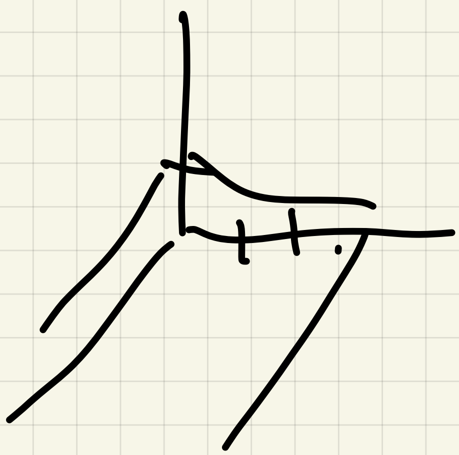
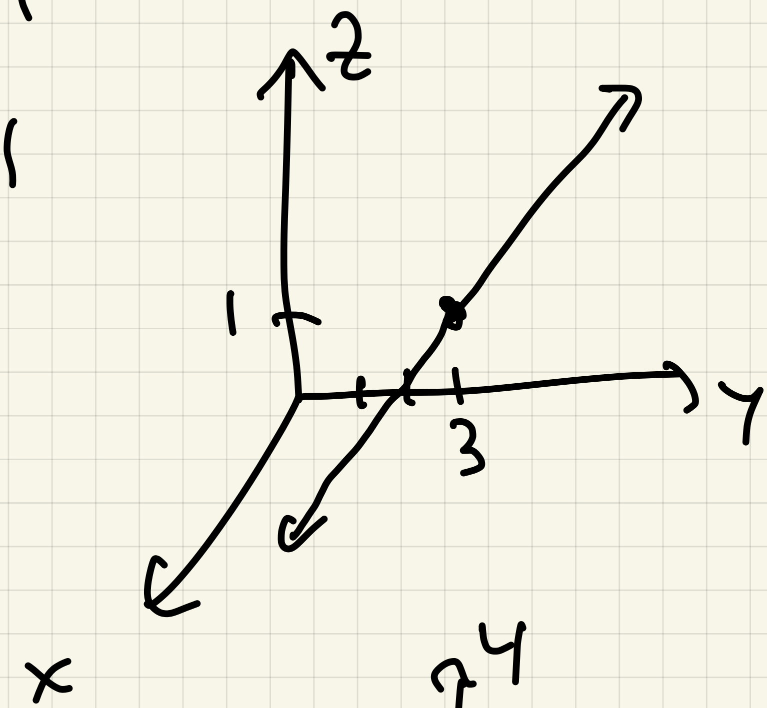
8/26/Calc3

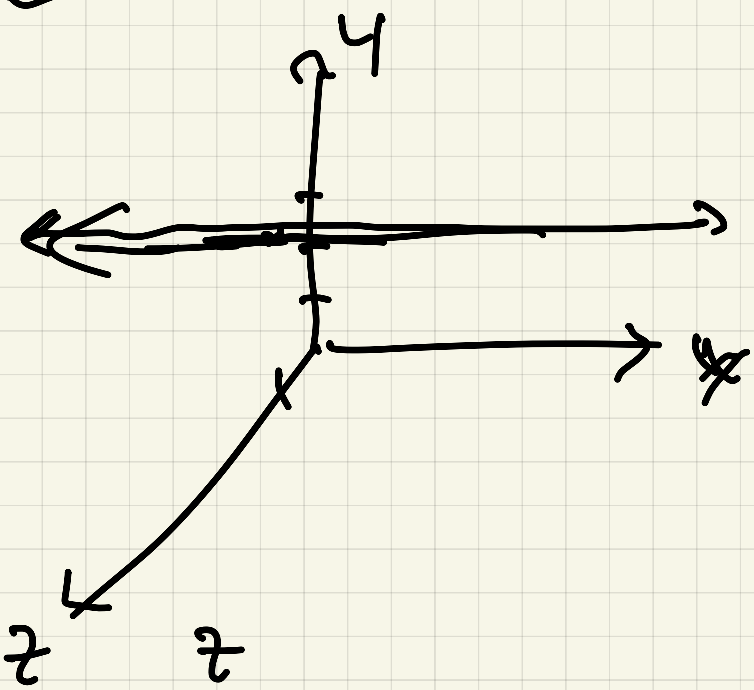
Quiz 1

1. $y=3,\ z=1$

(a)

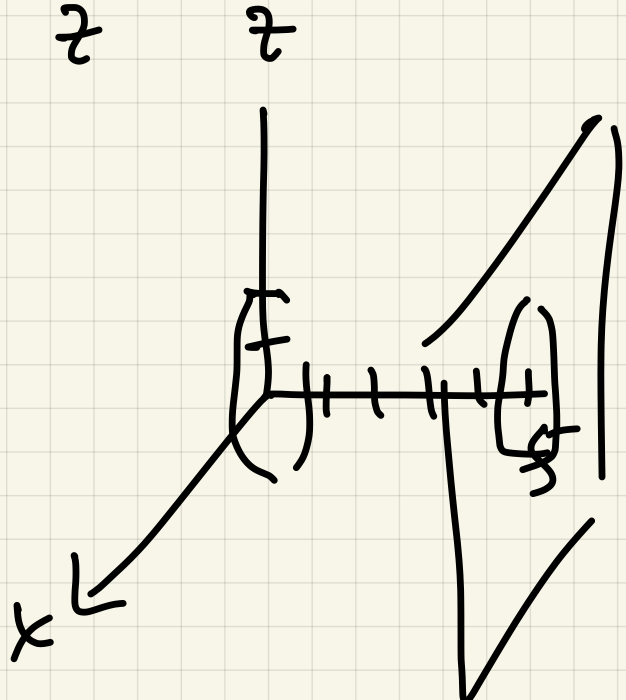






(b)

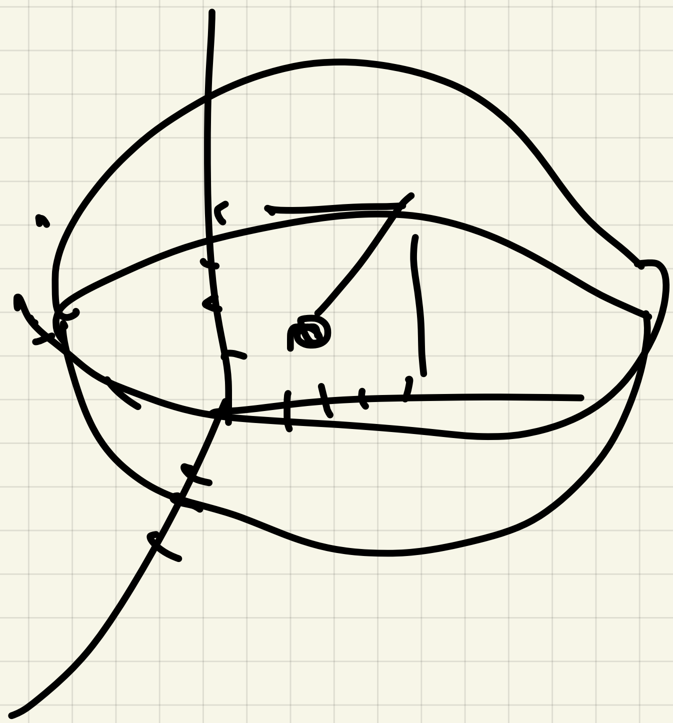
$\underline{x^2+z^2=4},\ y=5$



2. (a)

$$(x-3)^2+(y-4)^2+(z-5)^2=49$$

(b)



hint:

$$dist((3,4,5),(0,0,0))$$

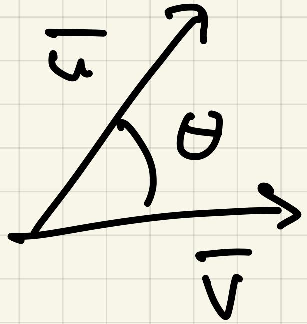
$$\sqrt{3^2+4^2+5^2}=\sqrt{50}$$

$$r=\sqrt{49}<\sqrt{50}$$

Outside

Last time dot product

$$\bar{u}\cdot\bar{v}$$



$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$$

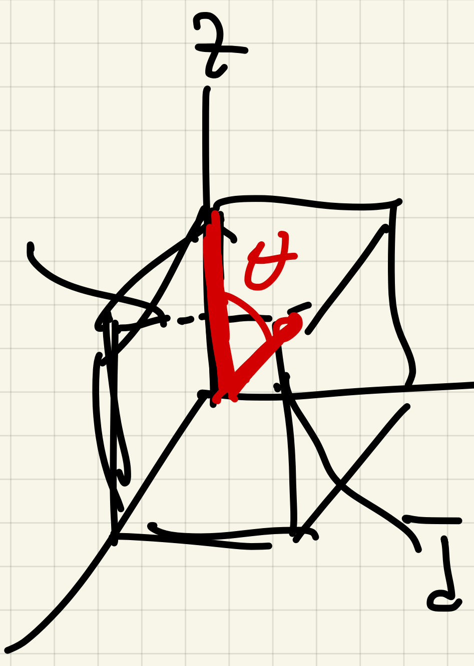
<u>Ex1</u> (a) We found that on unit cube, two diagonals meet at $60°$

$\triangle$ equilateral

$\therefore$

$\angle = 60°$

(b)



find angle between edge of cube and long diagonal

$$\bar{e} = \langle 0, 0, 1 \rangle$$

$$\bar{d} = \langle 1, 1, 1 \rangle$$

$$\cos\theta = \frac{\bar{e} \cdot \bar{d}}{|\bar{e}||\bar{d}|} = \frac{1}{1 \cdot \sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = \cos^{-1} \frac{1}{\sqrt{3}}$$

<u>Special case</u>:

$$\bar{u} \perp \bar{v} \Leftrightarrow \theta = 90^\circ \Leftrightarrow \cos\theta = 0$$

$$\Updownarrow$$

$$\bar{u} \cdot \bar{v} = 0$$

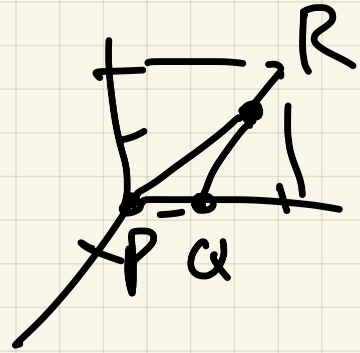
perpendicular (orthogonal, normal)

<u>Ex 2</u>

$P = (0, 0, 0)$

$Q = (0, 1, 0)$

$R = (1, 2, 2)$



Is ΔRQR right Δ?, acute Δ?, obtuse Δ?,

$\vec{PQ} = \langle 0,1,0 \rangle$

$\vec{PR} = \langle 1,2,2 \rangle$

$\vec{QR} = \langle 1,1,2 \rangle$

$\boxed{\text{not right}}$

$\angle QPR$ is $\theta = \dfrac{\vec{PQ} \cdot \vec{PR}}{|PQ||PR|} = \dfrac{2}{1 \cdot 3} = \dfrac{2}{3}$

$\theta = \cos^{-1} 2/3$

$(0,-1,0)$

$\angle PQR$

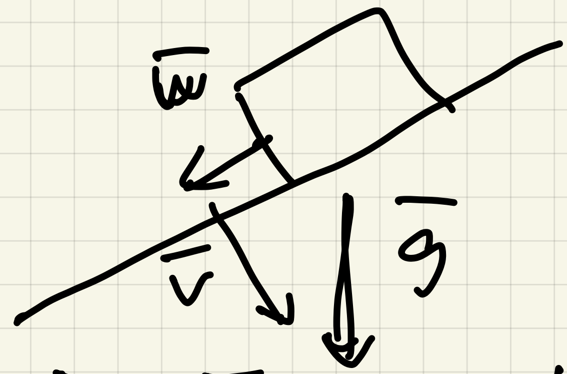
$\cos\theta = \dfrac{QP \cdot QR}{|QP||QR|}$

$= \dfrac{-1}{1 \cdot \sqrt{6}} < 0$

So $\theta > 90^\circ$

So Δ is <u>obtuse</u>.

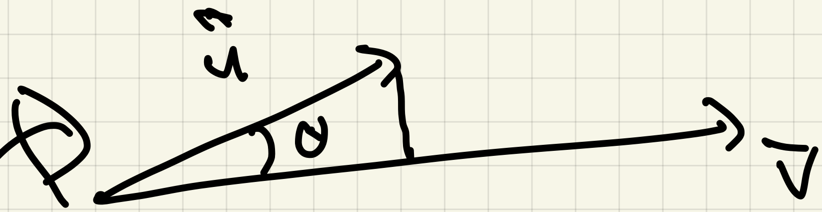
Projections:



motivation:

Find: $\bar v, \bar w$ so that

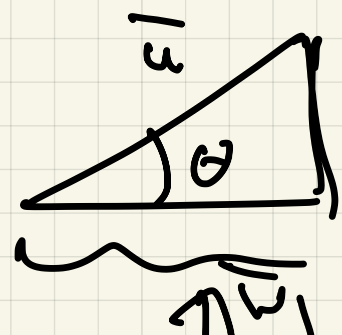
$$\bar g = \bar v + \bar w$$



Goal: write $\bar u = \bar w_1 + \bar w_2$

$w_1 \parallel \bar v$

$w_2 \perp \bar v$



$\bar w_1 = \mathrm{Proj}_{\bar v} \bar u$

How to compute it?

Picture: $|\bar w_1| = |\bar u| \cdot \cos\theta$

direction of $\bar w_1$ is $\dfrac{\bar v}{|\bar v|}$

so $\overline{w}_1 = |\overline{u}| \cos\theta \cdot \frac{\overline{v}}{|\overline{v}|}$

$\Vert$

$$\mathrm{Proj}_{\overline{v}}\overline{u} = \cancel{|\overline{u}|} \frac{\overline{u}\cdot\overline{v}}{\cancel{|\overline{u}|}|\overline{v}|} \cdot \frac{\overline{v}}{|v|}$$

$$= \frac{\overline{u}\cdot\overline{v}}{|v|^2}\,\overline{v}$$