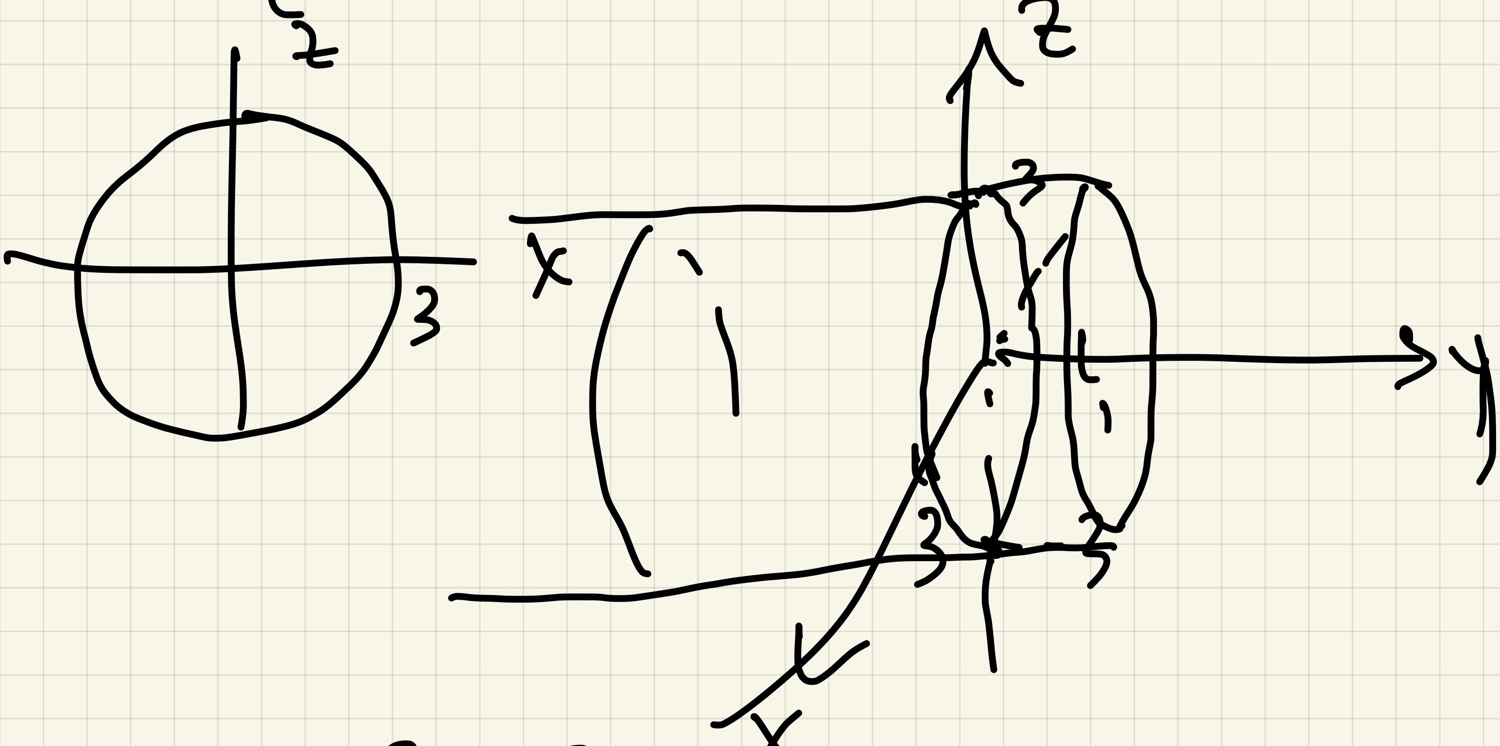
8/20 / Calc 3

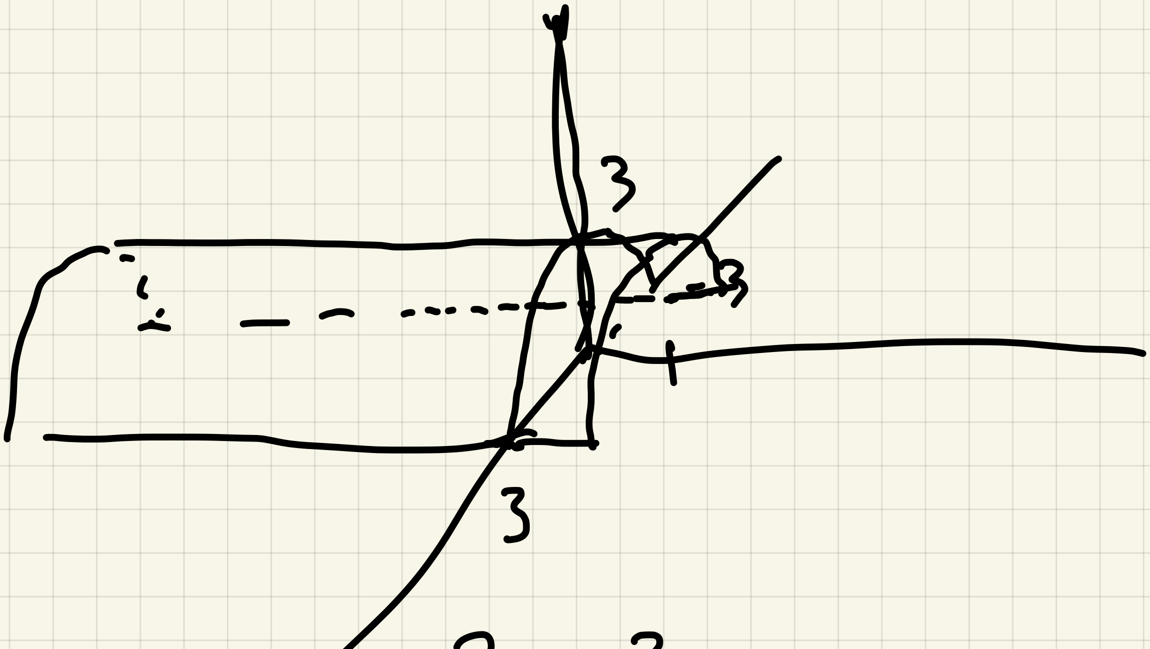
Last time: describe sets in $\mathbb{R}^3$

Ex 1 sketch

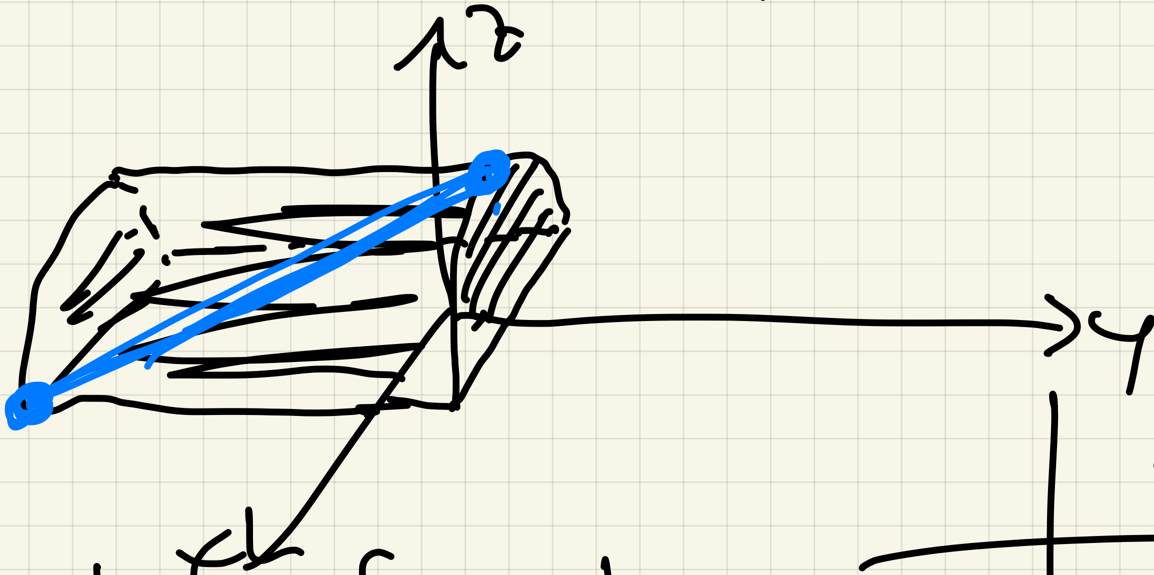
(a) $x^2+z^2=9, \ y \le 1$

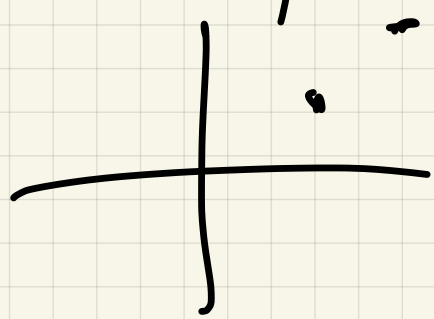


(b) $x^2+z^2=9, \ y \le 1, \ z \geq 0$



(c) $x^2+z^2 \le 9,\ -5 \le y \le 1,\ z \ge 0$





Distance formula

The distance between $P_1=(x_1,y_1,z_1)$ and $P_2=(x_1,y_2,z_2)$ is

$$|\overline{P_1P_2}| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

Ex2 dist from $(0,1,3)$ to $(3,-5,0)$ is

(a)

$$d = \sqrt{(0-3)^2 + (1--5)^2 + (3-0)^2}$$

$$= \sqrt{9 + 36 + 9} = \sqrt{54}$$

(b)

dist from $(0,1,3)$ to xy-plane is 3

dist from $(0,1,3)$ to yz-plane is 0

dist from $(0,1,3)$ to xz-plane is 1

Standard equation for sphere

The sphere of radius r

centered at (a, b, c)

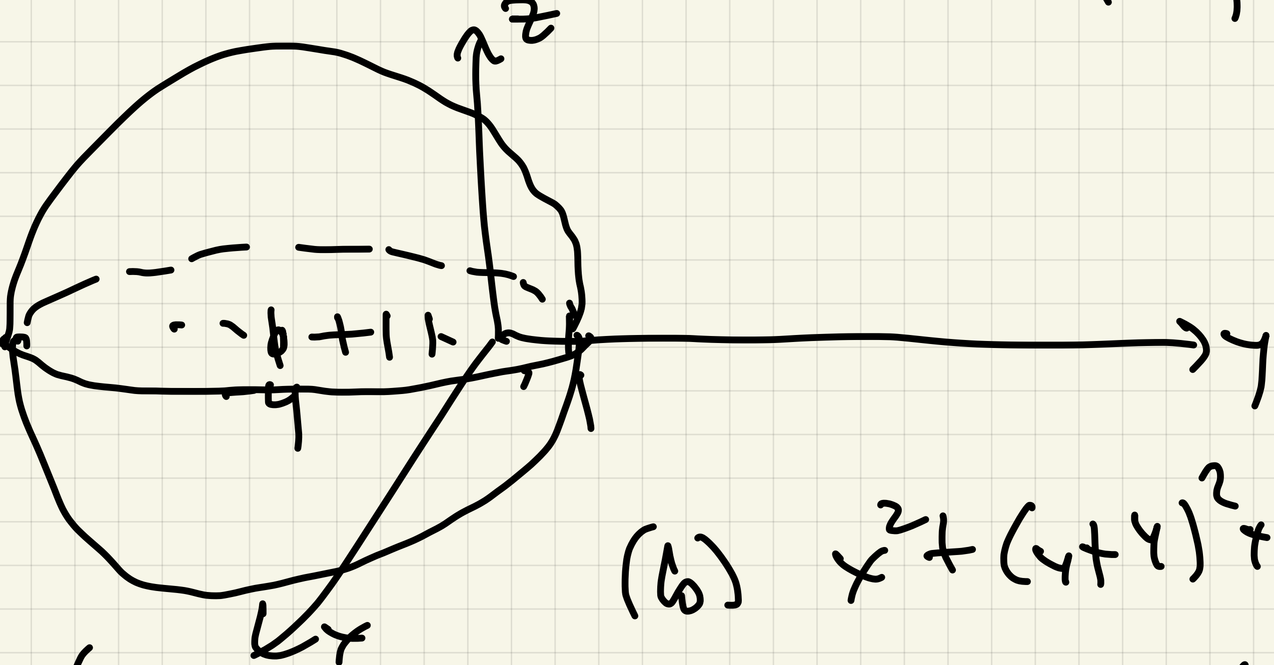
has equation

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

<u>Ex3</u> Sketch

(a) $x^2+(y+4)^2+z^2=25$

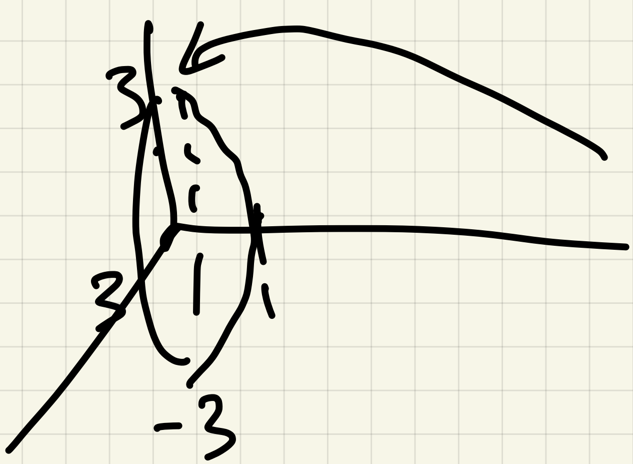
$r=5$, center $(0,-4,0)$



$(0,-4,0)$

(b) $x^2+(y+4)^2+z^2=25$

$y \geq 0$



$x^2+\underbrace{4^2}_{16}+z^2=25$

$x^2+z^2=9$

Ex4 Find center/radius of sphere

(a) $x^2+y^2+z^2+10x-6y-7z=60$

Idea complete square

$x^2+10x+25 \quad y^2-6y+9 \quad z^2-7z+\frac{49}{4}=$

$60+25+9+\frac{49}{4}$

$(x+5)^2+(y-3)^2+(z-\frac{7}{2})^2=$

$94+\frac{49}{4}=\frac{425}{4}$

Center $(-5,3,+\frac{7}{2})$ $\quad r=\frac{\sqrt{425}}{2}$

§11.2 Vectors $\mathbf{v}$, $\vec{v}$

A <u>vector</u> $\bar{v}$ is a directed line segment from pt A to B

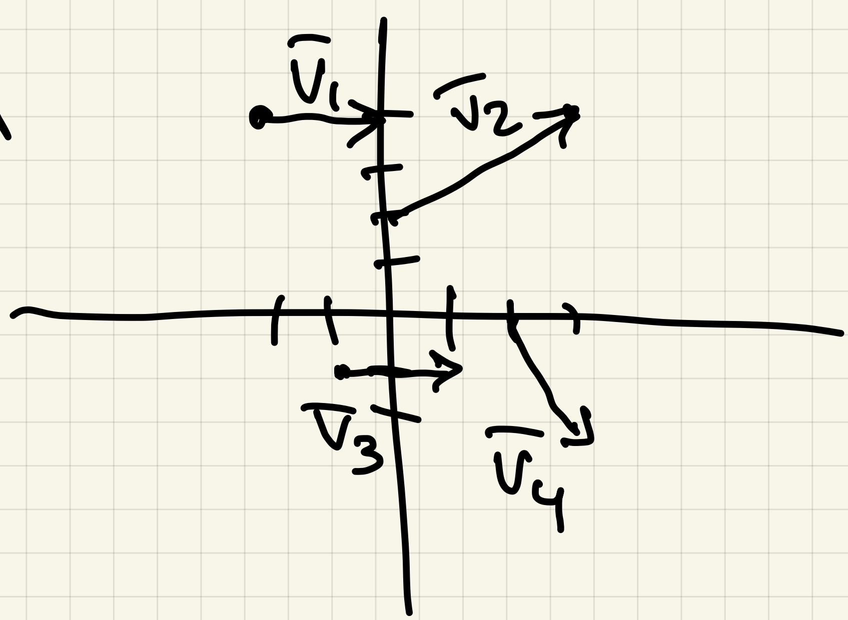
<u>initial point</u> (A)   <u>terminal point</u> (B)

It's <u>length</u> is $|AB|$

Two vectors are <u>equal</u> if they have same length and direction

<u>Ex1</u>

In $\mathbb{R}^2$ (a)

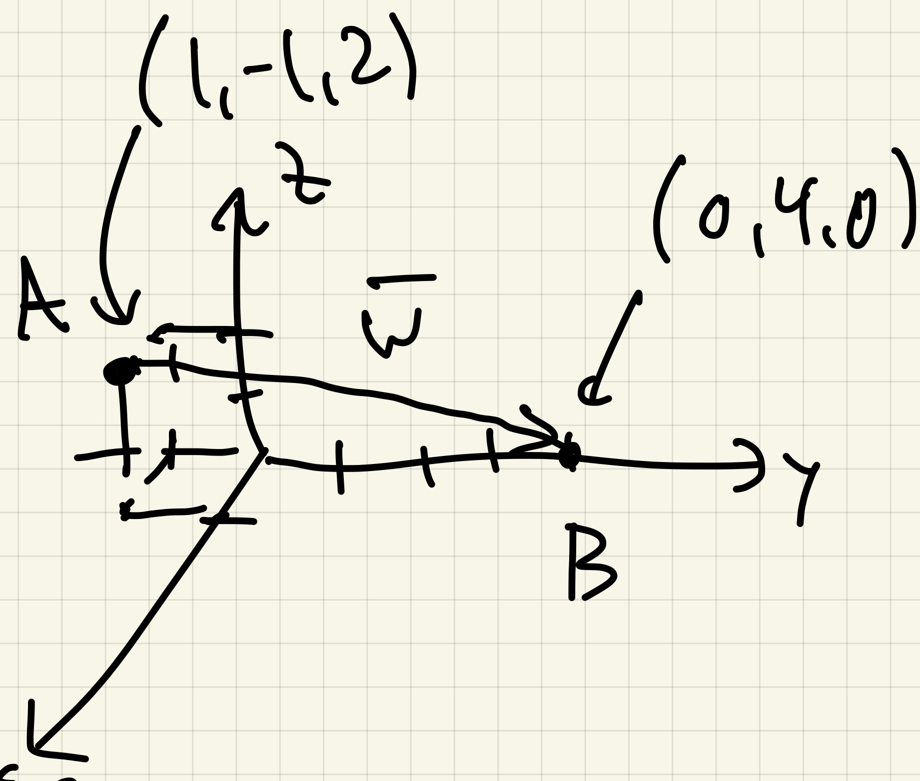


$\|\bar{v}_1\| = 2$

$\|\bar{v}_2\| = \sqrt{13}$

$\|v_4\| = \sqrt{5}$

(b)



Component form

Given vector $\bar{v}$, can find equal vector with initial point = origin and terminal point B

component form is given by entries for B

$(a, b) \rightarrow \bar{v} = \langle a, b \rangle$

$(a, b, c) \quad \bar{v} = \langle a, b, c \rangle$

Ex2 (a) $\bar{v}_1 = \langle 2, 0 \rangle$

$\bar{v}_2 = \langle 3, 2 \rangle$

$\bar{v}_4 = \langle 1, -2 \rangle$

(b) $\bar{w} = \langle -1, 5, -2 \rangle$

just subtract coords of init/term pts

Vector operations

If $\bar{u} = \langle u_1, u_2 \rangle$, $\bar{v} = \langle v_1, v_2 \rangle$,

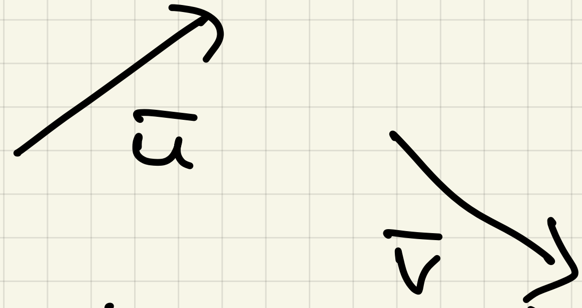
$k \in \mathbb{R}$ (scalar)

1. sum $\bar{u} + \bar{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$
2. difference $\bar{u} - \bar{v} = \langle u_1 - v_1, u_2 - v_2 \rangle$
3. scalar multiple

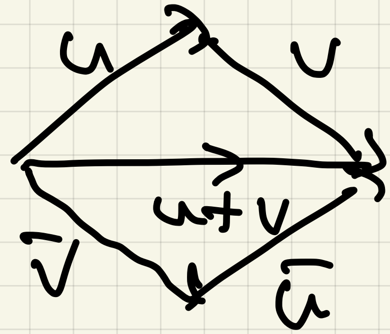
$$k \cdot \bar{u} = \langle k \cdot u_1, k \cdot u_2 \rangle$$

Geometry visual interpretation

addition "tip to tail"

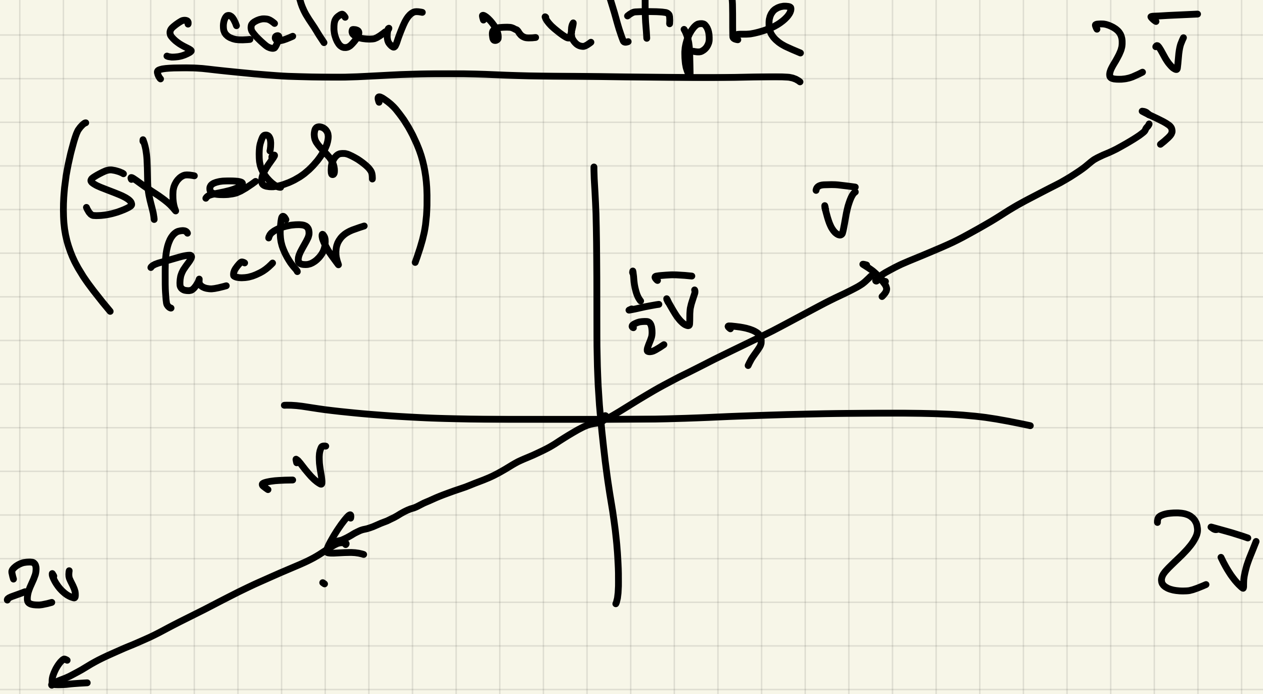


$\bar{u}+\bar{v}$



scalar multiple

(stretch factor)



Ex2 For $\bar{u}, \bar{v}$, find

(a) components of $\bar{u}, \bar{v}$

(b) $\bar{u}+\bar{v}$

(c) $\overline{u-v}$

(d) $-\bar{v}$

(e) $\bar{u}-3\bar{v}$

(f) sketch each

(0,8)

Ex) $\bar{u} = \langle 3, 2 \rangle$

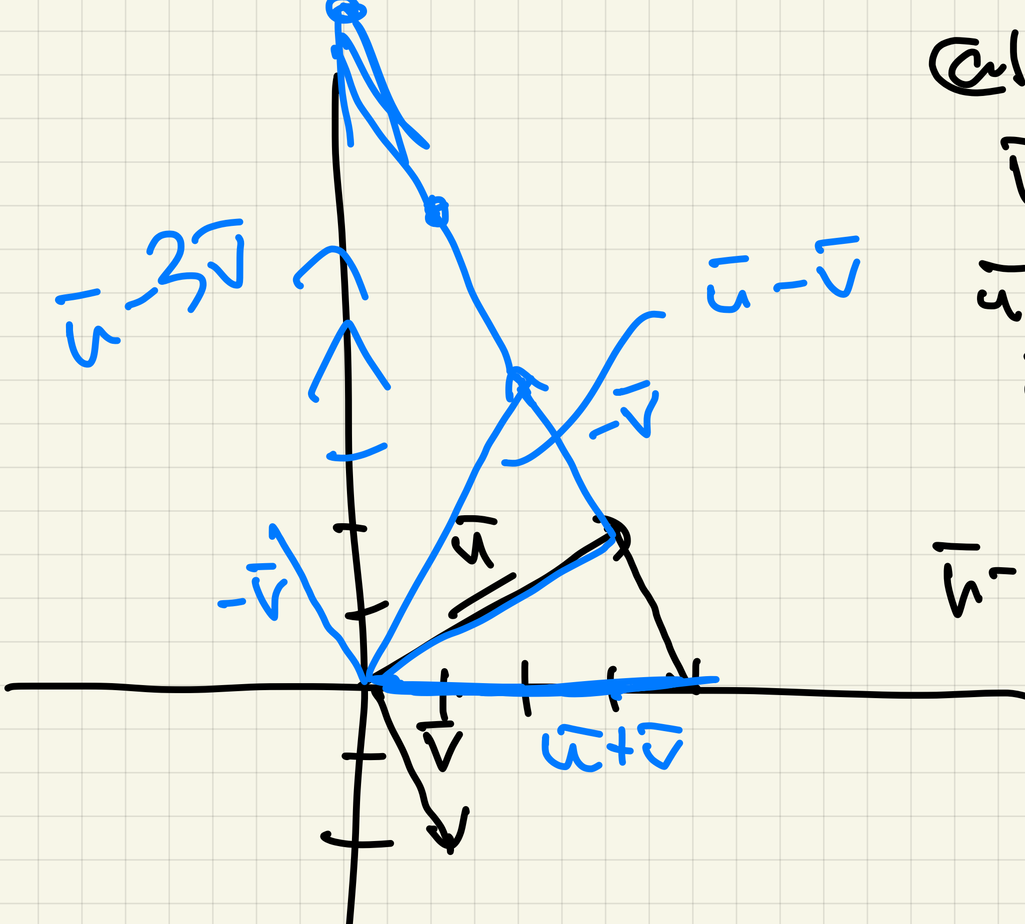
$\bar{v} = \langle 1, -2 \rangle$

$\bar{u} + \bar{v} = \langle 4, 0 \rangle$

$\bar{u} - \bar{v} = \langle 2, 4 \rangle$

$-\bar{v} = \langle -1, 2 \rangle$

$\bar{u} - 3\bar{v} = \langle 0, 8 \rangle$



<u>Defn</u> A <u>unit vector</u> is a vector of length 1

<u>Ex3</u>

$\langle 1, 0 \rangle$ $\qquad$ $\langle 0, -1 \rangle$

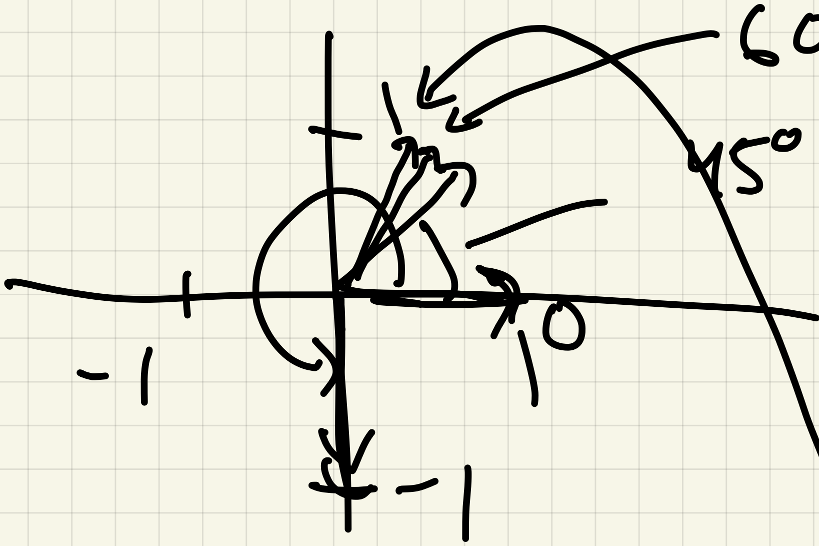
$\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\langle \frac{3}{5}, \frac{4}{5} \rangle$

$\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

60°

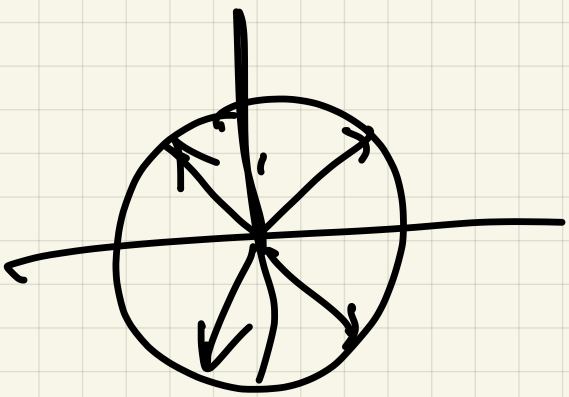
45° = $\pi/4$ rad



angle arctan $4/3$

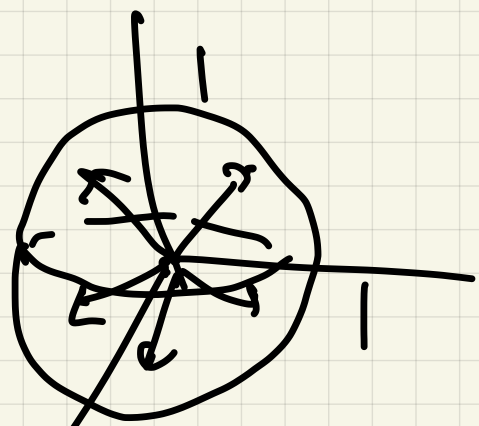
$\frac{3\pi}{2}$

<u>Note</u> in 2D, unit vectors $\updownarrow$ unit circle



<u>1, 3))</u>

$\bar{v} = \langle \frac{3}{7}, \frac{6}{7}, \frac{-2}{7} \rangle$



<u>Standard unit vectors:</u>

2D

$i = \langle 1, 0 \rangle$

$j = \langle 0, 1 \rangle$

3D

$i = \langle 1, 0, 0 \rangle$

$j = \langle 0, 1, 0 \rangle$

$k = \langle 0, 0, 1 \rangle$

Decomposition into length and direction

If $\bar{v}$ is a vector, then

$$\bar{v} = \|\bar{v}\| \cdot \frac{\bar{v}}{\|\bar{v}\|}$$

length

direction