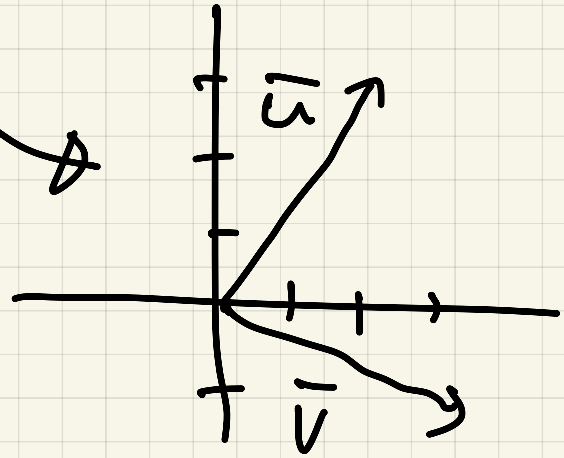
8/29 Calc3

Quiz 2

1(a)



$\bar{u} = \langle 2, 3 \rangle$ (b)

$\bar{v} = \langle 3, -1 \rangle$ (c)

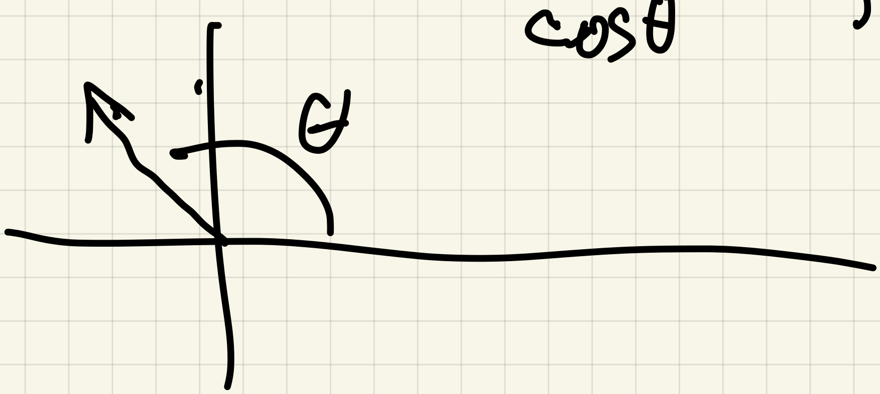
$\bar{u} + \bar{v} = \langle 5, 2 \rangle$

$\bar{u} - 4\bar{v} = \langle -10, 7 \rangle$

2. $\bar{w} = \langle -4, 3 \rangle$

(a) $|\bar{w}| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$

(b) $\dfrac{\bar{w}}{|\bar{w}|} = \langle \underset{\cos\theta}{-4/5}, \underset{\sin\theta}{3/5} \rangle$



(c) $\theta = \cos^{-1}(-4/5)$

$\theta = \pi - \sin^{-1}(3/5)$

$\theta = \pi + \tan^{-1}(-3/4)$

Last time determinants

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

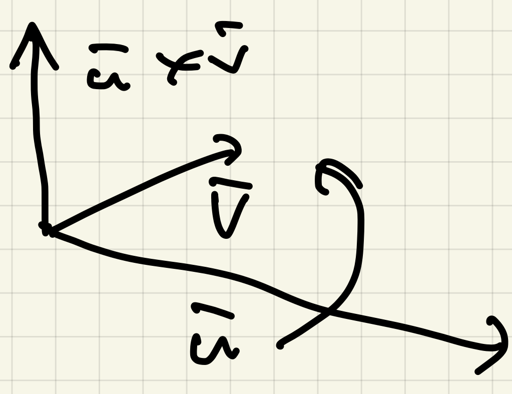
cross product

$$\bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

algebraic properties

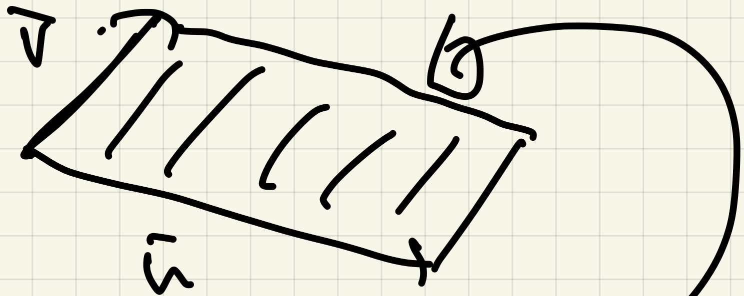
<u>Geometry property</u>

(A)



$\bar{u} \times \bar{v}$ is $\perp$ to $\bar{u}, \bar{v}$ via right hand rule

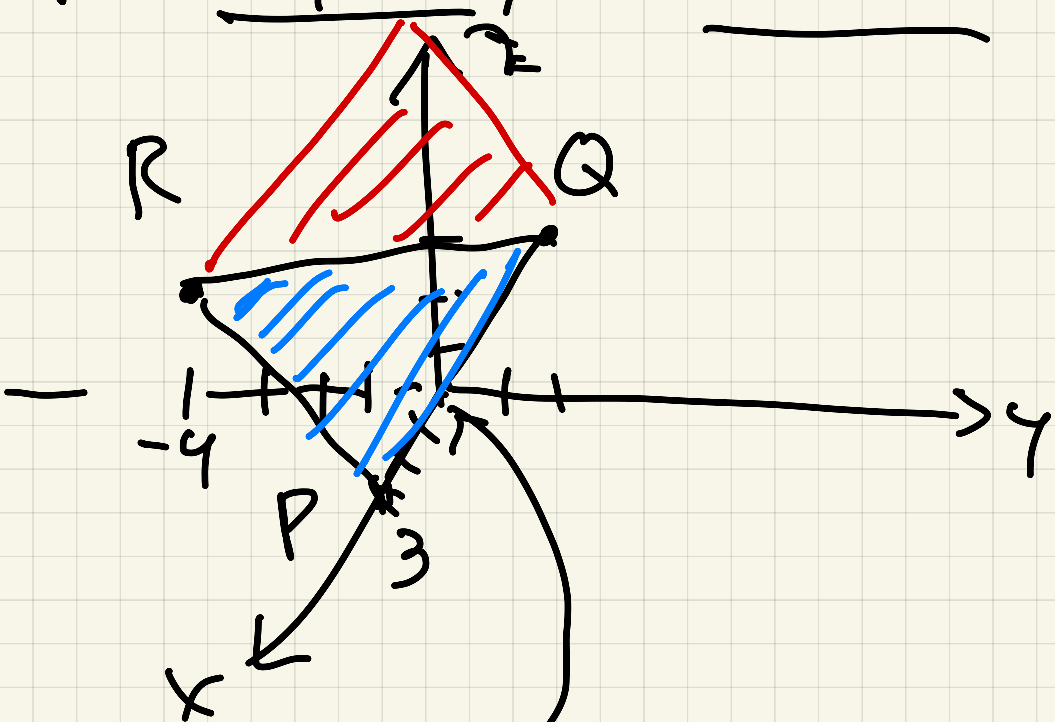
(B)

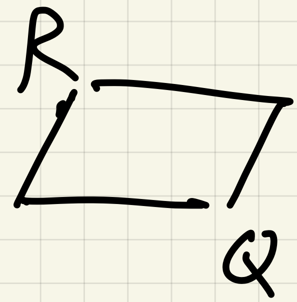


$|\bar u \times \bar v| = $ Area ▱

Ex) Find area of Δ with vertices

$P = \underline{(3,0,0)}$, $Q = \underline{(0,2,3)}$, $R = (0,-4,2)$



Area of Δ = $\frac{1}{2}$ Area P 

$= \frac{1}{2} \left| \overrightarrow{PQ} \times \overrightarrow{PR} \right|$

$$\overline{PQ} = \langle -3, 2, 3 \rangle$$
$$\overline{PR} = \langle -3, -4, 2 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -3 & 2 & 3 \\ -3 & -4 & 2 \end{vmatrix} =$$

$$16\hat{i} - j(3) + 18k = \langle 16, -3, 18 \rangle$$

Area of $\Delta = \frac{1}{2} \left| \langle 16, -3, 18 \rangle \right| =$

$$\frac{1}{2}\sqrt{256 + 9 + 324}$$
$$= \frac{1}{2}\sqrt{589}$$

$$i \begin{vmatrix} 2 & 3 \\ -4 & 2 \end{vmatrix} - j \begin{vmatrix} -3 & 3 \\ -3 & 2 \end{vmatrix} + k \begin{vmatrix} -3 & 2 \\ -3 & -4 \end{vmatrix}$$

$$2 \cdot 2 - (-3 \cdot 4)$$
$$4 + 12 = 16$$

Defn

Triple scalar product of $\bar{u}, \bar{v}, \bar{w}$ is $(\bar{u} \times \bar{v}) \cdot \bar{w}$

Note for easy calculation

$$(\bar{u} \times \bar{v}) \cdot \bar{w} = \bar{u} \cdot (\bar{v} \times \bar{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Ex1 $\bar{u} = \langle 2, 0, 0 \rangle$

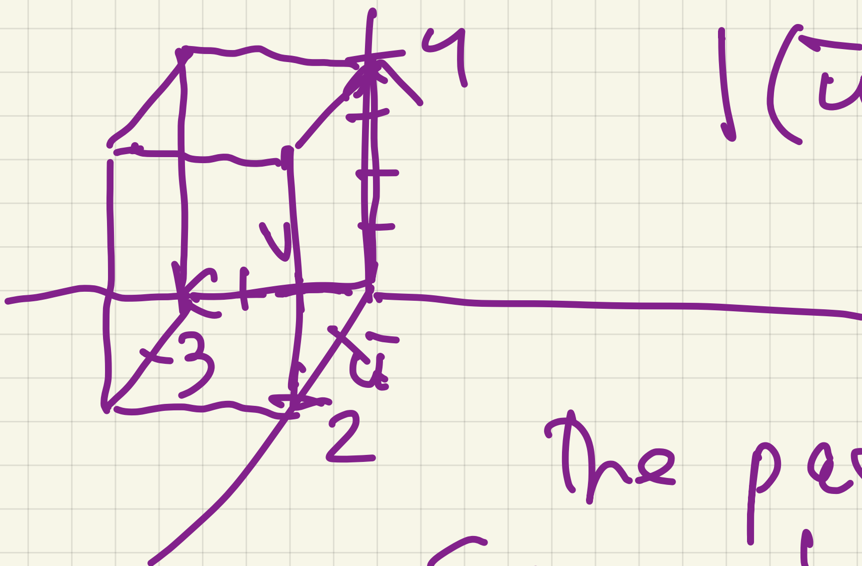
$\bar{v} = \langle 0, -3, 0 \rangle$

$\bar{w} = \langle 0, 0, 4 \rangle$

$\Downarrow$

$$(\bar{u}\times\bar{v})\cdot\bar{w} = \begin{vmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{vmatrix} = -24$$

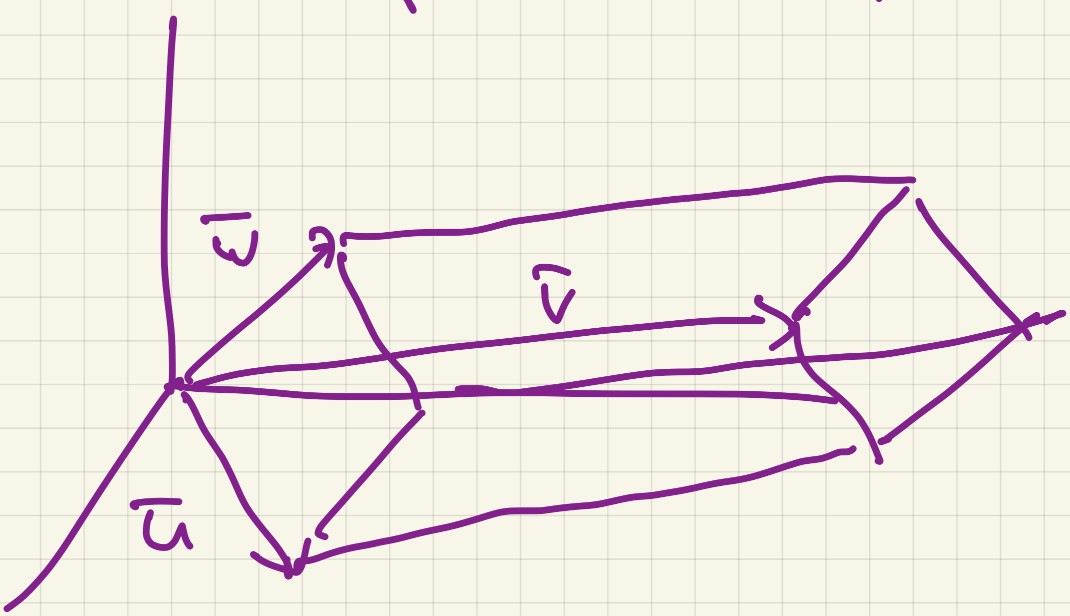
Geometric interpretation



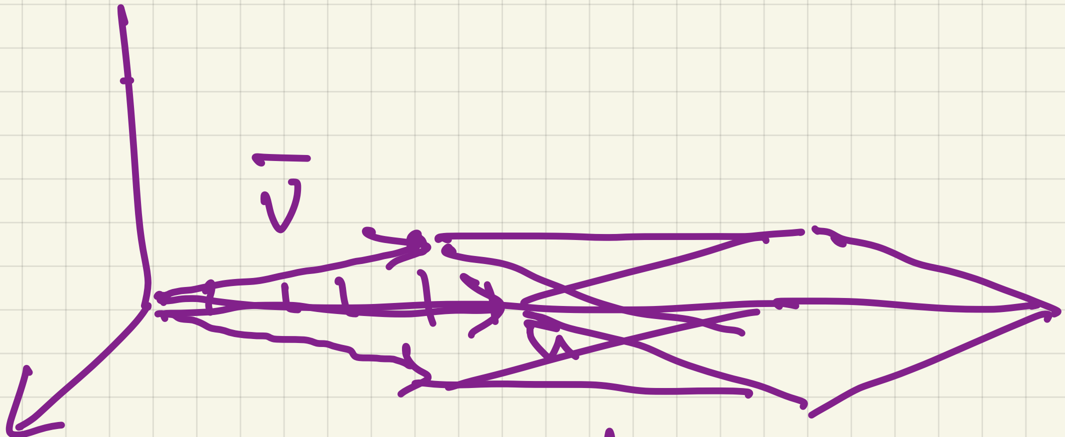
$|(\bar{u}\times\bar{v})\cdot\bar{w}|$

$\parallel$

volume of the parallelapiped spanned by $\bar{u}, \bar{v}, \bar{w}$.



Ex 2



$\bar{u} = \langle 0,5,0 \rangle$

$\bar{v} = \langle 0,4,1 \rangle$

$\bar{w} = \langle 1,4,0 \rangle$

volume

$$\begin{vmatrix} 0 & 5 & 0 \\ 0 & 4 & 1 \\ 1 & 4 & 0 \end{vmatrix} =$$

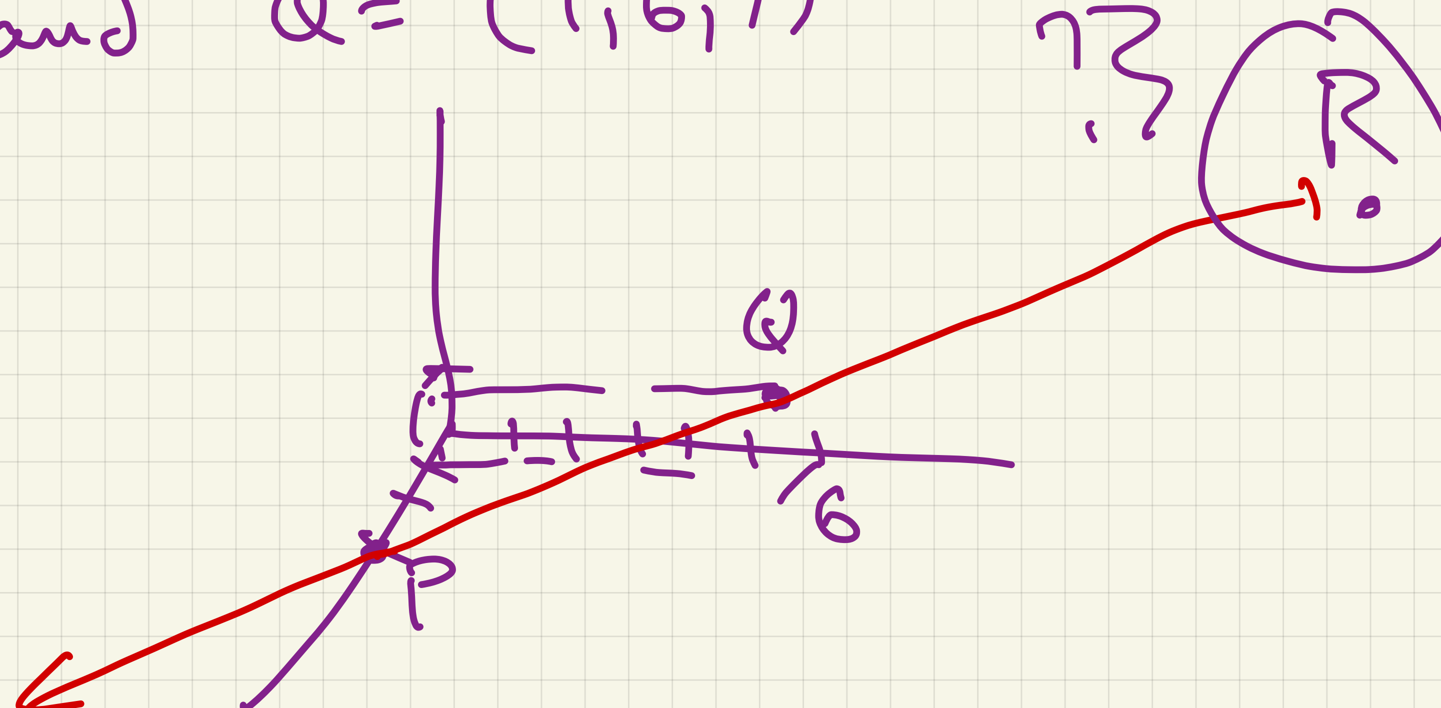
$$-5 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -5(-1) = 5.$$

§11.5 Lines and planes in $\mathbb{R}^3$

(Know lines in $\mathbb{R}^2$

$y = mx+b$, m = slope, b = y-int)

Lines: A line L passes through $P = (3,0,0)$ and $Q = (1,6,1)$



Is $R = (-7,30,5)$ is on L?

i.e. is $\overrightarrow{PQ} \parallel \overrightarrow{PR}$??

$$\overrightarrow{PQ} = \langle -2, 6, 1 \rangle \qquad \overrightarrow{PR} = \langle -10, 30, 5 \rangle$$

yes. $5\,\overrightarrow{PQ} = \overrightarrow{PR}$

Can determine all points on L by same method:

$R = \langle x, y, z \rangle$

$\overrightarrow{PQ} \parallel \overrightarrow{PR}$

Is there a scalar t with

$t\,\overrightarrow{PQ} = \overrightarrow{PR}$??

$t\langle -2, 6, 1 \rangle = \langle x-3, y, z \rangle$

$$\boxed{\begin{aligned} x &= 3-2t \\ y &= 6t \\ z &= 1t \end{aligned}}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3-2t \\ 6t \\ t \end{pmatrix} \quad t \text{ in } \mathbb{R}$$

<u>Parametric equations of a line</u>

If L is parallel to $\vec{v} = \langle a, b, c \rangle$ and

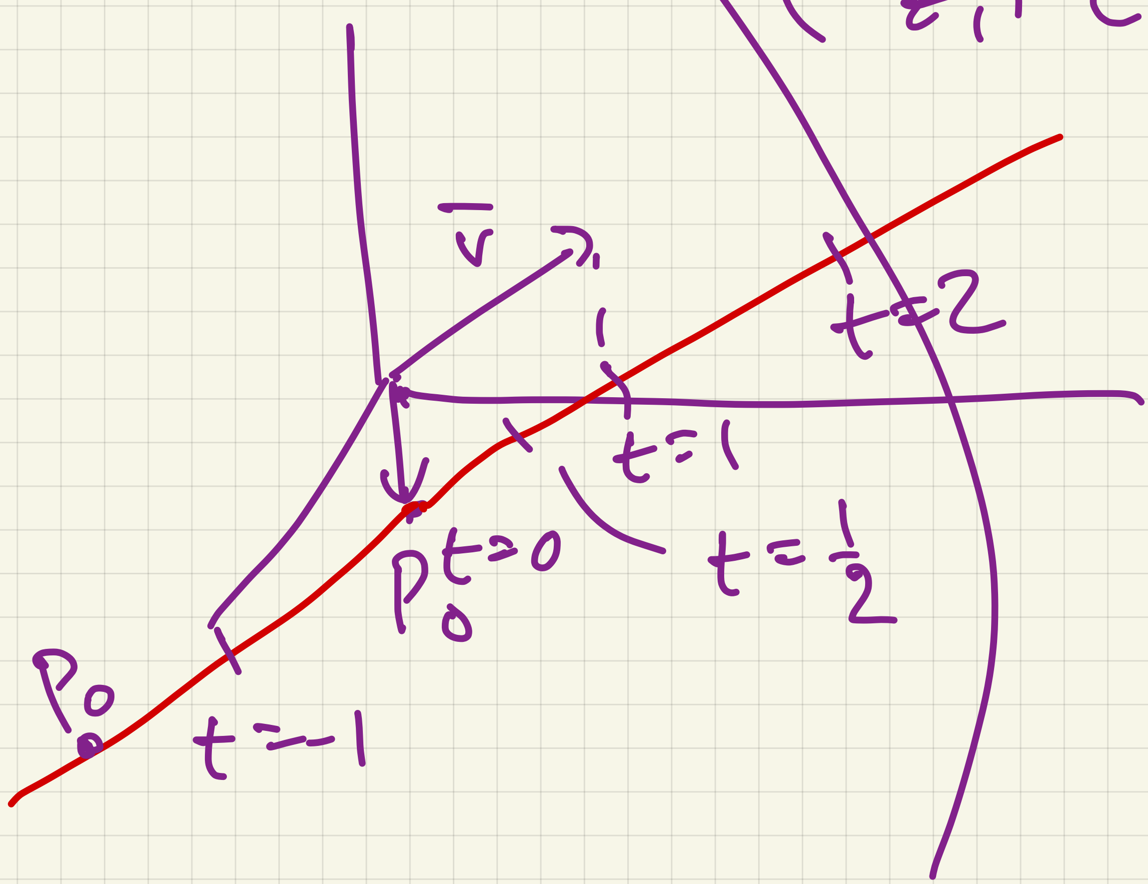
$P_0 = (x_1, y_1, z_1)$ is on L,

Then L has parametric equations

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + t \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$= P_0 + t\bar{v}$$

$$= \begin{pmatrix} x_1 + at \\ y_1 + bt \\ z_1 + ct \end{pmatrix}$$



$P_0 + t\vec{v}$

Remarks Not like $\mathbb{R}^2$?

(1) because of parameter t

(2) Many descriptions are same line.

Ex $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} + t \begin{pmatrix} 16 \\ -48 \\ -8 \end{pmatrix}$

same line from first ex.

Ex L_1 is line $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+2t \\ 2+3t \\ 4-5t \end{pmatrix}$

Is $Q = \begin{pmatrix} -5 \\ -7 \\ 19 \end{pmatrix}$ on L_1?

yes, take $t = -3$.