Exam 3

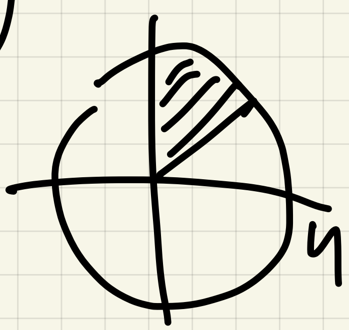
1 (a)

(b) $\displaystyle\int_0^2 \int_0^{y^2} \frac{2}{1+2y^3}\, dx\, dy =$

(c) $\displaystyle\int_0^2 \frac{2y^2}{1+2y^3}\, dy = \frac{2}{3} \ln(1+2y^3)\Big|_0^2 =$

$\frac{2}{3} \ln 17$

2

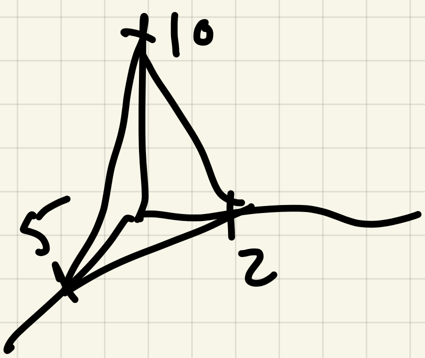


$\displaystyle\int_{\pi/4}^{\pi/2} \int_0^4 (16-r^2)\, r\, dr\, d\theta =$

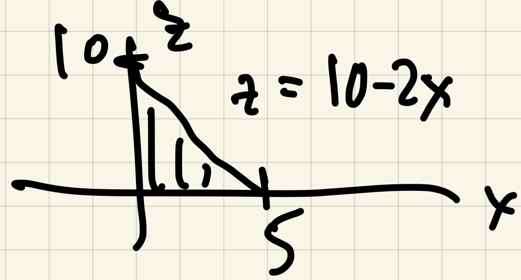
$\displaystyle= \int_{\pi/4}^{\pi/2} 8r^2 - \frac{r^4}{4}\Big|_0^4 =$

$\displaystyle\int_{\pi/4}^{\pi/2} 128 - 64 = \int_{\pi/4}^{\pi/2} 64\, d\theta = 16\pi$

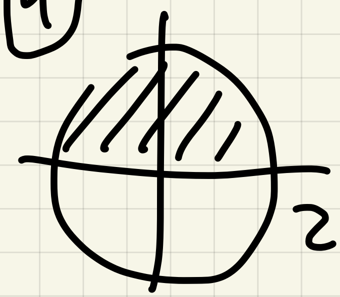
3



$\displaystyle\int_0^5 \int_0^{10-2x} \int_0^{\frac{10-2x-z}{5}} dy\, dz\, dx$



$\boxed{4}$



$$\int_0^{\pi}\int_0^{\pi/2}\int_0^2 \rho\cos\phi\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{\pi}\int_0^{\pi/2} \frac{\rho^4}{4}\Big|_0^2 = \int_0^{\pi}\int_0^{\pi/2} 4\cos\phi\sin\phi =$$

$$\int_0^{\pi} 2\sin^2\phi\Big|_0^{\pi/2} = \int_0^{\pi} 2\,d\theta = 2\pi.$$

$\boxed{5}$ (a) $x^2+y^2 = 18-x^2-y^2 \Rightarrow$

$x^2+y^2 = 9 \quad r = 3$

(b) $$\int_0^{2\pi}\int_0^3\int_{r^2}^{9-r^2} r\,dz\,dr\,d\theta$$

$\boxed{6}$ (a) $r(t) = \langle 2t, 3t, 2+5t\rangle \quad 0\le t\le 1$

(b) $$\int_0^1 (3t-2t+4(2+5t))\sqrt{42}\,dt$$

$$=\sqrt{42}\int 21t+8 = \frac{21}{2}t^2+8t\Big|_0^1\sqrt{42} =$$

$$\frac{21+16}{2} = \frac{37\sqrt{42}}{2}$$

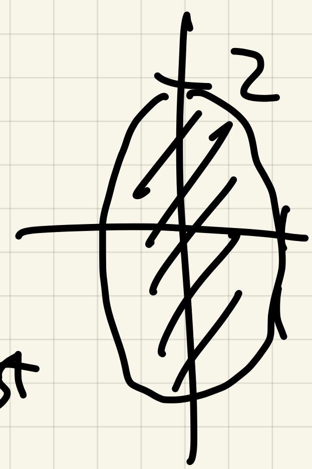
(c) $$\int_0^1 -(3t)(2t)\cdot 3 + (2+5t)\cdot 5 \, dt =$$

$$\int_0^1 -18t^2 + 25t + 10 \, dt =$$

$$-6t^3 + \frac{25}{2}t^2 + 10t \Big|_0^1 =$$

$$4 + \frac{25}{2} = \frac{33}{2}$$

$\boxed{7}$ $\nabla f = \langle y, x \rangle = 0$ at $(0,0)$, $f(0,0) = 0$



Boundary : $x = \cos t$
$y = 2\sin t$

$f = xy = 2\sin t \cos t \quad 0 \le t \le 2\pi$

$f' = 2(\cos^2 t - \sin^2 t) = 0$ at

$t = \pi/4, \; 3\pi/4, \; 5\pi/4, \; 7\pi/4$

$f(\pi/4) = 1, \quad f(3\pi/4) = -1$

$f(5\pi/4) = 1 \quad f(7\pi/4) = -1$

abs max at $\pm\left(\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right)$ / abs min $\pm\left(\frac{1}{\sqrt{2}}, \frac{-2}{\sqrt{2}}\right)$

OR $y = 2\sqrt{1-x^2}$

so $xy = 2x\sqrt{1-x^2} \to$

$$f'(x) = 2\sqrt{1-x^2} - \frac{2x^2}{\sqrt{1-x^2}} =$$

$$\frac{2-4x^2}{\sqrt{1-x^2}} = 0 \to x = \pm\frac{1}{\sqrt{2}} \to,$$

$f\left(\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right) = 1$ max

$f\left(-\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right) = -1$ min

$y = -2\sqrt{1-x^2}$ for bottom $\to$

$f\left(-\frac{1}{\sqrt{2}}, -\frac{2}{\sqrt{2}}\right) = +1$ max

$f\left(+\frac{1}{\sqrt{2}}, \frac{-2}{\sqrt{2}}\right) = -1$ min