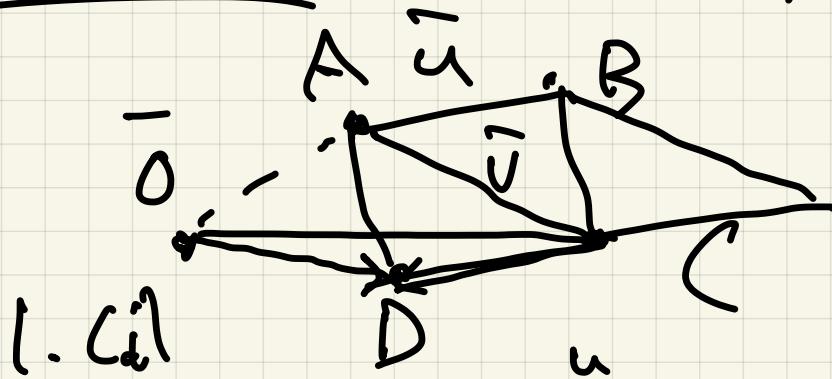


2/14/ Calc 3 Exam

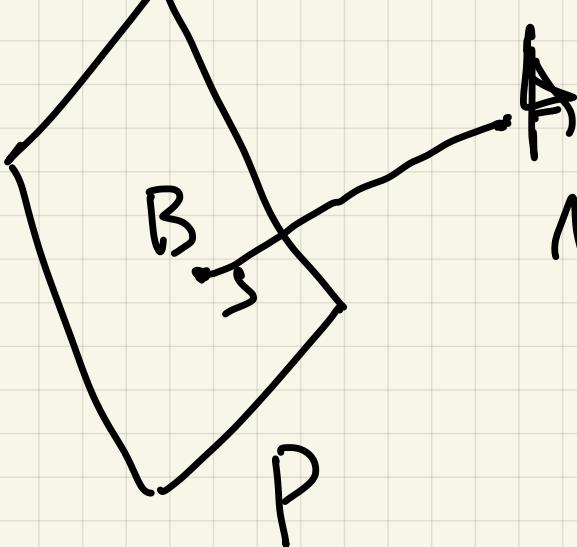


$$\overrightarrow{OC} = \overrightarrow{AB}$$

$$\langle 1, -5, 4 \rangle$$

$$\frac{\overrightarrow{OA} + \overrightarrow{AC} - \overrightarrow{AB}}{\sqrt{}}$$

2 (e)



$$l = (s_1 - 3, 11)$$

13.1 functions of 2 variables

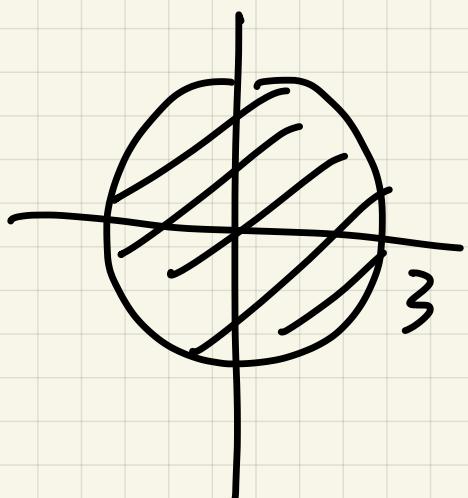
$$z = f(x, y)$$

Domain / range / graph

$$\text{Ex} \quad z = f(x, y) = \sqrt{9 - x^2 - y^2}$$

domain  $9 - x^2 - y^2 \geq 0$

$$9 \geq x^2 + y^2$$

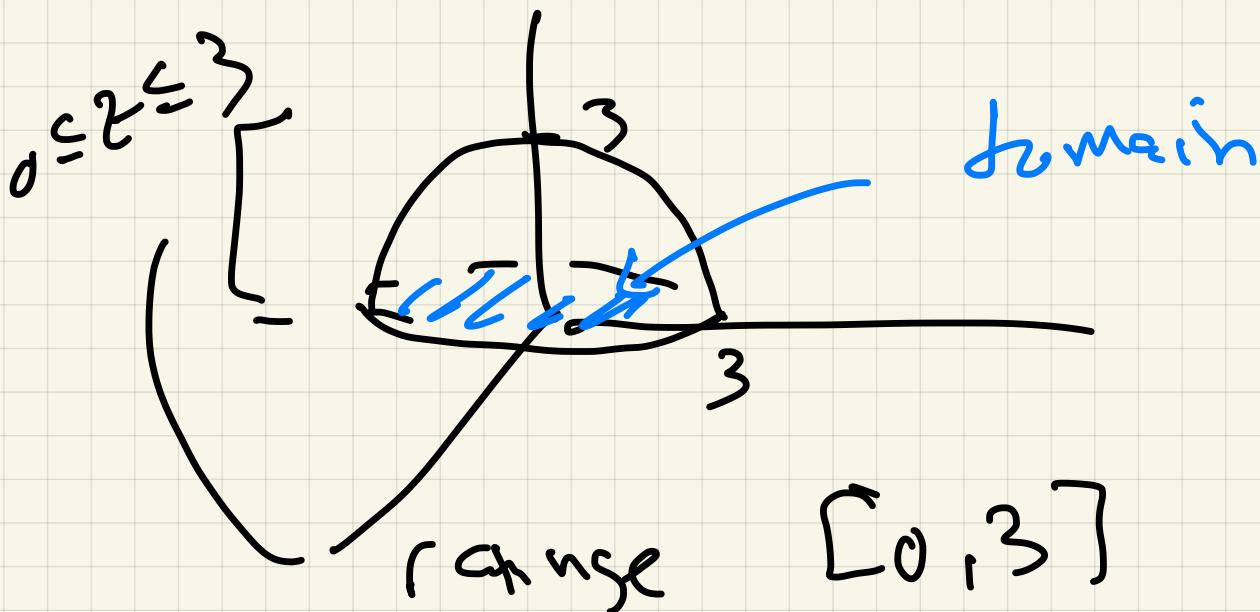


Graph

$$z = \sqrt{9 - x^2 - y^2}$$

$$z^2 = 9 - x^2 - y^2$$

$$x^2 + y^2 + z^2 = 9, \quad z \geq 0$$

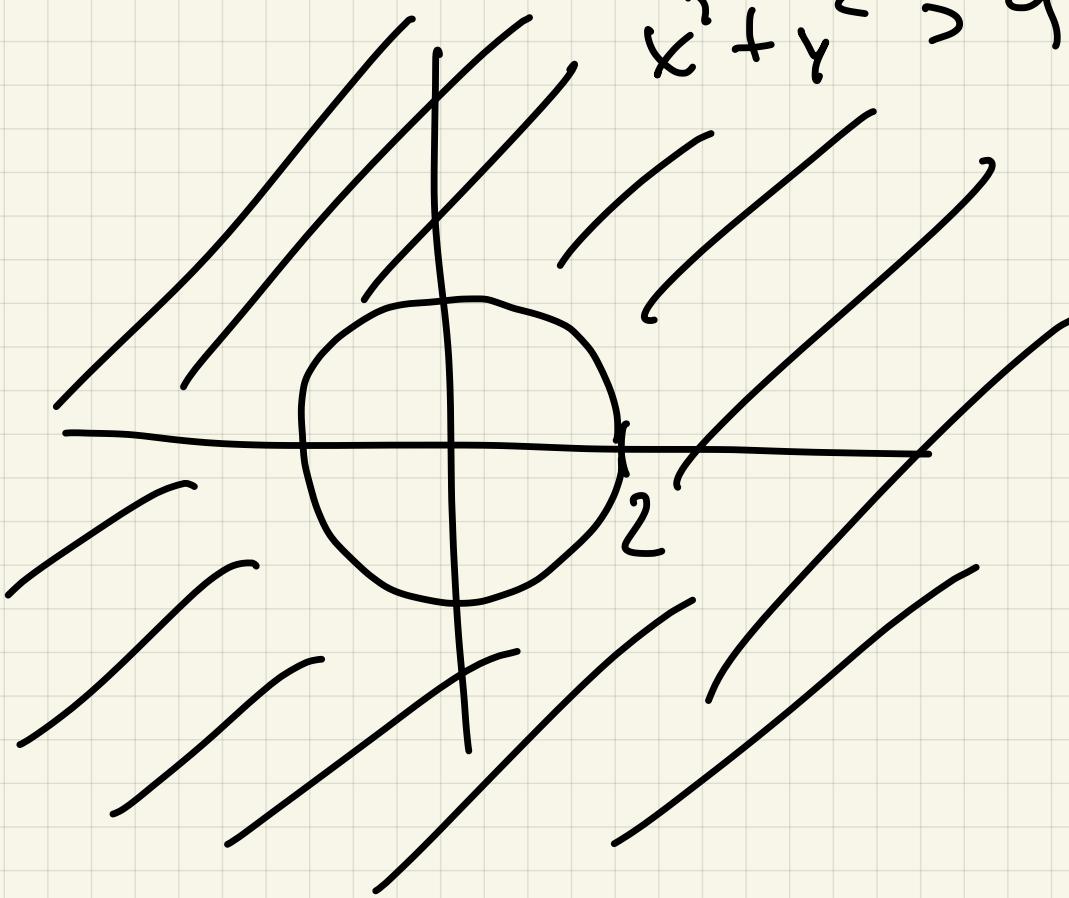


$$\underline{\text{Ex2}} \quad f(x,y) = \frac{1}{\sqrt{x^2+y^2-4}}$$

Domain

$$x^2 + y^2 - 4 > 0$$

$$x^2 + y^2 > 4$$



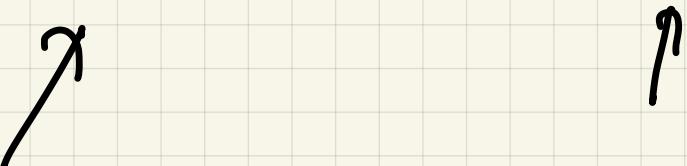
Defn : Domain  $f = \{(x,y) : f(x,y)$  is defined

Graph :  $\{(x,y,z) : z = f(x,y)\}$

Range :  $\{z : z = f(x,y) \text{ some } (x,y)\}$

Graph: Can make a contour map by sketching

$$\{ (x, y) : z = f(x, y) = \text{constant} \}$$

  
level sets

$$f(x, y) = \frac{1}{\sqrt{x^2 + y^2 - 4}} = 0$$

$$\underline{z = 0} : \text{empty}$$

$$\underline{z = 1} \quad \frac{1}{\sqrt{x^2 + y^2 - 4}} = 1 \leftarrow 5$$

$$x^2 + y^2 - 4 = 1$$

$$x^2 + y^2 = 5$$

$$\underline{z = 5} \quad x^2 + y^2 - 4 = \frac{1}{25}$$

$$x^2 + y^2 = 4 + \frac{1}{25}$$

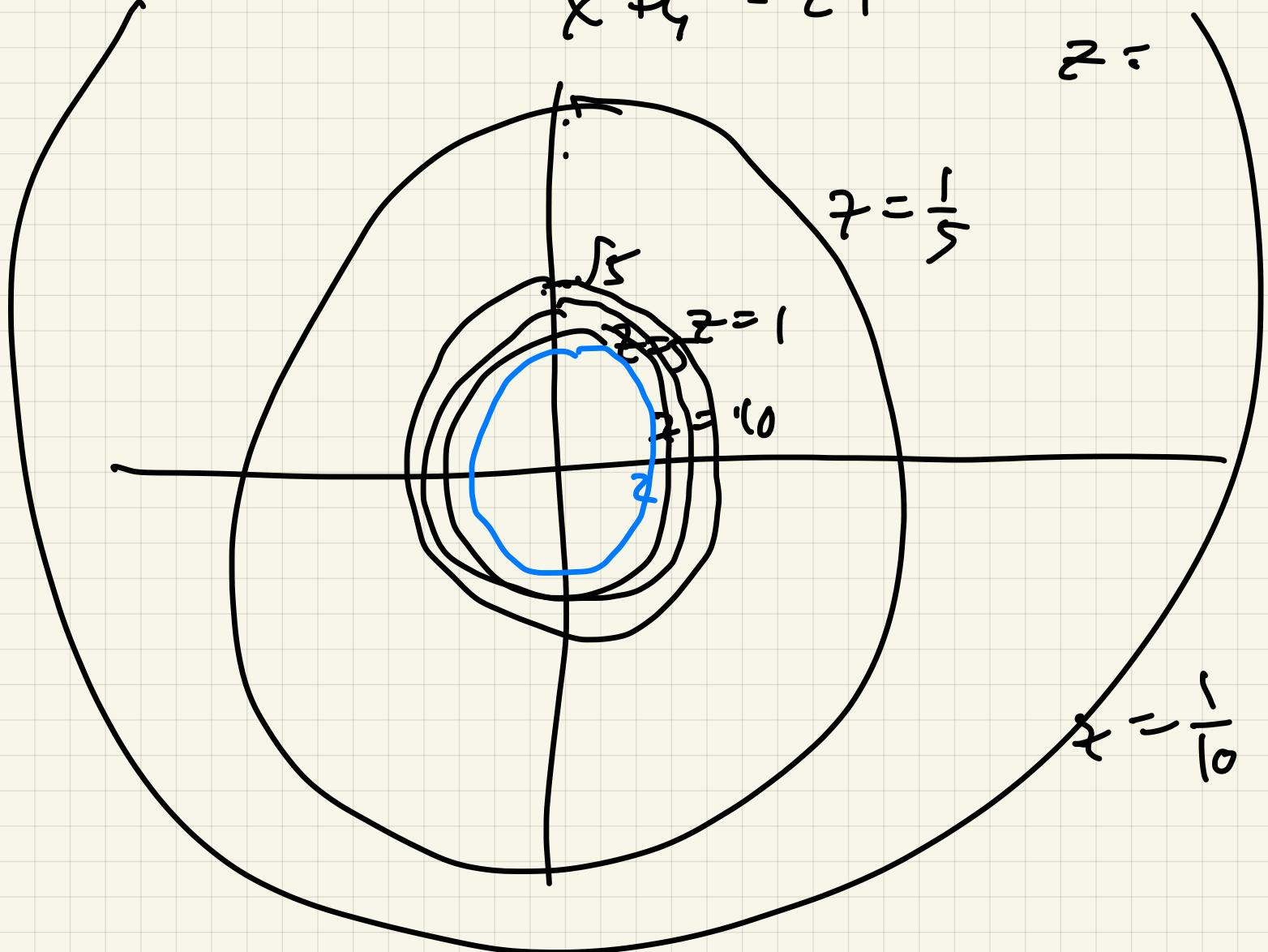
$$x^2 + y^2 = 4 + \frac{1}{100}$$

$$\underline{z = 10}$$

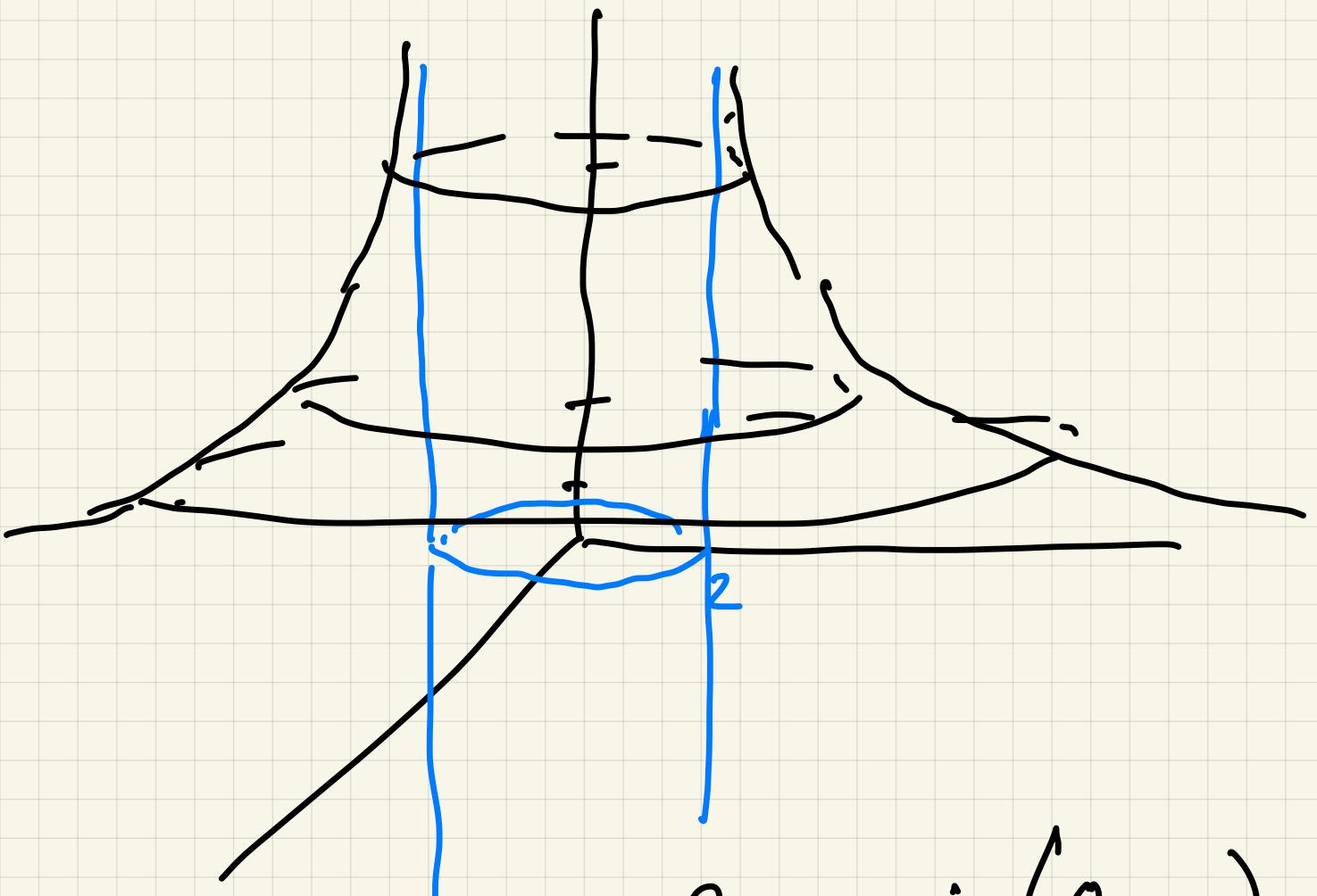
$$z = \frac{1}{5}$$

$$x^2 + y^2 - 4 = 25$$

$$x^2 + y^2 = 29$$



Put together



Range :  $(0, \infty)$

Ex 3  $f(x_1, y) = -2x + 2$

Domain =  $\mathbb{R}^2$  = (all  $(x, y)$ )

Range  $(-\infty, \infty)$

Graph? Level sets

$$z = -2x + 2$$

$$z = 0 : x = 1$$

$$-1 = -2x + 2$$

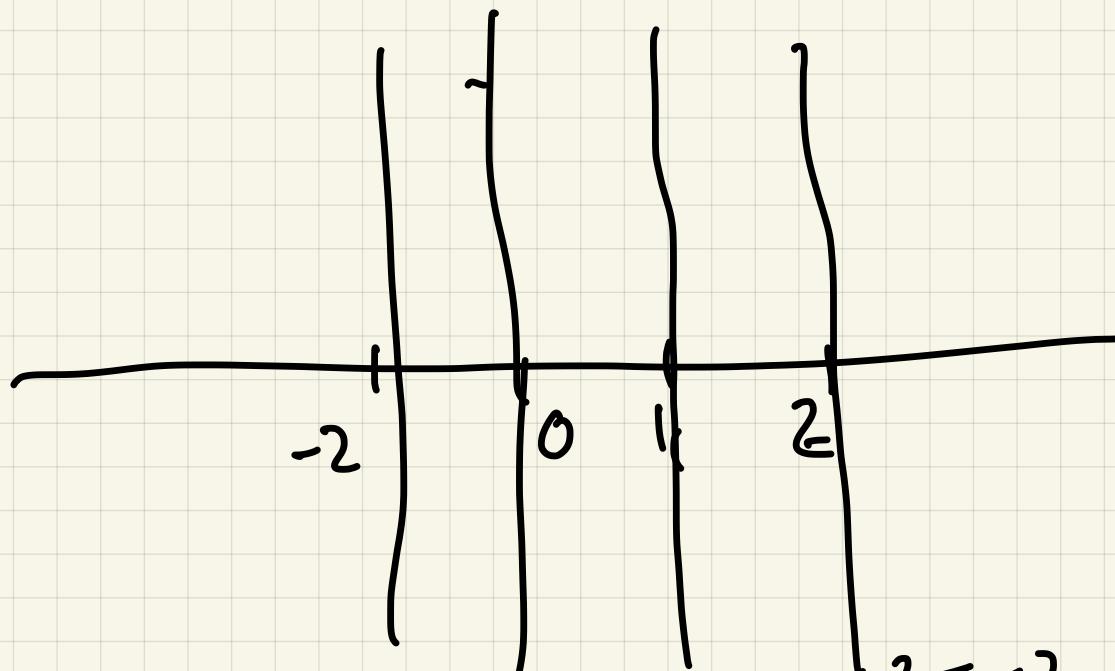
$$z = -1 : x = \frac{3}{2}$$

$$-3 = -2x$$

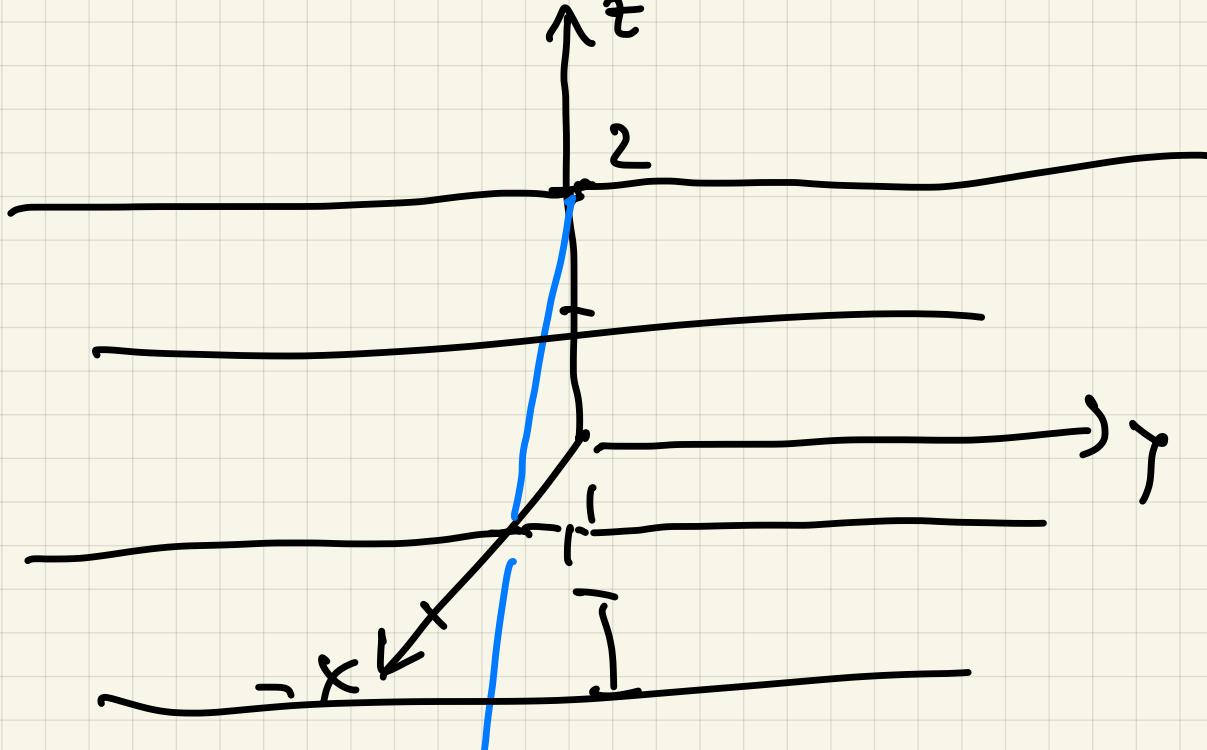
$$z = -2 : x = 2$$

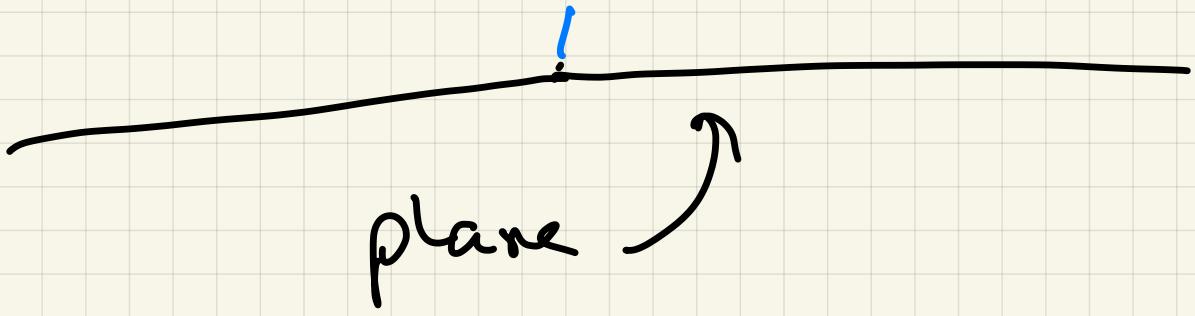
$$\frac{3}{2} = x$$

$$x = 2 \quad ; \quad x = 0$$



$$t = \frac{z+2}{2} = \frac{z}{2} + 1 \quad z = 0 \quad z = -2$$





of course!  $z = -2x + 2$



$2x + z = 2$   
plane, normal vector  $\langle 2, 0, 1 \rangle$

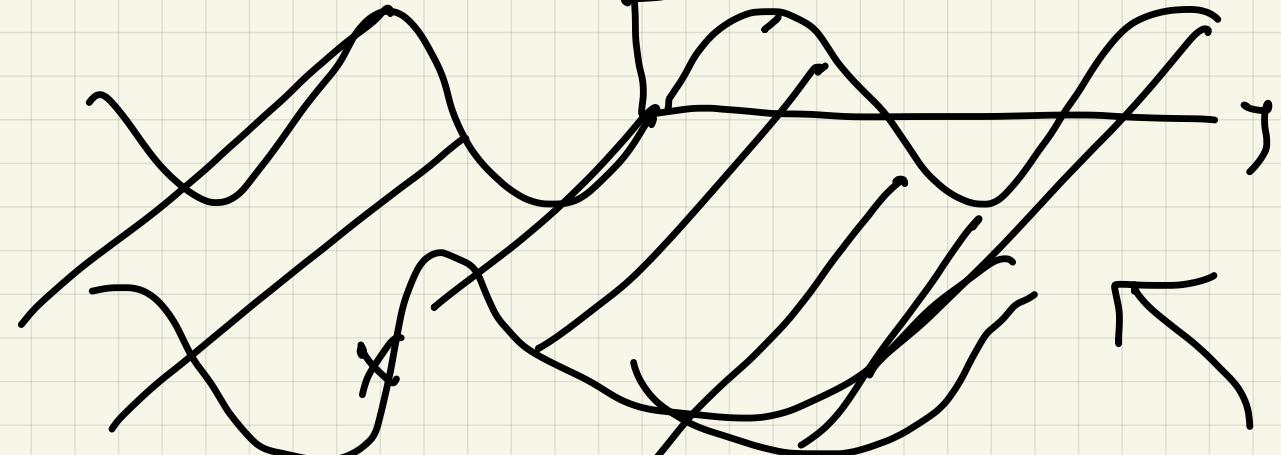
Ex 9

$$z = \sin(y)$$

$$\text{Dom} = \mathbb{R}^2$$

(a) cylinder ( $\S(1.6)$ ) /

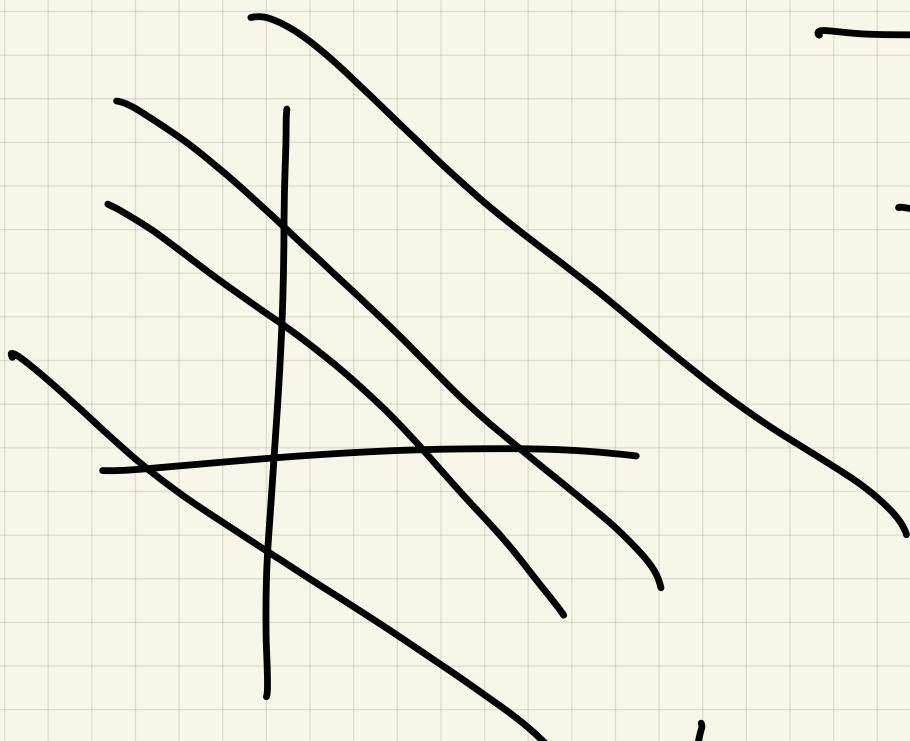
$$\text{range} = [-1, 1]$$



$$(6) \quad z = \sin(x+y)$$

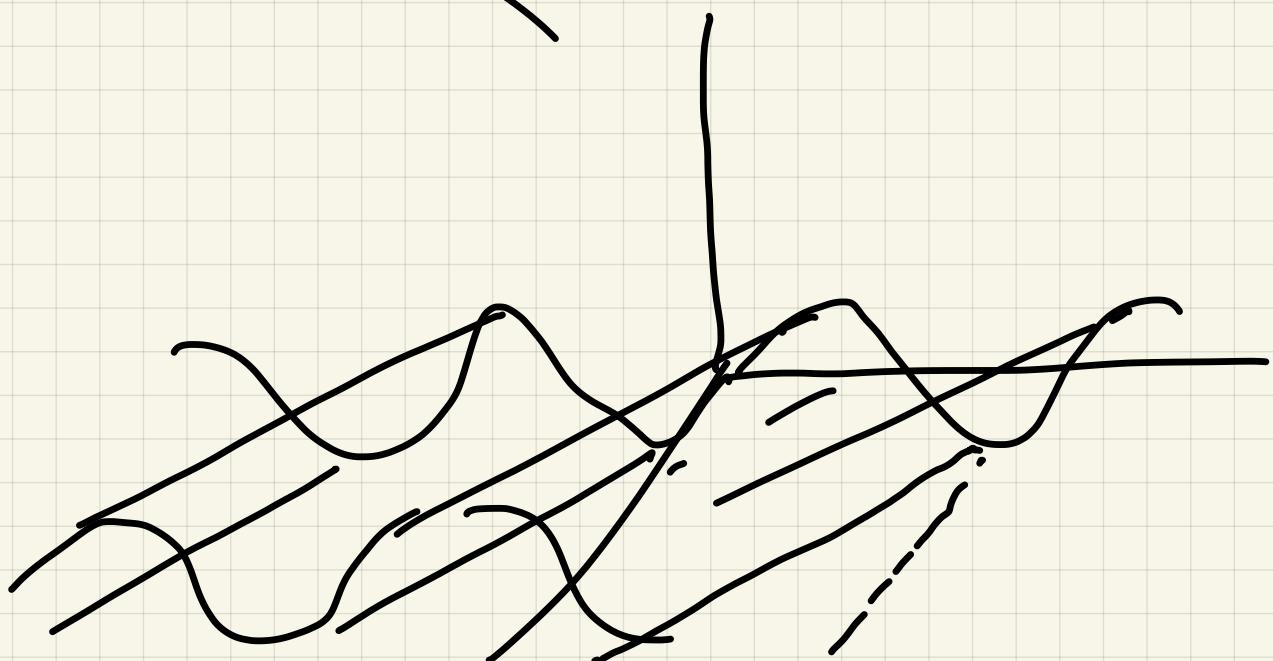
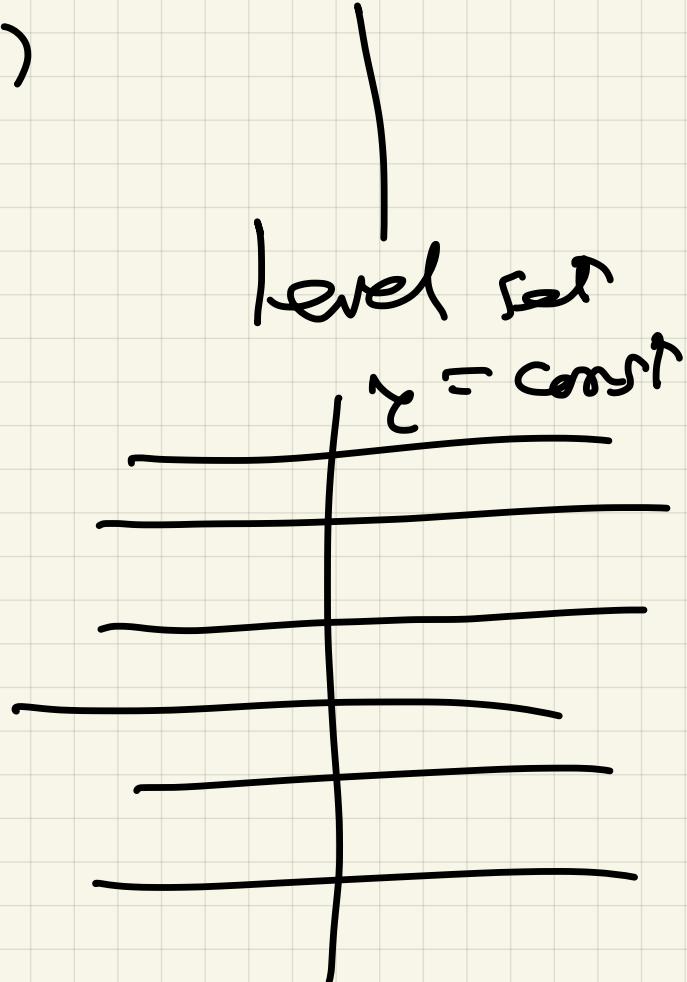
level sets

$$x+y = \text{const}$$



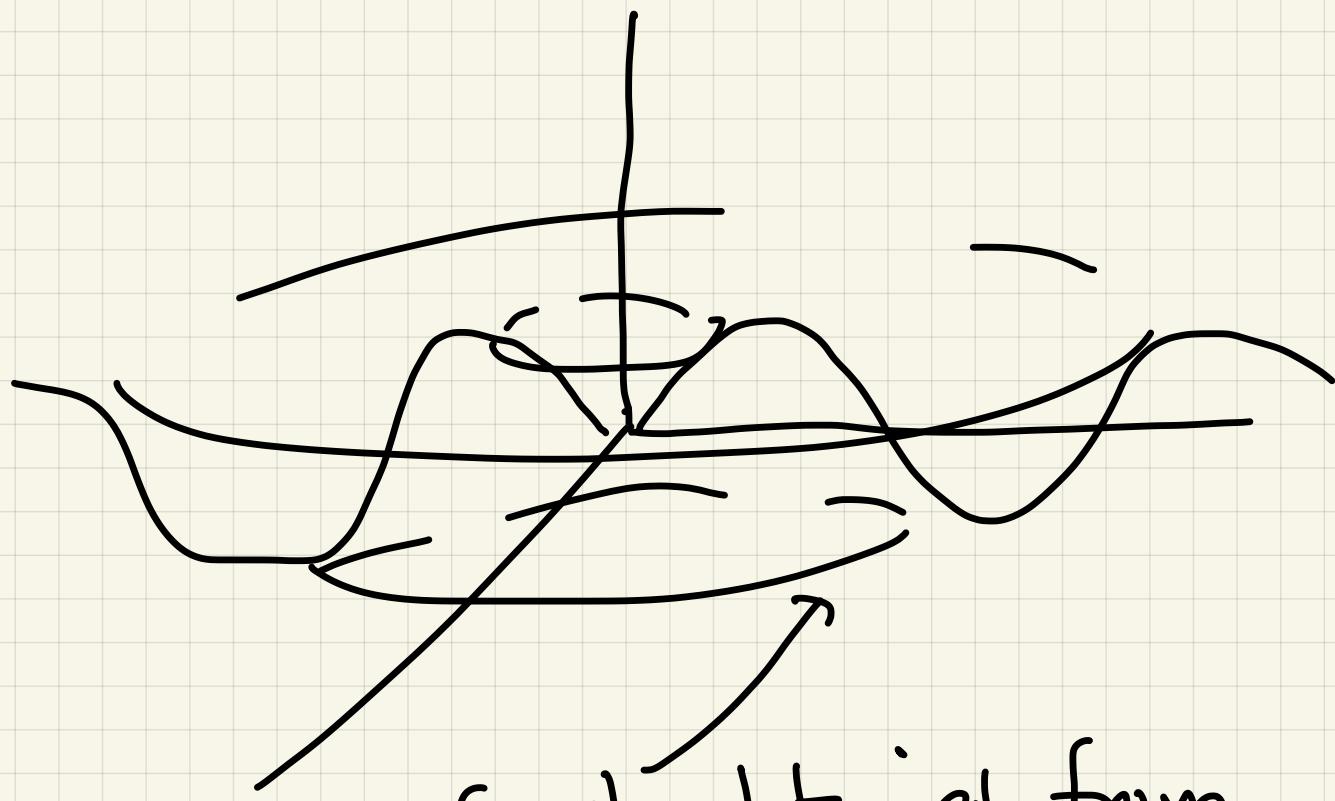
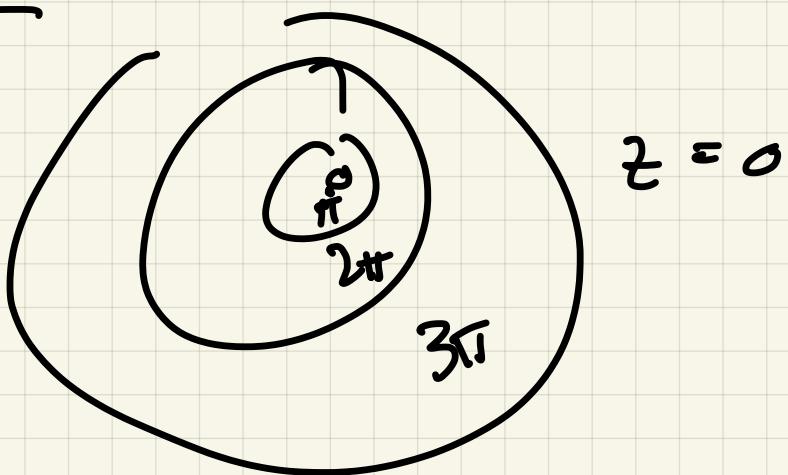
level set

$$z = \text{const}$$



$$(c) z = \sin(\sqrt{x^2 + y^2})$$

Level sets:  $z = \text{const}$



Graph obtained from  
revolving  $z = \sin y$  ( $y \geq 0$ )  
about the  $z$ -axis.