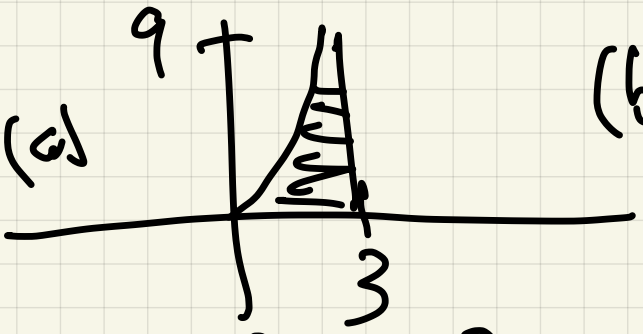
Exam 3 (y-ellow)

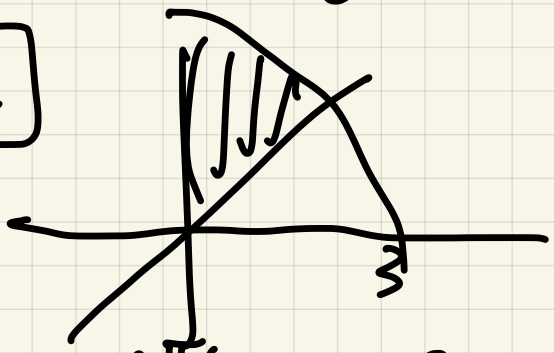
1 (a)



(b) $\int_0^3 \int_0^{x^2} \frac{2}{1+x^3}\, dy\, dx$

(c) $= \int_0^3 \frac{2x^2}{1+x^3}\, dx = \frac{2}{3}\ln(1+x^3)\Big|_0^3 = \frac{2}{3}\ln(28)$

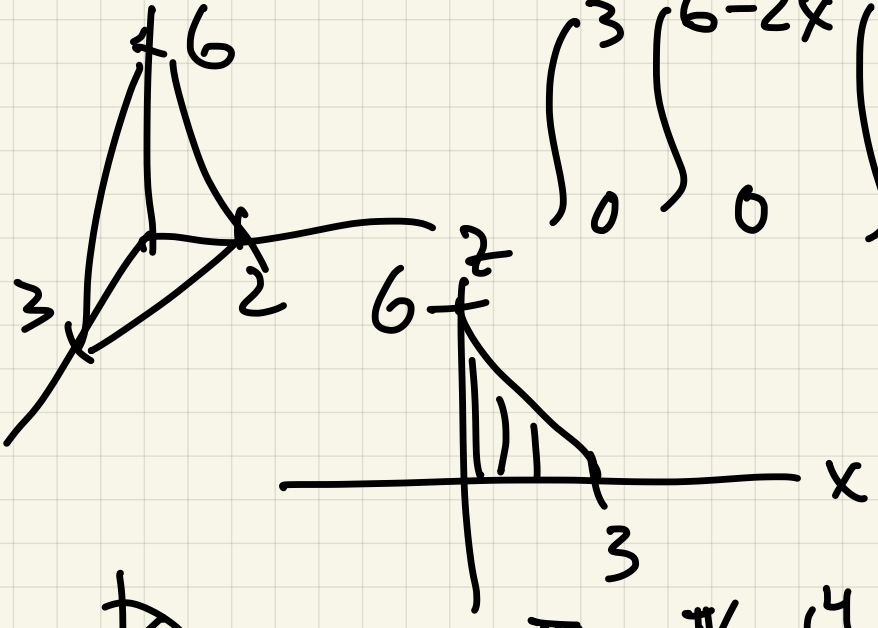
2



$\int_{\pi/4}^{\pi/2} \int_0^3 (9-r^2)\, r\, dr\, d\theta =$

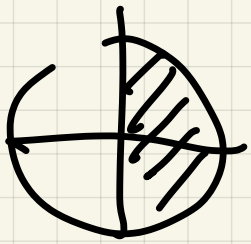
$\int_{\pi/4}^{\pi/2} \frac{9r^2}{2} - \frac{r^4}{4}\Big|_0^3 = \int_{\pi/4}^{\pi/2} \frac{81}{4}\, d\theta = \frac{81\pi}{16}$

3



$\int_0^3 \int_0^{6-2x} \int_0^{\frac{6-2x-z}{3}} dy\, dz\, dx$

4



$\Rightarrow \int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} \int_0^4 \rho\cos\phi\, \rho^2\sin\phi\, d\rho^4 =$

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{64\cos\phi\sin\phi} = \int_{-\pi/2}^{\pi/2} 32\sin^2\phi \Big|_{-\pi/2}^{\pi/2} =$$

$$\int_{-\pi/2}^{\pi/2} 32\,d\theta = 32\pi$$

5 (a) $x^2+y^2 = 50 - x^2 - y^2$

$x^2+y^2 = 25 \Rightarrow r = 5$

(b) $\int_0^{2\pi}\int_0^{5}\int_{r^2}^{50-r^2} r\,dz\,dr\,d\theta$

6 (a) $r(t) = \langle 2t, 4t, 1+5t\rangle \quad 0 \le t \le 1$

(b) $\int_0^1 (2t - 4t + 2(1+5t))\sqrt{45}\,dt =$

$$\sqrt{45}\int_0^1 8t+2\,dt = \sqrt{45}\,\left.4t^2+2t\right|_0^1 = 6\sqrt{45}$$

(c) $\int_0^1 -\langle 2t\rangle\langle 4t\rangle\cdot 2 + \langle 1+5t\rangle\cdot 5\,dt =$

$$\int_0^1 -16t^2 + 25t + 5\,dt =$$

$$\left. -16/3\,t^3 + 25t^2/2 + 5t\right|_0^1 =$$

$$-16/3 + 25/2 + 5 = \frac{-32 + 75 + 30}{6} = \frac{73}{6}$$

7. $\nabla f = \langle y, x \rangle = 0$ at $(0,0)$ $f(0,0) = 0$

Boundary: $r(t) = (3\cos t, \sin t)$ $0 \le t \le 2\pi$

$f(r(t)) = 3 \sin t \cos t$

$f'(t) = 3(\cos^2 t - \sin^2 t) = 0$ at

$t = \pm \pi/4, \pm 3\pi/4$

$f(3/\sqrt{2}, 1/\sqrt{2}) = \frac{3}{2}$ ← abs max

$f(3/\sqrt{2}, -1/\sqrt{2}) = -3/2$ ← abs min

$f(-3/\sqrt{2}, 1/\sqrt{2}) = -3/2$ ← abs min

$f(-3/\sqrt{2}, -1/\sqrt{2}) = 3/2$

Also can solve boundary as

$x = 3\sqrt{1-y^2} \Rightarrow f(y) = 3y\sqrt{1-y^2}$

and find crit pts where $f'(y) = 0$.

$y = \pm \frac{1}{\sqrt{2}}$ etc.