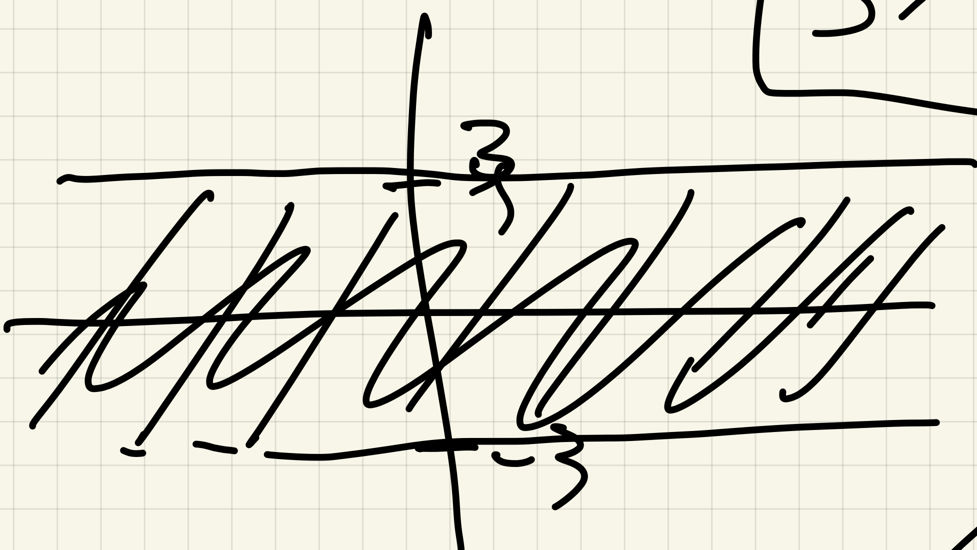
2/25 | Calc3 Quiz 8 avg 75

$$z = f(x,y) = \cancel{-}\sqrt{9-y^2}$$

a) $9 - y^2 \geq 0$ $\quad 9 \geq y^2$

$$\boxed{3 \geq |y|}$$

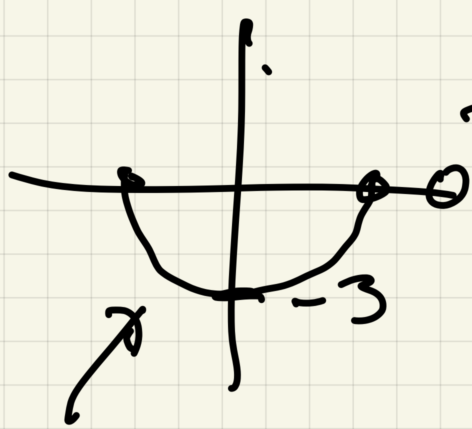


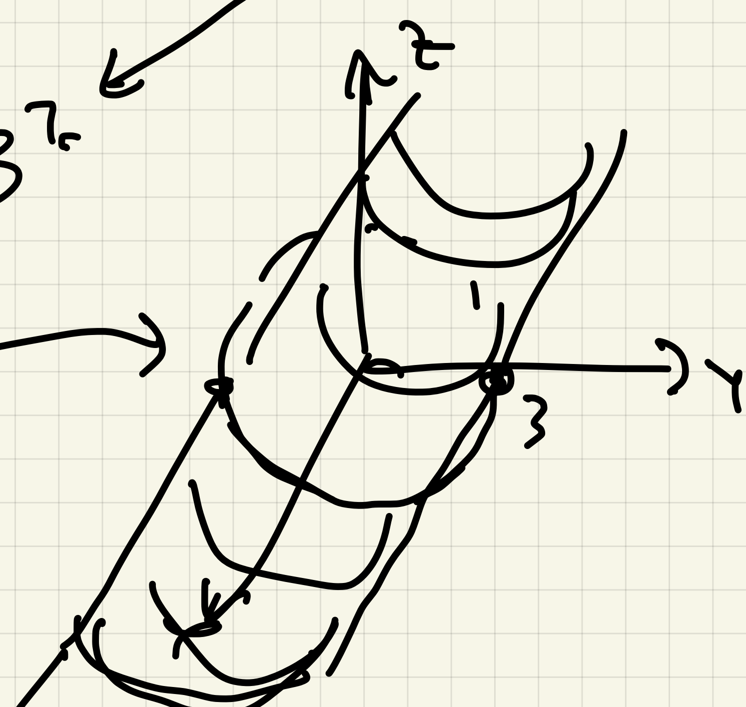
b) $z \leq 0$

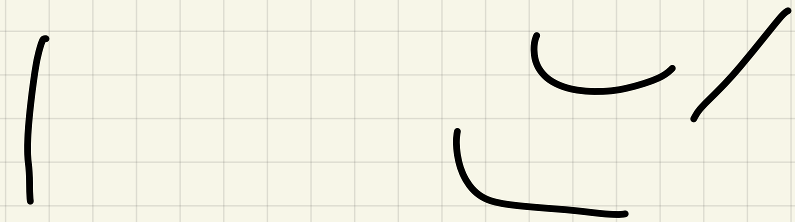
$$z^2 = 9 - y^2$$

$$z^2 + y^2 = 9 = 3^2$$

c)







$-3 \le z \le 0$ $\quad [-3, 0]$

range

§ 13.4 Chain Rule

If $z = f(x,y)$, $\quad x = g(t)$
$\qquad\qquad\qquad\quad y = h(t)$

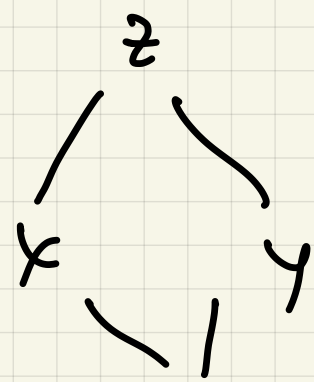
$$\Downarrow$$

$$z = f(x,y) = f(\underset{x}{g(t)}, \underset{y}{h(t)})$$

a function of t:

Chain rule: $$\underbrace{\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}}$$

Dependency diagram



t

<u>Ex1</u>: $z = x^2 + xy + y^3$

$\boxed{x = t^2,}$ $y = t^3$

(a) Find $\frac{dz}{dt}$ with chain rule

(b) check answer by substitution

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt} =$$

$$= (2x+y)(2t) + (x+3y^2)(3t^2)$$

$$= \underbrace{(2t^2+t^3)(2t)} + \underbrace{(t^2+3t^6)(3t^2)}$$

$$= 4t^3 + \underbrace{2t^4 + 3t^4} + 9t^8$$

$$= 4t^3 + 5t^4 + 9t^8$$

<u>Check</u>: $z = x^2 + xy + y^3$

$x = t^2$
$y = t^3$

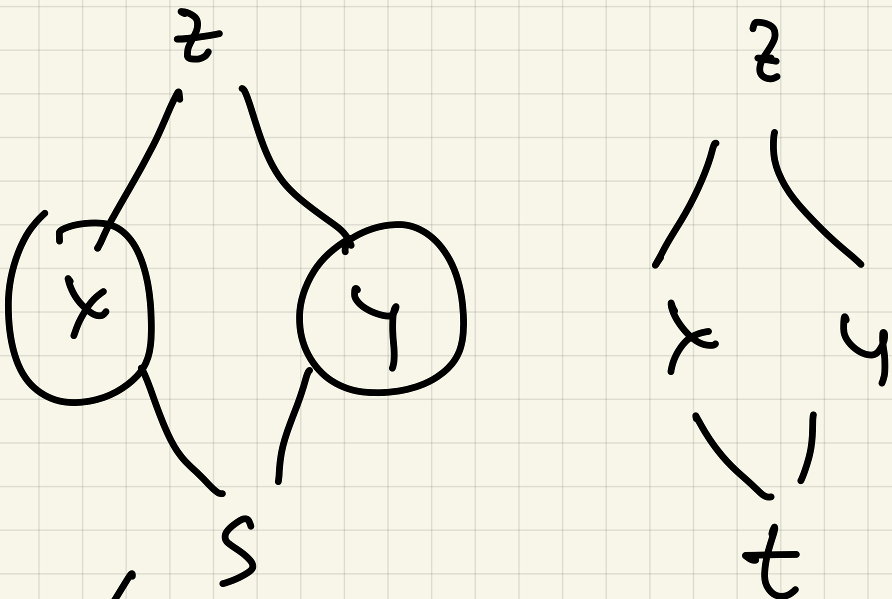
$= \quad z = t^4 + t^5 + t^9$ ✓

$\Rightarrow \quad \frac{dz}{dt} = 4t^3 + 5t^4 + 9t^8$

<u>Chain rule</u>:

$z = f(x, y), \quad x = g(s, t)$

$\qquad\qquad\qquad\; y = h(s, t)$



- $\partial z/\partial s = \partial z/\partial x \cdot \partial x/\partial s + \partial z/\partial y \cdot \partial y/\partial s$

- $z_t = z_x x_t + z_y y_t$

<u>Ex 2</u> $\qquad (\arctan x)' = \frac{1}{1+x^2}$

$$z = \arctan\left(\frac{y}{x}\right)$$

$$\boxed{x = r\cos\theta} \qquad \left(\begin{array}{l} s = r \\ t = \theta \end{array}\right)$$

$$\boxed{y = r\sin\theta}$$

Find $\partial z/\partial r$, $\partial z/\partial \theta$

$$\partial z/\partial r = \underbrace{\partial z/\partial x}\ \underbrace{\partial x/\partial r} + \partial z/\partial y\ \partial y/\partial r$$

$$\frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(\frac{-y}{x^2}\right)\cos\theta +$$

$y x^{-1}$

$$\frac{1}{1+\left(\frac{y}{x}\right)^2}\left(\frac{1}{x}\right)\sin\theta =$$

$$\frac{-y}{x^2+y^2}\cos\theta + \frac{x}{x^2+y^2}\sin\theta$$

$$\frac{-r\sin\theta}{}\cos\theta + \frac{r\cos\theta}{}\sin\theta$$

r^2 $\qquad$ r^2

$= 0$!

$$\partial z/\partial\theta = \underbrace{\partial z/\partial x}\cdot \partial x/\partial\theta + \partial z/\partial y \cdot \partial y/\partial\theta$$

$$= \frac{-y}{\cancel{x^2+y^2}}(-r\sin\theta) + \frac{x}{x^2+y^2}\, r\cos\theta$$

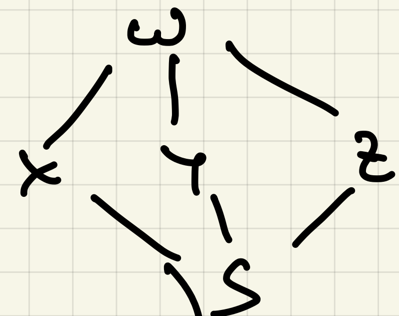
$$= \frac{r^2\sin^2\theta}{r^2} + \frac{r^2\cos^2\theta}{r^2} = \frac{r^2}{r^2} = 1 \;!$$

Why simple answers?

$$z = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}\right)$$

$$= \arctan(\tan\theta) = \theta$$

$$\boxed{z = \theta}$$



<u>Chain Rule</u>: $w = k(x, y, z)$

x, y, z functions of s, t

$\Downarrow$

w is a function of s, t,

$$w_s = w_x \cdot x_s + w_y \cdot y_s + w_z \cdot z_s$$

$$w_t = w_x x_t + w_y y_t + w_z z_t$$

Implicit differentiation

Calc1 $y = \sqrt{25 - x^2}$ $\Rightarrow$ explicit

$$\frac{1}{\cancel{2}}(25 - x^2)^{-1/2} \cdot (-\cancel{2}x) = \frac{-x}{\sqrt{25 - x^2}}$$

Also: $y^2 = 25 - x^2$ $\leftarrow$ diff wrt x

$$y^2 + x^2 = 25$$

Think of $y = f(x)$

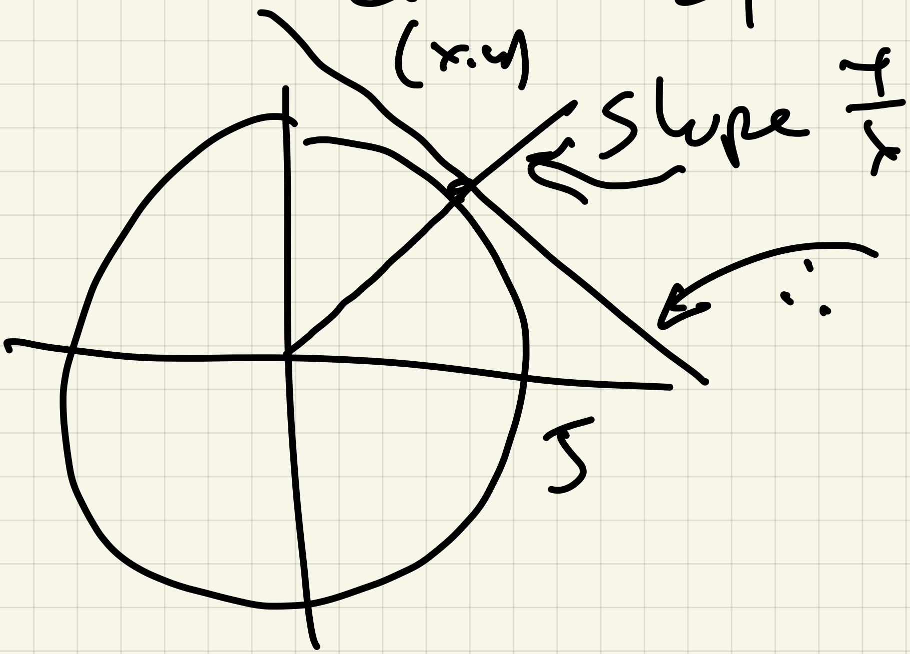
$$\Downarrow$$

$$2y \cdot \frac{dy}{dx} + 2x = 0$$

$$\Downarrow$$

$$2y\,\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y} \checkmark$$



slope $= -\frac{x}{y}$

Same ideas apply here:

<u>Ex D</u> We computed earlier that

$$\boxed{z = \sqrt{9-(x+1)^2-(y-2)^2}}$$

then $\partial z/\partial x\,(0,0) = -1/2$ ✓

$\partial z/\partial y\,(0,0) = 1$

(Visualized with a picture)

half sphere:

$(x+1)^2 + (y-2)^2 + z^2 = 3^2$

Ex Compute $\partial z/\partial x\,(0,0)$,

$\partial z/\partial y\,(0,0)$

implicitly!

$(x+1)^2 + (y-2)^2 + z^2 = 3^2$

Apply $\partial/\partial x$ $\quad 2(x+1)' + 0 + 2z \cdot \frac{\partial z}{\partial x} = 0$

$$\Downarrow$$

$$\partial z/\partial x = \frac{-2(x+1)}{2z} = -\frac{(x+1)}{z}\Big|_{(0,0)} =$$

$$= -\frac{1}{2}$$

$x = 0, y = 0$

$z = 2$

$\partial/\partial y$: $0 + 2(y-2)' + 2z\frac{\partial z}{\partial y} = 0 \Rightarrow$

$$\partial z / \partial y = \frac{-\cancel{x}(y-2)}{\cancel{x} z} = \frac{-(y-2)}{z}\Bigg|_{\substack{(0,0) \\ z=2}} =$$

$$\frac{-(-2)}{2} = 1 \checkmark$$

<u>Ex2</u> If $x^3 + xy + z^2 + yz^3 - 4 = 0$ defines a function $z = f(x,y)$ near $(1,1,1)$

find $\partial z / \partial x$ & $\partial z / \partial y$ at $(1,1,1)$

$$\underline{x}^3 + \underline{xy} + \underline{z}^2 + \boxed{y \cdot z^3} - 4 = 0$$

$$\partial / \partial x : 3x^2 + y + 2z \, \partial z / \partial x + y \, 3z^2 \cdot \frac{\partial z}{\partial x} = 0$$

$$\partial z / \partial x = \frac{-3x^2 - y}{2z + 3yz^2}\Bigg|_{(1,1,1)} = \frac{-4}{5}$$

$$\frac{\partial}{\partial y}: \; 0 + x + 2z\underline{\underline{\frac{\partial z}{\partial y}}} + 1 \cdot z^3 + y \cdot 3z^2 \underline{\underline{\frac{\partial z}{\partial y}}} = 0$$

$$\text{so } \; \partial z/\partial y = \left.\frac{-x - z^3}{2z + 3yz^2}\right|_{(1,1,1)} = \frac{-2}{5}$$