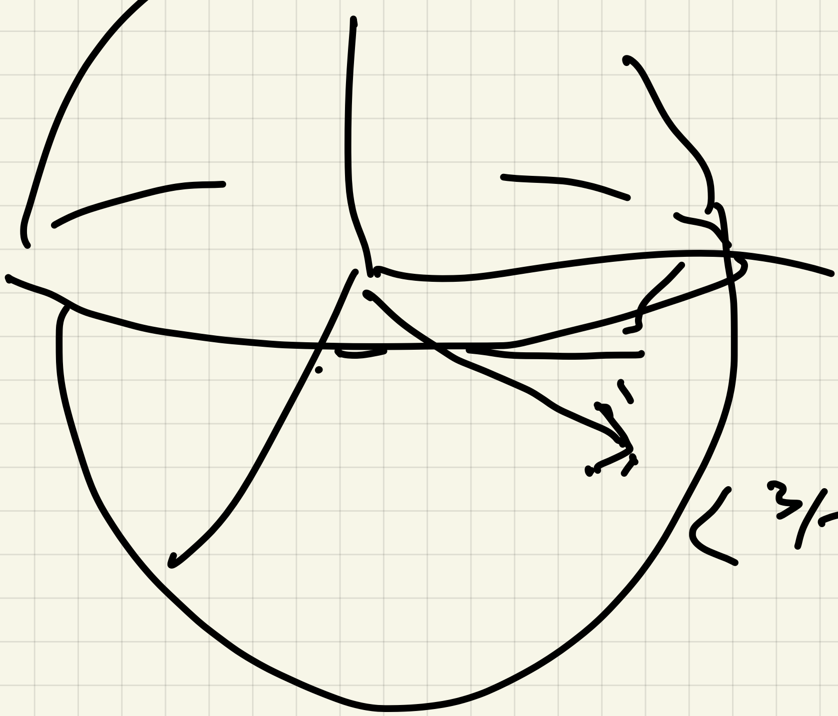
$\Updownarrow$

angles

$\Updownarrow$

unit vectors

<u>Ex)</u> $\bar{V} = \langle 3/7, 6/7, -2/7\rangle$



<u>Notation</u>: <u>Standard unit vectors</u>

$\mathbb{R}^2$	$\mathbb{R}^3$
$\hat{i} = \langle 1, 0 \rangle = e_1$	$\hat{i} = \langle 1, 0, 0 \rangle = e_1$
$\hat{j} = \langle 0, 1 \rangle = e_2$	$\hat{j} = \langle 0, 1, 0 \rangle = e_2$
	$k = \langle 0, 0, 1 \rangle = e_3$

<u>Note</u>: $\bar{v} = \langle a, b, c \rangle =$

$= a\hat{i} + b\hat{j} + c\hat{k}$

<u>The point</u>: unit vectors

$\Downarrow$

direction

$\left(\frac{0}{0}\right)$

Thm 11.2 c scalar

v vector

$$|c\cdot\bar{v}| = |c||\bar{v}|$$

abs value

length vector

Consequence:

Decomposition into length/direction

$\bar{v}\neq 0, \Rightarrow |\bar{v}|\neq 0 \Rightarrow$

$$\bar{v} = |\bar{v}|\cdot \frac{\bar{v}}{|v|}$$

length

unit vector (direction)

Thm 11.2: $\left|\frac{1}{|\bar{v}|}\bar{v}\right| = \left|\frac{1}{|\bar{v}|}\right|\cdot|\bar{v}|$

$= \frac{1}{|\bar{v}|} \cdot |v| = 1$ ✓

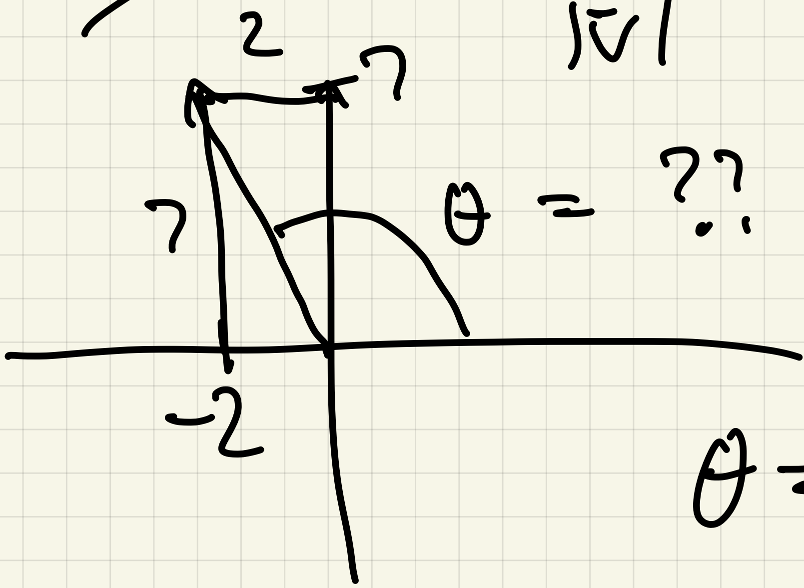
Ex 8

Decompose into length / direction:

(e) $\bar{v} = \langle -2, 7 \rangle = -2i + 7j$

length $|\bar{v}| = \sqrt{4 + 49} = \sqrt{53}$

direction $u = \frac{\bar{v}}{|\bar{v}|} = \langle -2/\sqrt{53}, 7/\sqrt{53} \rangle$



$\theta = \arctan -7/2$

no: use reference:

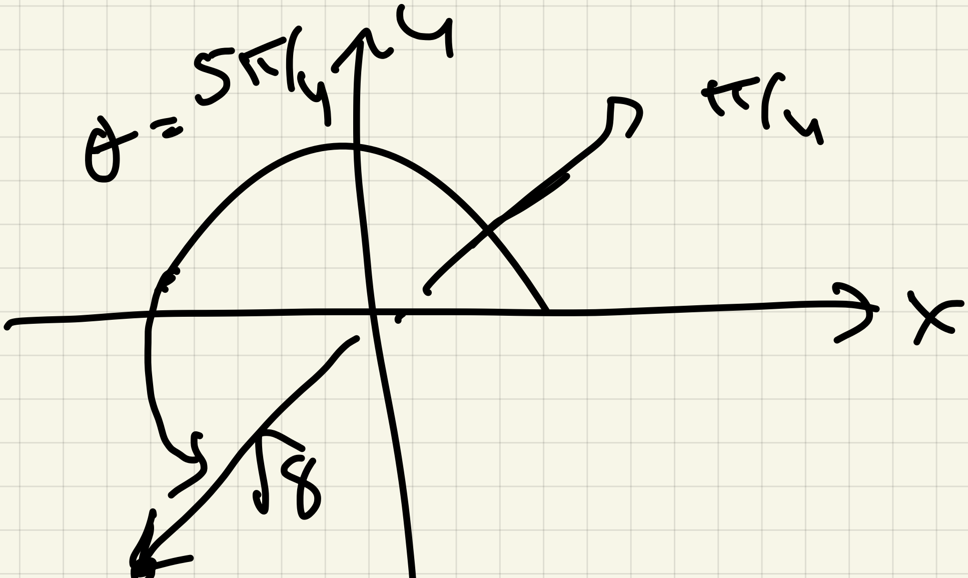
$\theta = \pi + \arctan -7/2$

Easier: $\theta = \arccos -2/\sqrt{53}$

$\theta = \arcsin \frac{7}{\sqrt{53}}$ ✗ <u>no</u>

$\theta = \pi - \arcsin \frac{7}{\sqrt{53}}$ yes

<u>Ex 9</u> Find the vector of length $\sqrt{8}$ making angle of $225^\circ = \frac{5\pi}{4}$ rads with positive x-axis



$$\bar{u} = |\bar{u}| \frac{\bar{u}}{|\bar{u}|}$$

$|\bar{u}| = \sqrt{8}$

$\frac{\bar{u}}{|\bar{u}|} = \langle \cos 225^\circ, \sin 225^\circ \rangle$

$$\sqrt{8} \cdot \langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$$

$=$

$$\left\langle -\frac{\sqrt{8}}{\sqrt{2}}, -\frac{\sqrt{8}}{\sqrt{2}} \right\rangle = \langle -2, -2 \rangle$$

Ex 10 $\quad \bar{u} = \langle 2, 3, 6 \rangle$ in $\mathbb{R}^3$

$$|\bar{u}| = \sqrt{2^2 + 3^2 + 6^2}$$

length $= \sqrt{4+9+36} = \sqrt{49} = 7$

direction: $\quad \frac{\bar{u}}{7} = \langle 2/7, 3/7, 6/7 \rangle$