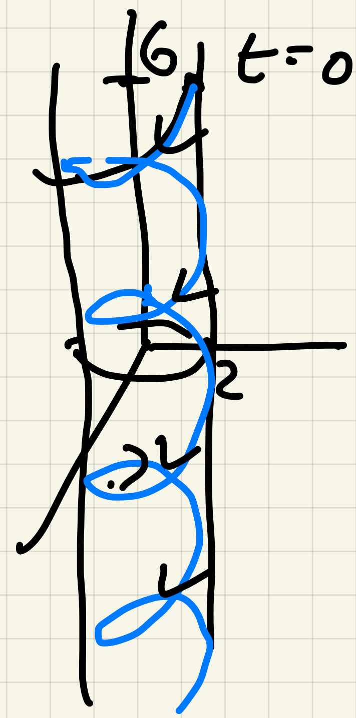
Exam 2

$\boxed{1}$ $\bar{r}(t) = \langle 2\sin t, 2\cos t, 6-5t \rangle$

(a) $\bar{v}(t) = \langle 2\cos t, -2\sin t, -5 \rangle$

$\bar{a}(t) = \langle -2\sin t, -2\cos t, 0 \rangle$

(b) $x^2 + y^2 = 4$ $\longrightarrow$



(c) $|\bar{v}| = \sqrt{4\cos^2 t + 4\sin^2 t + 25} = \sqrt{29}$

so $L = \int_0^7 \sqrt{29}\,dt = 7\sqrt{29}$

(d) $\bar{v} = \langle 2, 0, -5 \rangle$, $\bar{a} = \langle 0, -2, 0 \rangle$

$$\begin{vmatrix} i & j & k \\ 2 & 0 & -5 \\ 0 & -2 & 0 \end{vmatrix} = \langle -10, 0, -4 \rangle$$

$$k = \frac{|\langle -10, 0, -4 \rangle|}{|\langle 2, 0, -5 \rangle|^3} = \frac{2\sqrt{29}}{(\sqrt{29})^3} = \frac{2}{29}$$

$\boxed{2}$ $\bar{a} = \langle 0, -6 \rangle \Rightarrow \bar{v} = \int \langle 0, -6 \rangle\, dt =$

(a) $\langle 0, -6t \rangle + \bar{C}$, $\bar{v}(0) = \langle 20, 0 \rangle \Rightarrow$

$\bar{v}(t) = \langle 20, -6t \rangle$.

(b) $\bar{r}(t) = \int \langle 20, -6t \rangle = \langle 20t, -3t^2 \rangle + \bar{D}$

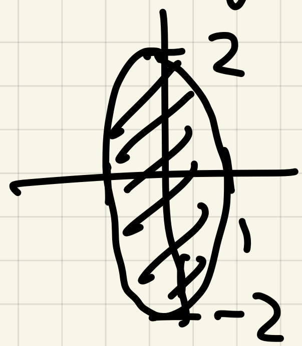
$\bar{r}(0) = \langle 0, 85 \rangle \Rightarrow \bar{r}(t) = \langle \underbrace{20t}_{x}, \underbrace{85 - 3t^2}_{y} \rangle$

(c) At sign, $\underset{20t}{x} = 120 \Rightarrow t = 6 \Rightarrow$

$y = 85 - 3\cdot 6^2 = -23 \Rightarrow$
ball falls short of sign.

$\boxed{3}$ (a) $1 - x^2 - \frac{y^2}{4} \geq 0$

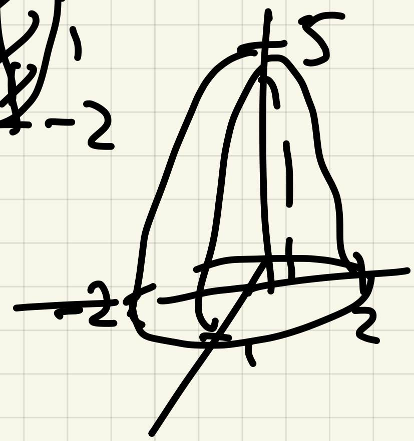
$D: x^2 + \frac{y}{4} \leq 1 \Rightarrow$



(b) $z = 5\sqrt{1 - x^2 - y^2/4} \Rightarrow$

$x^2 + y^2/4 + z^2/25 = 1$

upper half ellipsoid



(c) Range $[0, 5]$

$\boxed{4}$ (a) $\lim\limits_{(x,y)\to(1,1)} \frac{9x^2 - y^2}{3x - y} = \frac{9-1}{3-1} = \frac{8}{2} = 4$

(b) $\lim\limits_{(x,y)\to(2,6)} \frac{\cancel{(3x-y)}(3x+y)}{\cancel{(3x-y)}} = \lim (3x+y) = 6+6 = 12$

$\boxed{5}$ $f = yz^3 + ye^{xy}$

(a) $f_x = y^2 e^{xy}$, $f_y = z^3 + e^{xy} + xye^{xy}$, $f_z = 3yz^2$

(b) $f_{xy} = 2ye^{xy} + xy^2 e^{xy}$

$f_{yx} = ye^{xy} + ye^{xy} + yxye^{xy}$

(c) $\nabla f(0,2,1) = \langle 4, 2, 6 \rangle$

(d) $D_u f(0,2,1) = \langle 4,2,6\rangle \cdot \frac{\langle 2,2,1\rangle}{3} = \frac{18}{3} = 6$

(e) $\nabla f / |\nabla f| = \frac{\langle 4,2,6\rangle}{\sqrt{56}} = \frac{\langle 2,1,3\rangle}{\sqrt{14}}$

(f) Many! $\frac{\langle 1,-2,0\rangle}{\sqrt{5}}$ is one

(g) $4(x)+2(y-2)+6(z-1)=0$

$4x+2y+6z=10 \Rightarrow 2x+y+3z=5$

$\boxed{6}$ $z^3+xz^2+2xy=16$

(a) $\partial/\partial x$: $3z^2z_x+z^2+2xz\cdot z_x+2y=0 \Rightarrow$

$$z_x=\left.\frac{-2y-z^2}{3z^2+2xz}\right|_{(3,2,1)}=\frac{-5}{9}$$

$\partial/\partial y$: $3z^2z_y+2xz\,z_y+2x=0 \Rightarrow$

$$z_y=\left.\frac{-2x}{3z^2+2xz}\right|_{(3,2,1)}=\frac{-6}{9}=-2/3$$

(b) $6z(z_y)^2+3z^2z_{yy}+2x(z_y)^2+2xz\,z_{yy}=0$

$\partial/\partial y$

$$\Rightarrow z_{yy}=\left.\frac{(-6z-2x)(z_y)^2}{3z^2+2xz}\right|_{(3,2,1)}=$$

$$\frac{-12\left(-2/3\right)^2}{9}=\frac{-48}{81}=\frac{-16}{27}$$