2/20/ Calc3 Quiz 7

1. $\bar{r}(t) = \langle 8t - t^3, \underline{4e^{5t+2}}, \underline{3\sin 2t} \rangle$

$\bar{v} = \bar{r}'(t) = \langle 8 - 3t^2, 20e^{5t+2}, 6\underline{\cos 2t} \rangle$

$\bar{a} = \bar{r}''(t) = \langle -6t, 100e^{5t+2}, -12\sin 2t \rangle$

2. $R'(t) = \bar{r}(t)$

$R(t) = \int \bar{r}(t)\,dx$

$\langle 4t^2 - \frac{t^4}{4}, \frac{4}{5}e^{8t+2}, -\frac{3}{2}\cos 2t \rangle + \bar{C}$

$R(0) = \langle 0, 0, 0 \rangle$

$\langle 0, \frac{4}{5}e^2, -\frac{3}{2} \rangle + \bar{C}$

$\bar{C} = \langle 0, -\frac{4}{5}e^2, +\frac{3}{2} \rangle$

$R(t) = \langle 4t^2 - \frac{t^4}{4}, \frac{4}{5}e^{5t+2} - \frac{4}{5}e^2, \frac{3}{2} - \frac{3}{2}\cos 2t \rangle$

13.2

Last time $z = f(x,y)$

Domain / graph / range

Limits:

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

Easy limits:

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt{e^{\tan(3\cos x + x^2)}} + \arcsin\left(\frac{y^2+1}{2}\right)}{x^2+y^2-50}$$

$$= \frac{\sqrt{e^{\tan 3}} + \pi/6}{-50}$$

Non-obvious limits

$$\boxed{\text{Ex 1} \quad \lim_{(x,y)\to(0,0)} \frac{2x+3y}{x+y} \quad \text{DNE}}$$

Ex 2

$$\lim_{(x,y)\to(0,0)} \frac{3x^2+3y^2+x^4-y^4}{x^2+y^2}$$

$$"$$

$$\lim_{(x,y)\to(0,0)} \frac{3+x^2-y^2}{1} = 3$$

Ex 3

$$\lim_{(x,y)\to(0,0)} \frac{\sin 4(x^2+y^4)}{x^2+y^4}$$

$$u = x^2+y^4$$

$$\| \quad \text{LH}$$

$$\lim_{u\to 0} \frac{\sin 4u}{u} =$$

$$\lim_{u\to 0} \frac{4\cos 4u}{1} = 4$$

Ex 4 $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

//

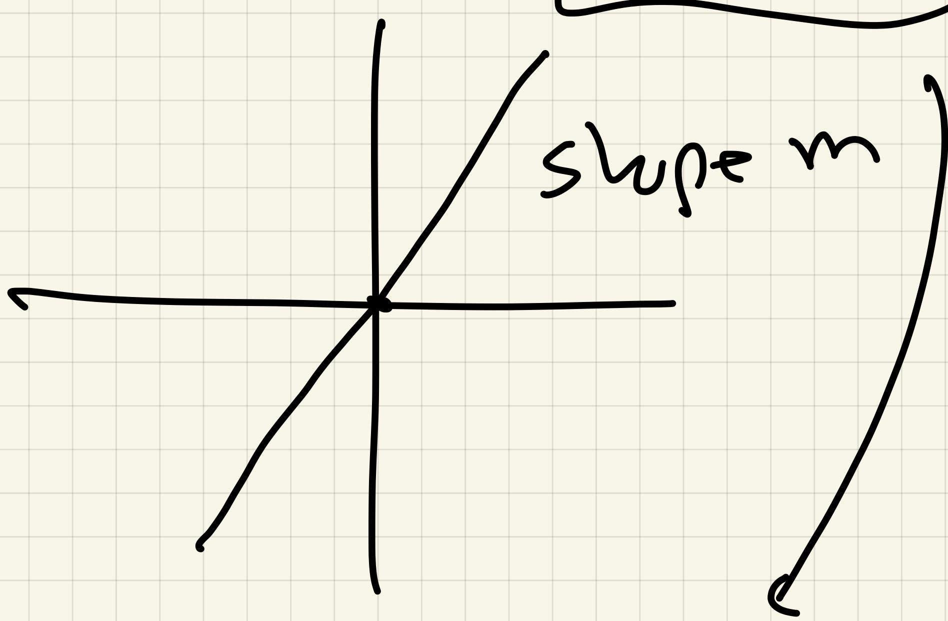
limit along x-axis: $y=0$

$$\lim_{x\to 0} \frac{0}{x^2} = 0$$

limit along y-axis: $x=0$

$$\lim_{y\to 0} \frac{0}{y^2} = 0$$

limit
Along line $\boxed{y = mx}$, m = slope



$$\lim_{x \to 0} \frac{m x^2}{x^2 + (mx)^2} \qquad \frac{xy}{x^2+y^2}$$

$\shortparallel \longleftarrow$ cancel x^2

$$\lim_{x \to 0} \frac{m}{1+m^2} = \frac{m}{1+m^2} \neq 0$$

if $m \neq 0$

So limit DNE

Ex 5 $\lim_{(x,y) \to (0,0)} \frac{x^2 y}{x^4 + y^2}$

Find limit

(a) along x-axis

(b) along y-axis

(c) along line $y = mx$

(d) along parabola.

(a) Set $y = 0$ $\quad \lim_{x \to 0} \frac{0}{x^4} = 0$

(b) $x = 0$: $\lim_{y \to 0} \frac{0}{y^2} = 0$

(c) $\lim_{x \to 0} \frac{x^2 \cdot mx}{x^4 + (mx)^2} =$

Set $y = mx$

$$\lim_{x \to 0} \frac{mx^3}{x^4 + m^2 x^2} \overset{LH}{=}$$

$$\lim_{x \to 0} \frac{3mx^2}{4x^3 + 2m^2 x} \overset{LH}{=}$$

$$\lim_{x \to 0} \frac{6mx}{12x^2 + 2m^2} = \frac{0}{2m^2} = 0$$

(d) Along parabola $\boxed{y = x^2}$

$$\lim_{x\to 0} \frac{x^2 y}{x^4+y^2} = \lim_{x\to 0} \frac{x^2 \cdot x^2}{x^4+x^4} =$$

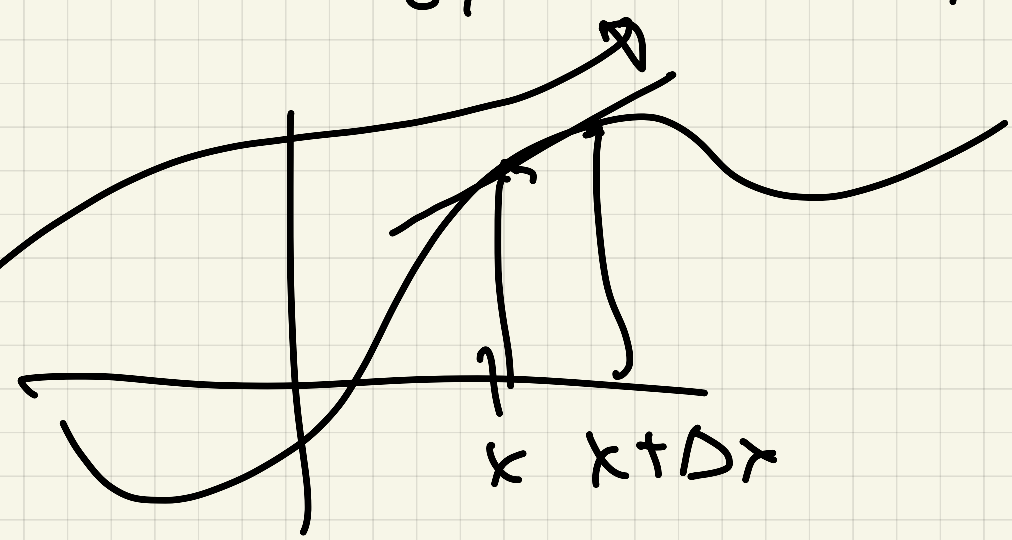
$$\lim_{x\to 0} \frac{x^4}{2x^4} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2} \,!$$

lim <u>DNE</u>

13.3

<u>Calc I</u>:

$$\frac{df}{dx} = \frac{dy}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



<u>Calc 3</u>: <u>Partial derivatives</u>

$$z = f(x,y)$$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} = \partial f/_{\partial x} = \partial z/_{\partial x} =$$

$$// \quad f_x = z_x$$

Partial derivative of f

with respect to x

$$\lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y} = \partial f/_{\partial y} = \partial z/_{\partial y}$$

$$f_y = z_y$$

<u>Easy to compute:</u>

<u>Ex</u> $f(x,y) = \underline{x}^2 + \underline{x^3 y}^4 + \frac{x}{y} + y^2$

$$\partial f/_{\partial x} = 2x + 3y^4x^2 + \frac{1}{y} + 0$$

$$\partial f/\partial y = 0 + 4x^3y^3 + \frac{-x}{y^2} + 2y$$

<u>Ex2</u>

$$f = x\,\ln y + e^{x^2y} + 7\sin(y^5 + 2x)$$

$$\partial f/\partial x = \ln y + 2xy\,e^{x^2y} + 2\cdot 7\cdot\cos(y^5+2x)$$

$5y^4$

$$\partial f/\partial y = \frac{x}{y} + x^2e^{x^2y} + 35y^4\cos(y^5+2x)$$

<u>Interpretation:</u>

<u>Calc</u>: $\frac{dy}{dx}$ = slope of tangent line to graph

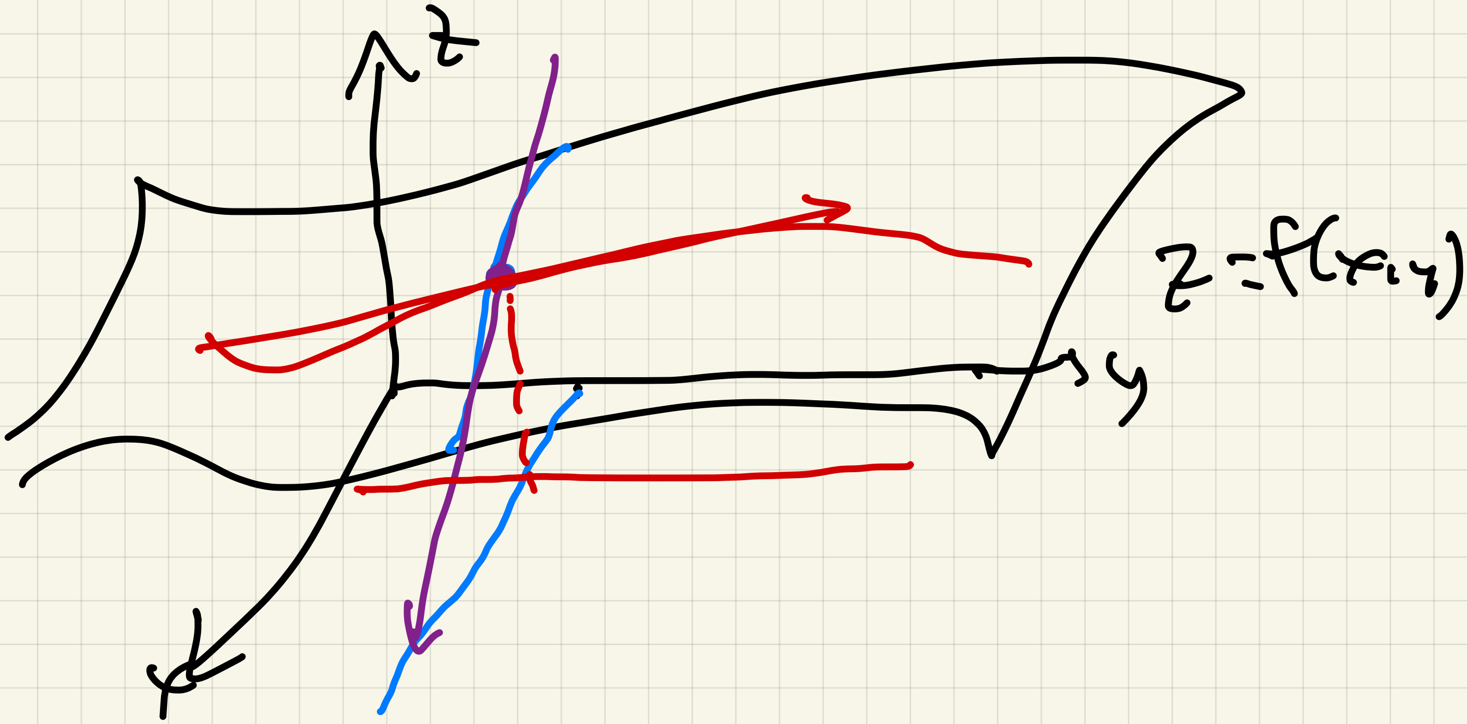
$z = f(x,y) \Rightarrow$

$\partial z/\partial x$ = slope of tangent line to y-trace of $z = f(x,y) =$

rate of change of z in the x-direction,

$\partial z / \partial y$ = slope of tangent line to ∂-trace of $z = f(x,y)$ =

rate of change of z in the y-direction



<u>Ex 3</u> Concrete example

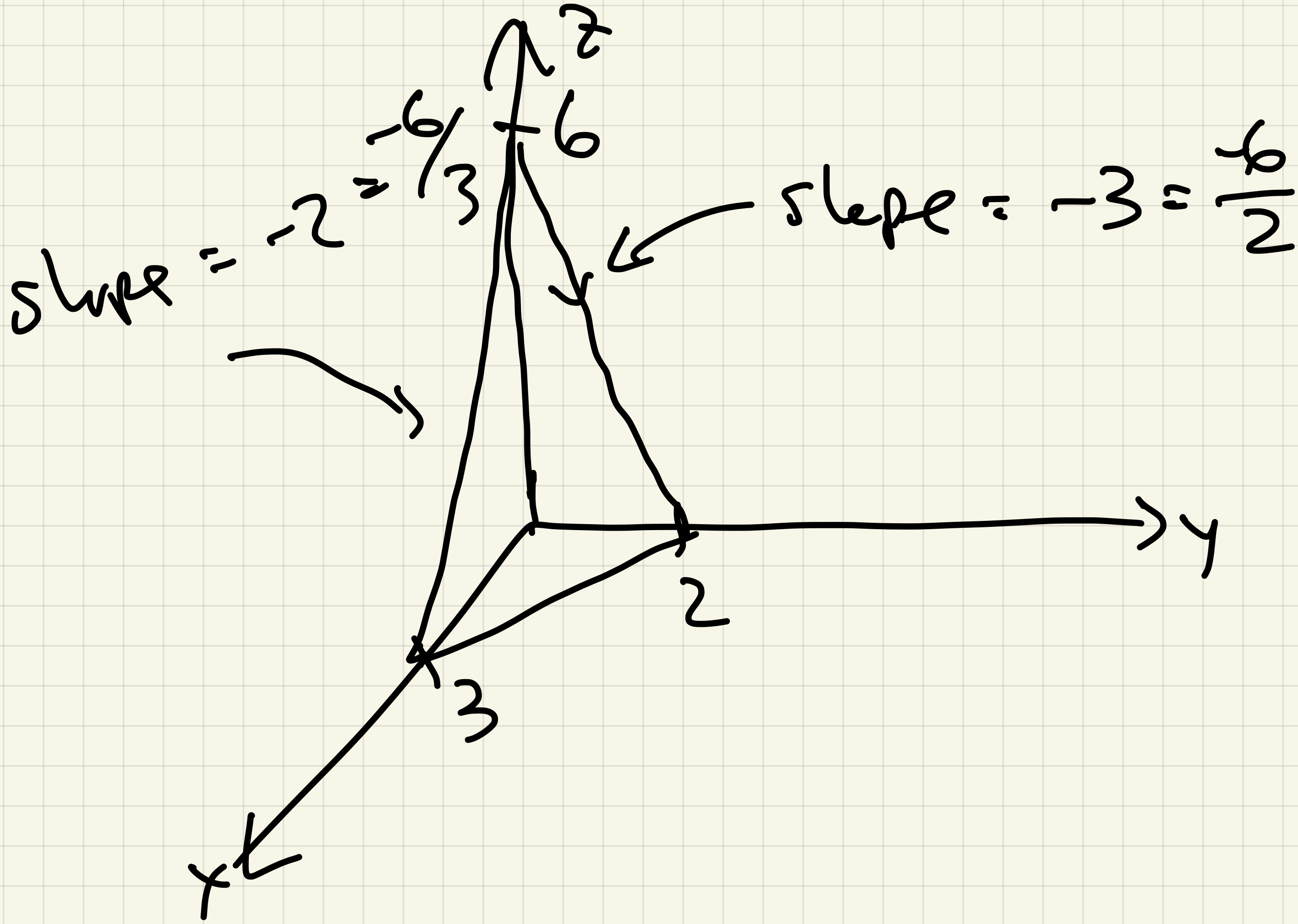
$$z = f(x,y) = 6 - 2x - 3y$$

<u>Sketch</u> surface, compute

$\partial z/\partial x$ and $\partial z/\partial y$ at $(0,0)$

$$\boxed{z = 6 - 2x - 3y}$$

$2x + 3y + z = 6$



$\partial z/\partial x = -2$

$\partial z/\partial y = -3$

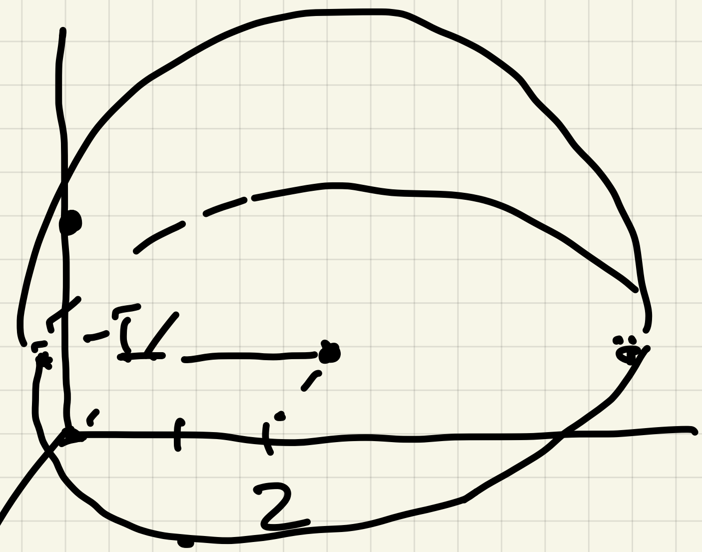
Ex4 Same for

$$z = f(x,y) = \underbrace{\sqrt{9-(x+1)^2-(y-2)^2}}$$

sketch:

$$z^2 = 9-(x+1)^2-(y-2)^2$$

$$(x+1)^2+(y-2)^2+z^2 = 3^2$$



$z \geq 0$

Visually, $\partial z/\partial y > 0$

$\partial z/\partial x < 0$

$$\partial z/\partial x = \frac{1}{2}\frac{-2(x+1)}{\sqrt{9-(x+1)^2-(y-2)^2}}\Bigg|_{(0,0)} = \frac{-2}{2\sqrt{4}} = -\frac{1}{2}$$

$$\partial z/\partial y = \frac{1}{\cancel{2}} \left. \frac{-\cancel{2}(y-2)}{\sqrt{9-(x+1)^2-(y-2)^2}} \right|_{(0,0)} = \frac{-(-2)}{\sqrt{4}} =$$

+1 > 0

Ex 5 Find ~~first~~ $\boxed{\text{first}}$ partial derivatives of

$$w = h(x,y,z) = \underline{3x^2y} - 5y\cos z + \underline{ze^{xy}} + \ln(xy)$$

$$\ln(xy) = \ln x + \ln y$$

$$\partial h/\partial x = 6xy + zye^{xy} + \frac{\cancel{y}}{x\cancel{y}}$$

$$\partial h/\partial y = 3x^2 - 5\cos z + zxe^{xy} + \frac{1}{y}$$

$$\partial h/\partial z = 5y\sin z + e^{xy}$$