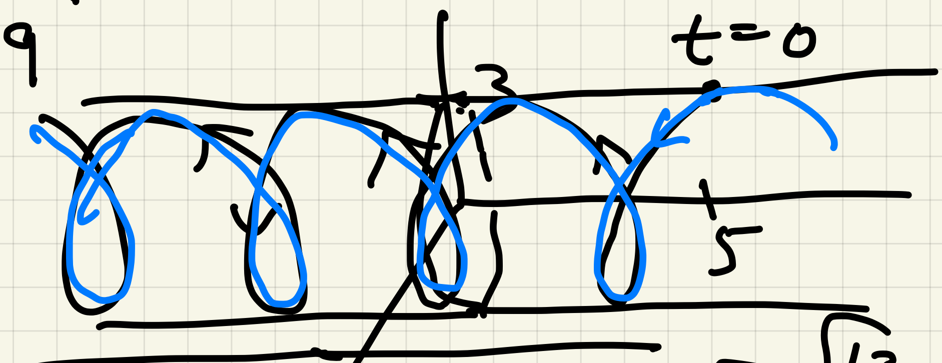
Exam 2

$\boxed{1}$ $\bar{r}(t) = \langle 3\sin t, 5-2t, 3\cos t\rangle$

(a) $\bar{v}(t) = \langle 3\cos t, -2, -3\sin t\rangle$

$\bar{a}(t) = \langle -3\sin t, 0, 3\cos t\rangle$

(b) $x^2+z^2=9$



(c) $|\bar{v}(t)| = \sqrt{9\cos^2 t + 4 + 9\sin^2 t} = \sqrt{13}$,

so $L = \int_0^5 |\bar{v}(t)|\,dt = \int_0^5 \sqrt{13}\,dt = 5\sqrt{13}$.

(d) $\bar{v}(0) = \langle 3,-2,0\rangle$, $\bar{a}(0) = \langle 0,0,3\rangle$

$$\begin{vmatrix} i & j & k \\ 3 & -2 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \langle -6,-9,0\rangle$$

$$K = \frac{|\bar{v}\times\bar{a}|}{|\bar{v}|^3} = \frac{|\langle -6,-9,0\rangle|}{|\langle 3,-2,0\rangle|^3} = \frac{3\sqrt{13}}{\sqrt{13}^3} = \frac{3}{13}.$$

$\boxed{2}$ $\bar{a} = \langle 0,-6\rangle \Rightarrow \bar{v} = \int \langle 0,-6\rangle\,dt =$

(a) $\langle 0,-6t\rangle + \bar{C}$, $\bar{v}(0) = \langle 25,0\rangle \Rightarrow$

$\bar{C} = \langle 25,0\rangle$, so $\bar{v}(t) = \langle 25,-6t\rangle$

(b) $\bar{r}(t) = \int \bar{v}(t)\,dt = \langle 25t, -3t^2\rangle + \bar{D}$

$\bar{r}(0) = \langle 0,50\rangle \Rightarrow \bar{r}(t) = \langle \underset{x}{25t}, \underset{y}{50-3t^2}\rangle$

(c) $x = 100$ when $t = 4$, then

$25t = x$

$y = 50 - 3\cdot 4^2 = 2$, $0 < 2 < 10$,

so ball hits sign

$\boxed{3}$ (a) $1 - x^2 - \frac{y^2}{9} \geq 0$

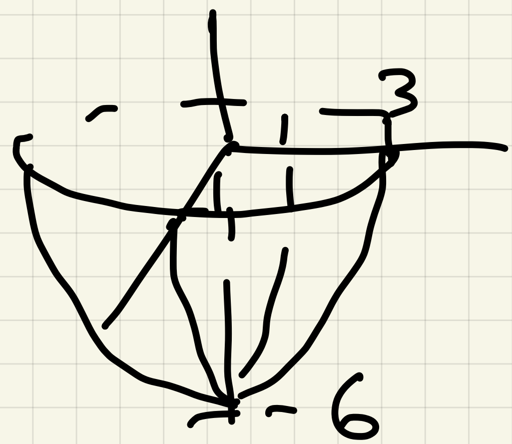
$\Downarrow$

$x^2 + y^2/9 \leq 1$

(b) $z = 5\sqrt{1 - x^2 - y^2/4} \Rightarrow$

$x^2 + \frac{y^2}{9} + \frac{z^2}{36} = 1$

lower half ellipsoid $\longrightarrow$



(c) range $z \in [-6, 0]$

$\boxed{4}$ (a) $\lim\limits_{(x,y)\to(1,1)} \frac{x^2 - 16y^2}{x - 4y} = \frac{1-16}{1-4} = \frac{-15}{-3} = 5$

(b) $\lim\limits_{(x,y)\to(8,2)} \frac{\cancel{(x-4y)}(x+4y)}{\cancel{(x-4y)}} = \lim (x+4y) = 8+8 = 16$

$\boxed{5}$ $f = xz^2 + xe^{xy}$

(a) $f_x = z^2 + e^{xy} + xye^{xy}$, $f_y = x^2 e^{xy}$, $f_z = 2xz$

(b) $f_{xy} = xe^{xy} + xe^{xy} + x^2 y e^{xy}$

$f_{yx} = 2xe^{xy} + x^2 y e^{xy}$

(c) $\nabla f(2,0,3) = \langle 10,4,12 \rangle$

(d) $D_u f = \langle 10,4,12 \rangle \cdot \langle \frac{1,2,2}{3} \rangle = \frac{42}{3} = 14$

(e) $\nabla f / |\nabla f| = \frac{\langle 10,4,12 \rangle}{\sqrt{260}} = \frac{\langle 5,2,6 \rangle}{\sqrt{65}}$

(f) Many! $\frac{\langle 0,3,-1 \rangle}{\sqrt{10}}$ is one.

(g) $10(x-2) + 4y + 12(z-3) = 0 \Rightarrow$

$10x + 4y + 12z = 56 \Rightarrow 5x + 2y + 6z = 28$

$\boxed{6}$ $z^2 + yz^3 + xy = 10$

(a) $\partial/\partial x$: $2z \cdot z_x + 3yz^2 z_x + y = 0 \Rightarrow$

$z_x = \frac{-y}{2z+3yz^2}\Big|_{(2,3,1)} = \frac{-3}{11}$

$\partial/\partial y$: $2z\, z_y + z^3 + 3yz^2 z_y + x = 0 \Rightarrow$

$z_y = \frac{-x-z^3}{2z+3yz^2}\Big|_{(2,3,1)} = \frac{-3}{11}$

(b) $\partial/\partial x$: $2(z_x)^2 + 2z \cdot z_{xx} + 6yz \cdot (z_x)^2$

$+ 3yz^2 \cdot z_{xx} = 0 \Rightarrow$

$z_{xx} = \frac{(-2-6yz)(z_x)^2}{2z+3yz^2}\Big|_{(2,3,1)} =$

$$\frac{-20\left(\frac{-3}{11}\right)^2}{11} = \frac{-180}{11^3} = \frac{-180}{1331}$$