Exam1

① (a) $3\bar{u} - \bar{v} = \langle 10, 11, -7 \rangle$

(b) $\text{Proj}_{\bar{v}} \bar{u} = \frac{\bar{u} \cdot \bar{v}}{|\bar{v}|^2} \bar{v} = \frac{0}{9} \langle 2, -2, 1 \rangle = \langle 0, 0, 0 \rangle$

(c) $\bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ 4 & 3 & -2 \\ 2 & -2 & 1 \end{vmatrix} = \langle -1, -8, -14 \rangle$

(d) $|(u \times v) \cdot w| = |\langle -1, -8, -14 \rangle \cdot \langle 1, -2, 4 \rangle| =$

$|-1 + 16 - 56| = |-41| = 41$

② (a) $\overline{AB} = \langle 0, 3, -2 \rangle, \ \overline{AC} = \langle 4, 3, -2 \rangle$

(b) $\overrightarrow{AB} \cdot \overline{AC} = 9 + 4 = 13 > 0 \Rightarrow$ acute

(c) $\overline{AB} \times \overline{AC} = \begin{vmatrix} i & j & k \\ 0 & 3 & -2 \\ 4 & 3 & -2 \end{vmatrix} = \langle 0, -8, -12 \rangle$

$= -4 \langle 0, 2, 3 \rangle$

(d) $\frac{\overline{AB} \times \overline{AC}}{|\overline{AB} \times \overline{AC}|} = \frac{-4\langle 0, 2, 3 \rangle}{|-4\langle 0, 2, 3 \rangle|} = \frac{-4\langle 0, 2, 3 \rangle}{4\sqrt{13}} = \langle 0, \frac{-2}{\sqrt{13}}, \frac{-3}{\sqrt{13}} \rangle$

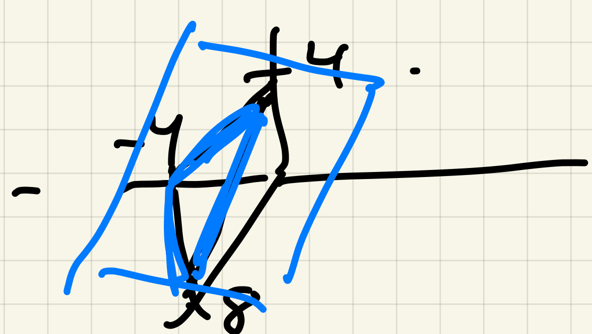
(e) $\frac{1}{2} |\overline{AB} \times \overline{AC}| = \frac{1}{2} \cdot 4\sqrt{13} = 2\sqrt{13}$

(f) $\underset{B}{\langle 0, 3, 0 \rangle} + \underset{\overline{AC}}{\langle 4, 3, -2 \rangle} = \langle 4, 6, -2 \rangle$

③ (a) $(x - 4) - 2(y - 3) + 2(z - 5) = 0 \Rightarrow$

$x - 2y + 2z = 8$

(b) $(8, 0, 0)$
$(0, -4, 0)$
$(0, 0, 4)$



(c) $n_2 = \langle 0,1,0 \rangle$,

$$\cos\theta = \frac{|n_1 \cdot n_2|}{|n_1||n_2|} = \frac{2}{3\cdot 1} = \frac{2}{3}.$$

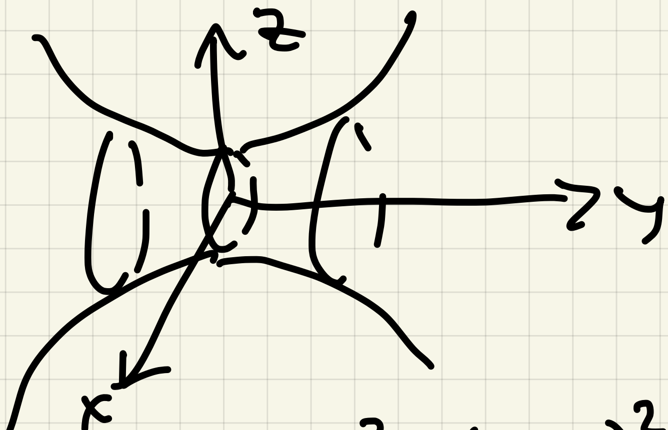
(d)
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7+t \\ -1-2t \\ 9+2t \end{pmatrix}$$

(e) $P_0 = (4,3,5) \Rightarrow \overline{AP_0} = \langle -3, 4, -4 \rangle$

$$\text{dist} = \frac{|\overline{AP_0} \cdot n_1|}{|n_1|} = \frac{|-3-8-8|}{3} = \frac{19}{3}$$

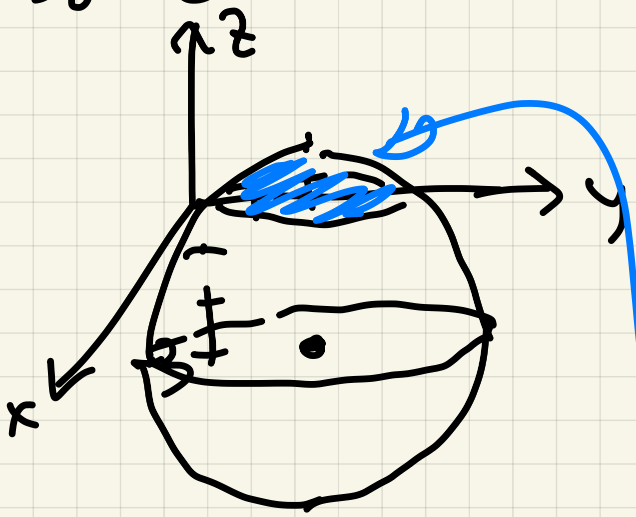
4

(a)



(b) $x^2 + (y-2)^2 + (z+3)^2 = 3+4+9 = 16$

so ctr $= (0,2,-3)$, $r = 4$



(c) The solid region inside the sphere but above the plane $z=0$

See blue shade

(d) $z<0 \Rightarrow$ circles, so (c)