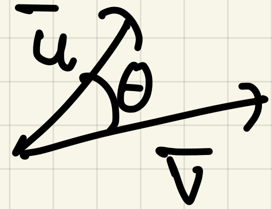
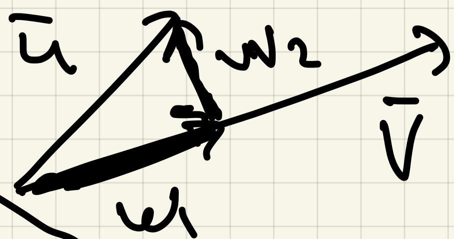
8/27/ Calc3

Last time



$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$$

<u>Projection</u>:



<u>Want</u>: $\boxed{\begin{array}{l} w_1 \parallel \bar{v} \\ \bar{w}_1 + \bar{w}_2 = \bar{u} \\ \bar{w}_2 \perp \bar{v} \end{array}}$

$\bar{w}_1 = \text{Proj}_{\bar{v}}\bar{u}$

<u>Thm</u> $\text{Proj}_{\bar{v}}\bar{u} = \dfrac{\bar{u}\cdot\bar{v}}{|\bar{v}|^2}\bar{v}$

$\bar{w}_2 = \bar{u} - \bar{w}_1$

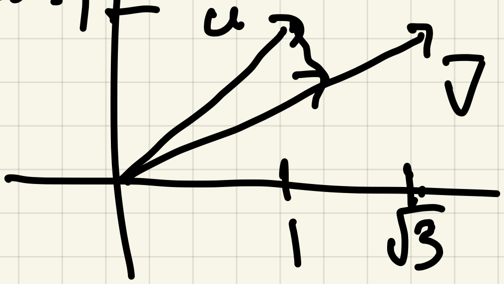
<u>Scalar component</u> $= |\bar{u}|\cos\theta$

<u>Ex1</u> Find the vector components of $\bar{u}$ <u>along $\bar{v}$</u> and <u>orthogonal to $\bar{v}$</u>

$\bar{w}_2$ $\qquad$ $\bar{w}_1$

and find scalar component.

(a)



$\bar{u} = \langle 1, 1 \rangle$

$\bar{v} = \langle \sqrt{3}, 1 \rangle$

$$\bar{w}_1 = \mathrm{proj}_{\bar{v}} \bar{u} = \frac{\bar{u} \cdot \bar{v}}{|\bar{v}|^2} \cdot \bar{v}$$

$$= \frac{\sqrt{3}+1}{2^2} \langle \sqrt{3}, 1 \rangle =$$

$$\left\langle \frac{3+\sqrt{3}}{4}, \frac{\sqrt{3}+1}{4} \right\rangle$$ <u>along $\bar{v}$</u>

<u>orthogonal to $\bar{v}$</u> :

$$\bar{w}_2 = \bar{u} - \bar{w}_1 =$$

$$\langle 1, 1 \rangle - \left\langle \frac{3+\sqrt{3}}{4}, \frac{\sqrt{3}+1}{4} \right\rangle =$$

$$\left\langle \frac{1-\sqrt{3}}{4}, \frac{3-\sqrt{3}}{4} \right\rangle$$

(b) $\bar{u} = \langle 3, -2, 7 \rangle$

(i) $\bar{v} = i = \langle 1, 0, 0 \rangle$

$$\bar{w}_1 = \text{Proj}_{\bar{v}} \bar{u} = \frac{\bar{u} \cdot \bar{v}}{|\bar{v}|^2} \bar{v}$$

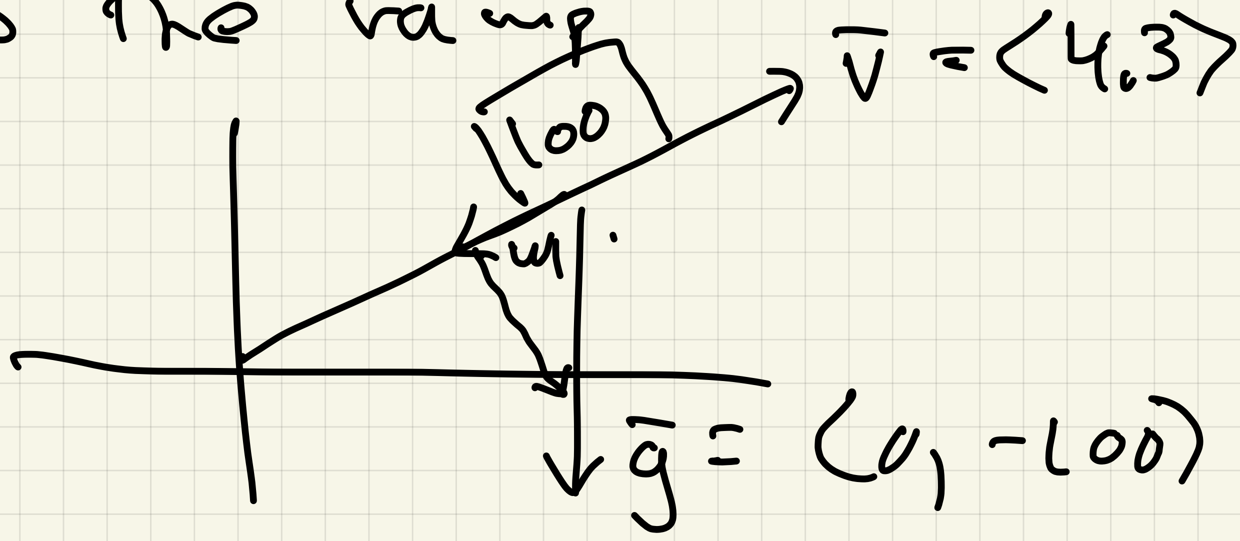
$$= \frac{3}{1^2} i = 3i$$

$$\bar{w}_2 = \langle 0, -2, 7 \rangle$$

scalar component $= 3$

Ex2 100 lb box sits on ramp of slope $3/4$

Find components of gravity force along and orthogonal to the ramp



$$\text{Proj}_{\bar{v}}\bar{g} = \frac{\bar{g}\cdot\bar{v}}{|\bar{v}|^2}\bar{v} = \frac{-300}{25}\langle 4,3\rangle$$

$$= -12\langle 4,3\rangle = \langle -48,-36\rangle$$

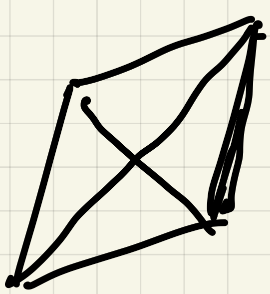
<u>Orthogonal</u>: $\bar{g} - \text{Proj}_{\bar{v}}\bar{g} =$

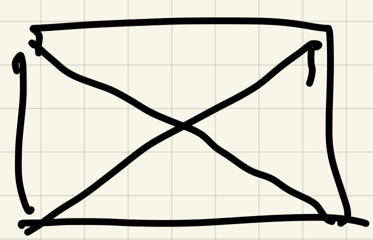
$$\langle 0,-100\rangle - \langle -48,-36\rangle =$$

$$\langle 48,-64\rangle$$

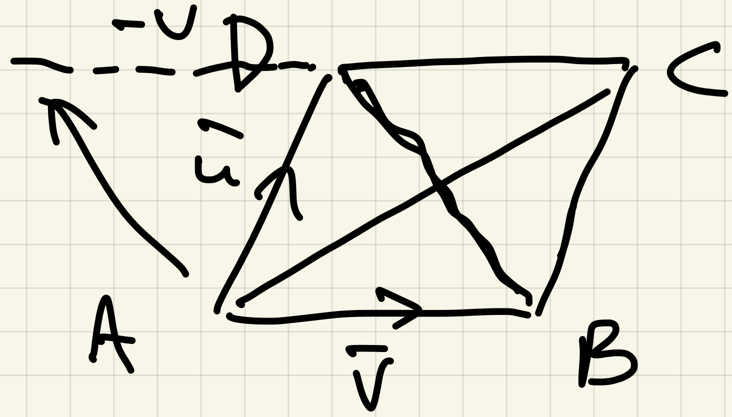
<u>Ex</u> (<u>Geometry</u>)

A parallelogram is a rectangle $\Leftrightarrow$ diagonals have the same length.





Idea:



$\bar u = \overrightarrow{AD}$

$\bar v = \overrightarrow{AB}$ $\rightarrow$

$\overrightarrow{AC} = \bar u + \bar v$

$\overrightarrow{BD} = \bar u - \bar v$

$$|\overrightarrow{AC}| = |\overrightarrow{BD}| \iff |\overrightarrow{AC}|^2 = |\overrightarrow{BD}|^2 \iff$$

$$|u+v|^2 = |u-v|^2 \iff$$

$$(u+v)\cdot(\bar u + v) = (u-v)\cdot(\bar u - v) \iff$$

$$u\cdot u + \underline{u\cdot v + v\cdot u} + v\cdot v =$$

$$u\cdot u - \underline{v\cdot u - u\cdot v} + v\cdot v \iff$$

$$2u\cdot v = -2u\cdot v \iff$$

$$4u\cdot v = 0 \iff$$

$$\bar u\cdot\bar v = 0$$

§ 11.4

Defn If $\bar{u} = \langle u_1, u_2, u_3 \rangle$ and $\bar{v} = \langle v_1, v_2, v_3 \rangle$ are vectors in $\mathbb{R}^3$, the cross product of $\bar{u}$ and $\bar{v}$ is

$$\bar{u} \times \bar{v} = \langle u_2 v_3 - u_3 v_2, -(u_1 v_3 - u_3 v_1), u_1 v_2 - u_2 v_1 \rangle$$

Hard to remember!

Easier to remember with determinants

2x2 determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = \det\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Ex) Cal $\begin{vmatrix} 2 & 1 \\ 3 & 7 \end{vmatrix} = 11$

$$\begin{vmatrix} 1 & 6 \\ 2 & 9 \end{vmatrix} = -3$$

3x3 determinant

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix}$$

$$- b \begin{vmatrix} d & f \\ g & i \end{vmatrix}$$

$$c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Ex3 (s)

$$\begin{vmatrix} 2 & 1 & 3 \\ 7 & 1 & 4 \\ 2 & 6 & 5 \end{vmatrix} =$$

$$2 \begin{vmatrix} 1 & 4 \\ 6 & 5 \end{vmatrix} - 1 \begin{vmatrix} 7 & 4 \\ 2 & 5 \end{vmatrix} + 3 \begin{vmatrix} 7 & 1 \\ 2 & 6 \end{vmatrix}$$

$$2(-19) - 1(27) + 3(40)$$

$$-38 - 27 + 120 = 55$$

Now the cross product formula becomes easy!

$$\bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} =$$

$$i(u_2 v_3 - u_3 v_2) - j - \cdots$$

<u>Ex3</u> $\bar{u} = \langle 1, 2, 3 \rangle$

$\bar{v} = \langle 1, 0, 2 \rangle$

(a)

$$\bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & 0 & 2 \end{vmatrix} =$$

$$i(4) - j(-1) + k(-2) = \langle 4, 1, -2 \rangle$$

(b) $\bar{v} \times \bar{u} = \begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ 1 & 2 & 3 \end{vmatrix} =$

$i(-4) - j(1) + k(2) =$
$\langle -4, -1, 2 \rangle$

(c) $\bar{u} \times \bar{u} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{vmatrix} = \bar{0}$

(d) $i \times j = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = k$

<u>Algebraic properties</u>

1. $\bar{u} \times \bar{v} = -\bar{v} \times \bar{u}$
2. $\bar{u} \times \bar{0} = \bar{0}$
3. $\bar{u} \times \bar{u} = \bar{0}$
4. $c(\bar{u} \times \bar{v}) = (c\bar{u}) \times \bar{v} = \bar{u} \times (c\bar{v})$
5. $\bar{u} \times (\bar{v} + \bar{w}) = \bar{u} \times \bar{v} + \bar{u} \times \bar{w}$

(6) $\bar{u}\cdot(v\times w) = (u\times v)\cdot w$

Geometric properties

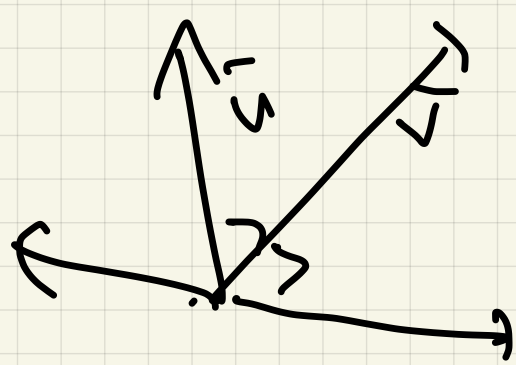
(A) $\bar{u}\times\bar{v} = \bar{0} \iff \bar{u} \parallel \bar{v}$

(i.e. there's a nonzero scalar c : $\bar{u} = c\bar{v}$)

(B) $\bar{u}\times\bar{v}$ is $\perp$ $\bar{u}$

and $\bar{u}\times\bar{v}$ is $\perp$ $\bar{v}$

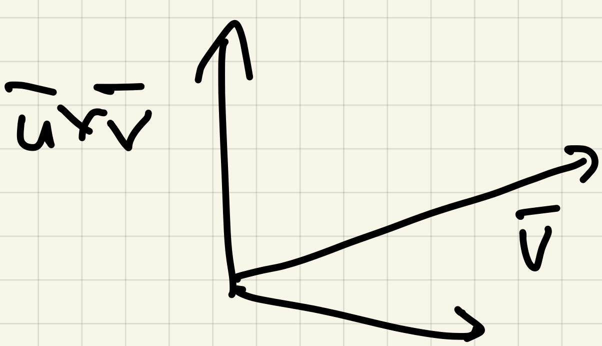
Moreover,

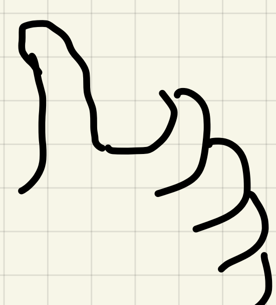


the

$\bar{u}\times\bar{v} \perp \bar{u}, \bar{v}$

and $\bar{u}, \bar{v}, \overline{u\times v}$ obey the right hand rule

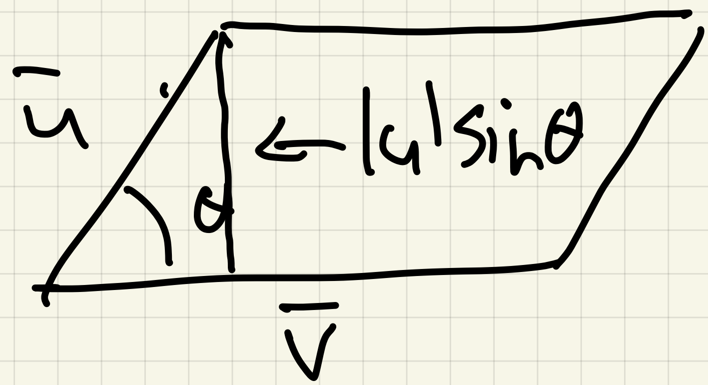




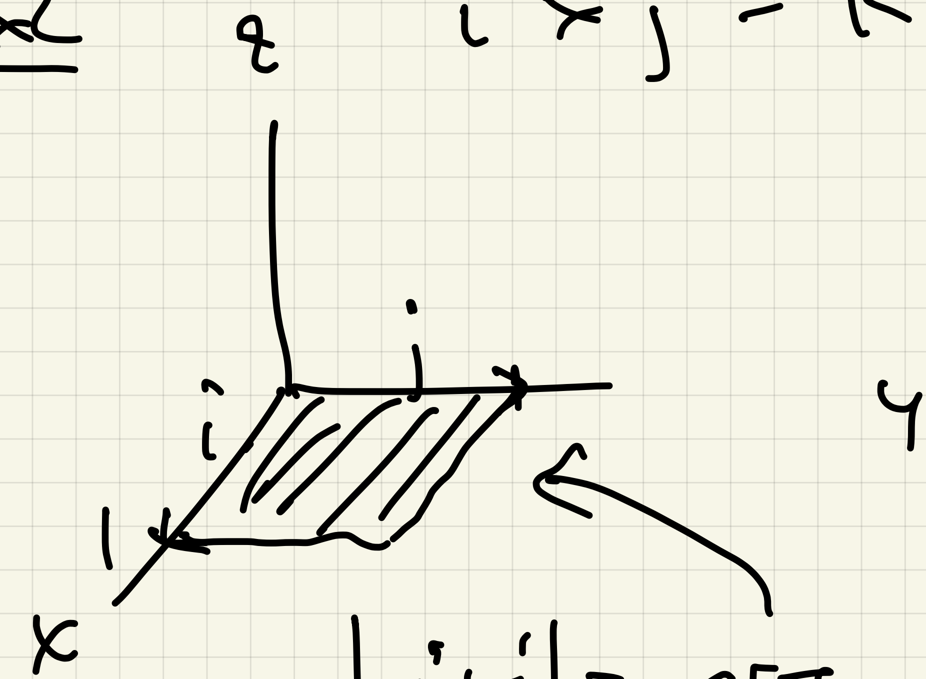
$\bar{u}$

(c) $|\bar{u} \times \bar{v}| = |\bar{u}||\bar{v}| \sin\theta$

$=$ Area of the parallelogram with edges $\bar{u}$ and $\bar{v}$.



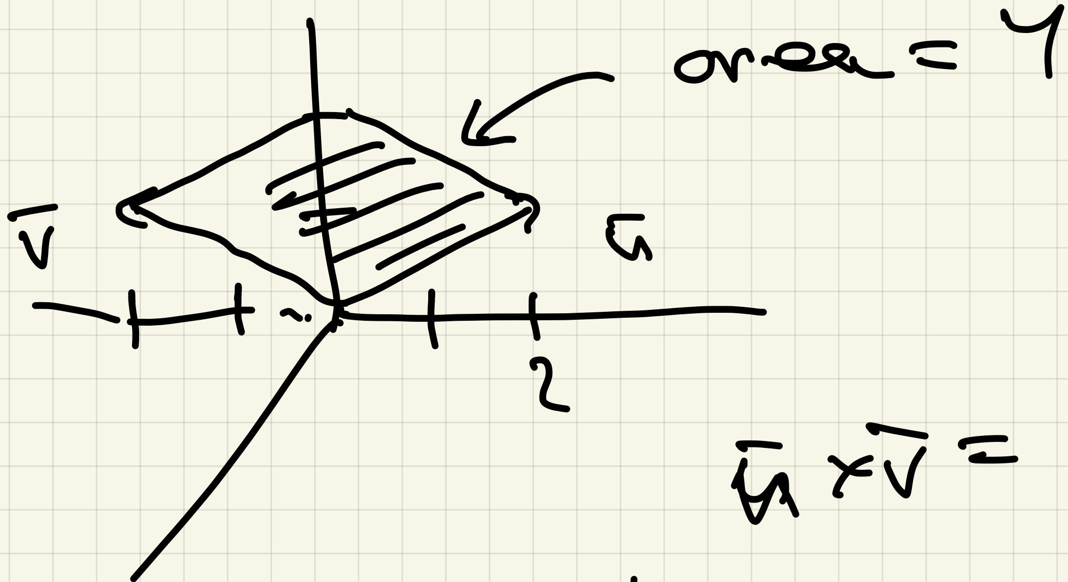
Ex2 $i \times j = k$



$|i \times j| =$ area $=$

Ex3 $\bar{u} = \langle 0, 2, 1 \rangle$

$\bar{v} = \langle 0, -2, 1 \rangle$

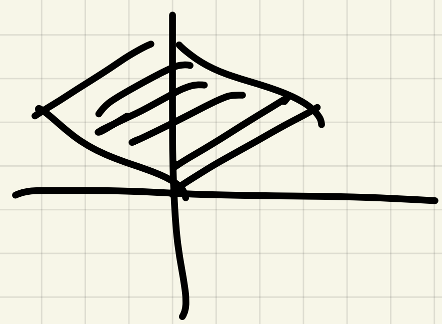


$\bar{u} \times \bar{v} = 4i$

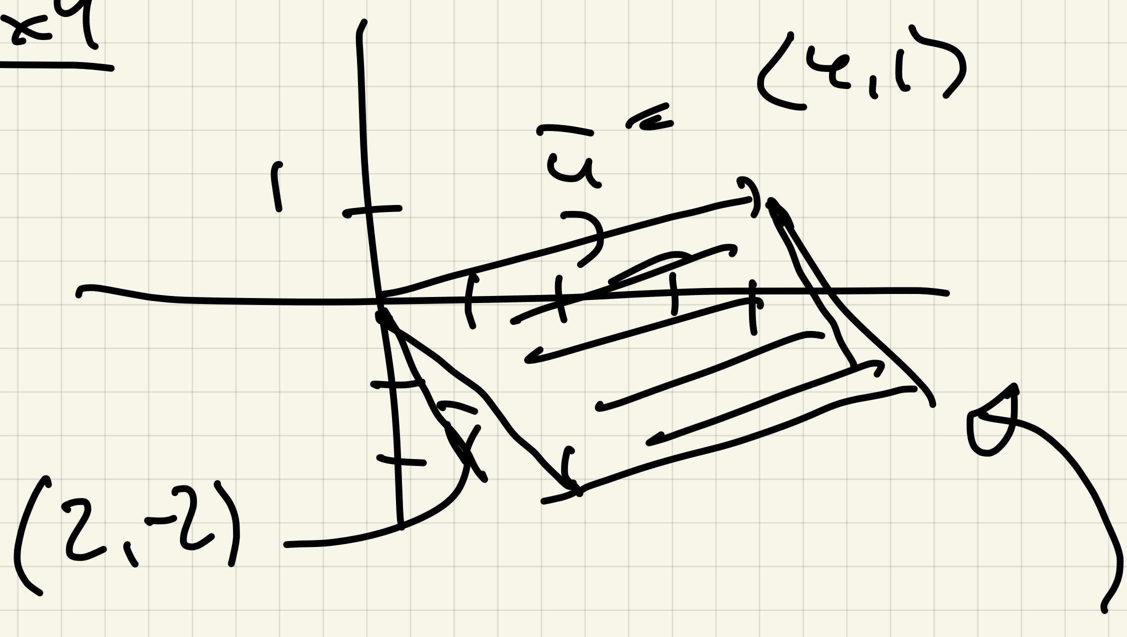
$$\bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 0 & -2 & 1 \end{vmatrix} =$$

$$i(4) + j\,0 + k\,0 = 4i \checkmark$$

$$\left\| \begin{matrix} 2 & 1 \\ -2 & 1 \end{matrix} \right\| = \text{Area}$$



Ex 4



$$\left\| \begin{matrix} 4 & 1 \\ 2 & -2 \end{matrix} \right\| = |-10| = 10$$