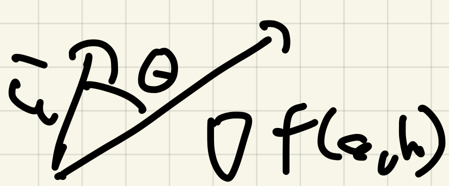
2/28 | Calc 3

Last time $z = f(x,y)$ function

$(a,b) \in \mathbb{R}^2$

Gradient $\nabla f(a,b) = \langle \frac{\partial f}{\partial x}(a,b), \frac{\partial f}{\partial y}(a,b) \rangle$

$\bar{u}$ = unit vector



(A) $D_u f(a,b) = \nabla f(a,b) \cdot \bar{u}$

$D_u f(a,b)$ = Directional deriv. of f in direction $\bar{u}$

$\nabla f(a,b) \cdot \bar{u} = |\nabla f(a,b)| \underline{\cos\theta}$

$\Downarrow$

(B) (1) $\nabla f(a,b)$ is direction of maximal increase

$\boxed{\theta = 0}$ $(D_u f(a,b) = |\nabla f(a,b)|)$

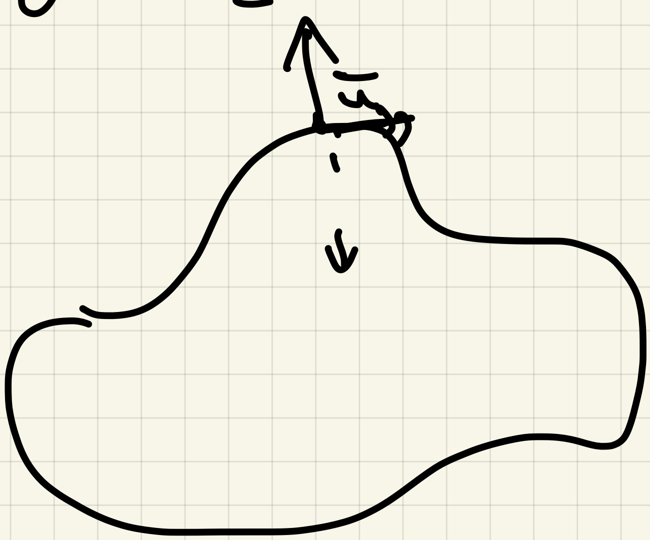
(2) $-\nabla f(a,b)$ is direction of maximal decrease

$\boxed{\theta = \pi}$ $\quad (D_u f(a,b) = -|\nabla f(a,b)|)$

(c) $\nabla f(a,b)$ is $\perp$ to level sets $f(x,y) = c = \text{constant}$

$$c = f(a,b)$$

$\theta = \pi/2$ $\quad D_u f(a,b) = 0$



$f(x,y) = c$

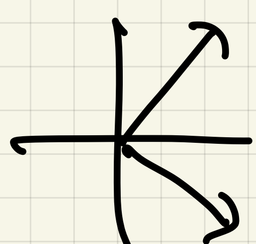
Ex1 $\quad f(x,y) = x^2 - 9y^2$

(a) $\nabla f(4,-1)$

(b) $D_u f(4,-1)$ in directions

$u_1 = \langle 1, 1 \rangle$

$u_2 = \langle 1, -1 \rangle$



(c) Direction of max incr/decr and value of $D_{\underline{u}} f(4,-1)$

(d) Sketch level set $f(x,y) = 7$ and $\nabla f(4,-1)$

Find tangent line to level curve at $(4,-1)$

(a) $\nabla f(x,y) = \langle 2x, -18y \rangle$

$\nabla f(4,-1) = \langle 8, 18 \rangle$

(b) $u = \langle 1, 1 \rangle$

$D_u f(4,-1) = \langle 8, 18 \rangle \cdot \dfrac{\langle 1, 1 \rangle}{\sqrt{2}} = \dfrac{26}{\sqrt{2}}$

$u = \langle 1, -1 \rangle$

$D_u f(4,-1) = \langle 8, 18 \rangle \cdot \dfrac{\langle 1, -1 \rangle}{\sqrt{2}} = \dfrac{-10}{\sqrt{2}}$

(c)
direction max incr

$$\nabla f(4,-1) = \frac{\langle 8,18\rangle}{|\langle 8,18\rangle|} =$$

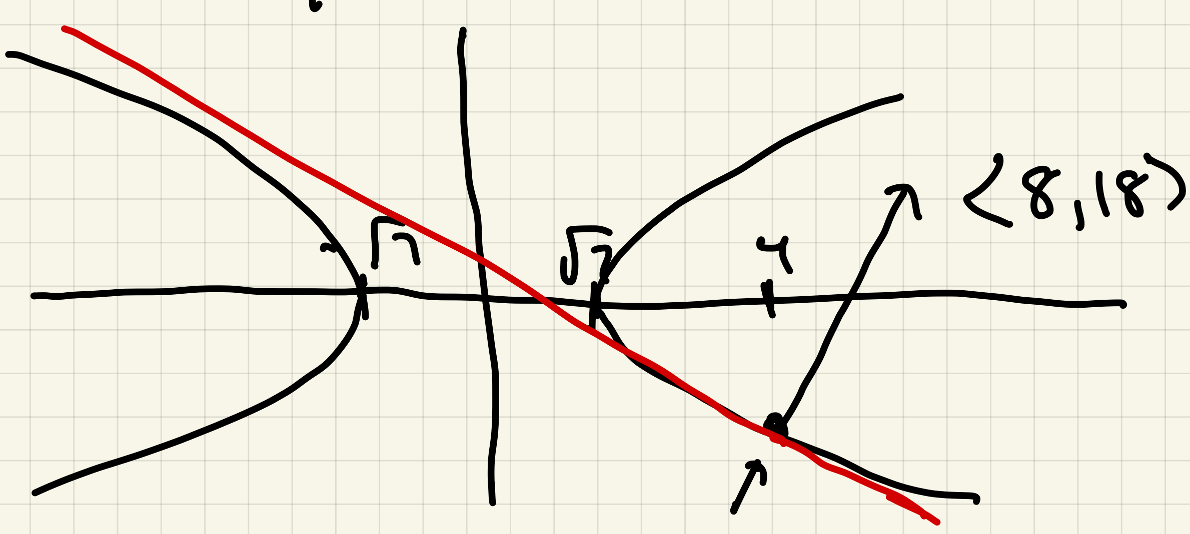
$$\frac{\langle 8,18\rangle}{2|\langle 4,9\rangle|} = \frac{\langle 8,18\rangle}{2\sqrt{97}}$$

and actual max rate of incr is $2\sqrt{97}$

dir max decr. is $-\frac{\langle 8,18\rangle}{2\sqrt{97}}$

max decr is $-2\sqrt{97}$

(d) $x^2 - 9y^2 = 7$ $(4,-1)$



(4,−1)

(x,y)

hyperbola:

$$\langle (x,y) - \langle 4,-1\rangle \rangle \perp \langle 8,18\rangle$$

$$\langle x-4, y+1\rangle \cdot \langle 8,18\rangle = 0$$

$$8(x-4) + 18(y+1) = 0$$

In general: If (a,b) is on level curve $f(x,y) = c$, then the tangent line to curve at (a,b) has

$$f_x(a,b)(x-a) + f_y(a,b)(y-b) = 0$$

Fact: (A) (B) (C) all

ok in 3D

$$w = f(x,y,z)$$

$$\nabla f = \langle \partial f/\partial x, \ \partial f/\partial y, \ \partial f/\partial z \rangle$$

$$D_u f(a,b,c) = \nabla f(a,b,c) \cdot \bar{u}$$

$$= |\nabla f(a,b,c)| \cos\theta$$

<u>Ex2</u> $w = f(x,y,z) = \underline{\underline{xe^{yz} - 2}}$

at $\underline{\underline{(2,0,-4)}}$

$$\nabla f(x,y,z) = \langle e^{yz}, xze^{yz}, xye^{yz} \rangle$$

$$\nabla f(2,0,-4) = \langle 1, -8, 0 \rangle$$

Directional derivative of f in direction $u = \langle 1,2,3 \rangle$ is

$$\langle 1,-8,0 \rangle \cdot \frac{\langle 1,2,3 \rangle}{\sqrt{14}} = \frac{-15}{\sqrt{14}}$$

(c) direction of max incr

is $\dfrac{\langle 1, -8, 0\rangle}{\sqrt{65}}$

Ex3 Find <u>normal vectors</u> to

surface $z = \underline{\underline{25 - x^2 - y^2}}$

at $(0,0,25)$, $\langle 0,3,16\rangle$

$\langle 0, -5, 0\rangle$

constant

$$g(x,y,z) = \boxed{x^2 + y^2 + z = 25}$$

$$x^2 + y^2 + z - 25 = 0$$

$$\nabla g = \langle 2x, 2y, 1\rangle$$

$$\nabla g\,(0,0,25) = \langle 0,0,1\rangle$$

$$\nabla g\,(0,3,16) = \langle 0,6,1\rangle$$

$$\nabla g\,(0,-5,0) = \langle 0,-10,1\rangle$$