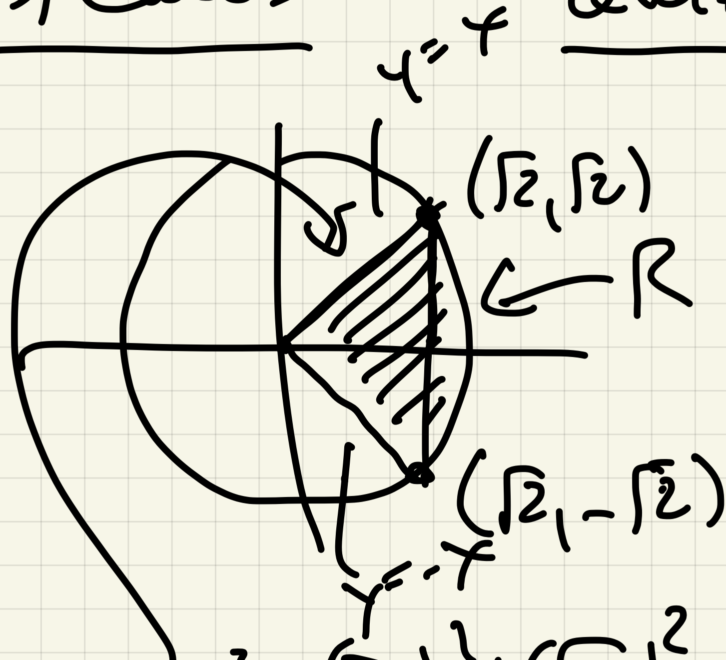
4/3 Calc 3 Quiz 14 avg 80%



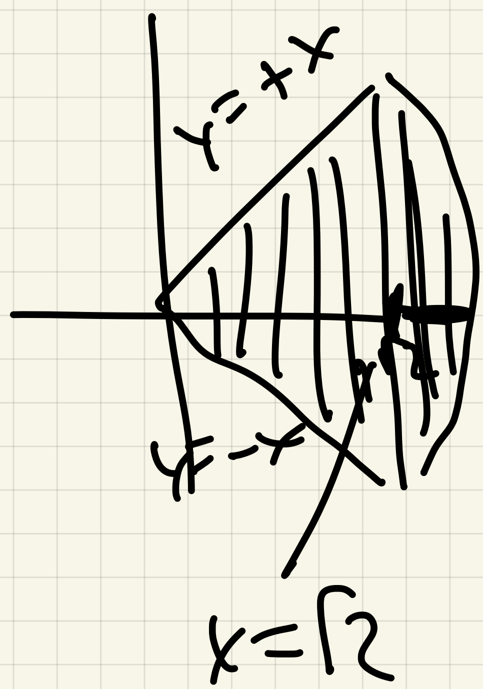
$$\iint_R x\,dA$$

$$r^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 2+2=4$$

$$\boxed{r=2}$$

$$x^2+y^2=4$$

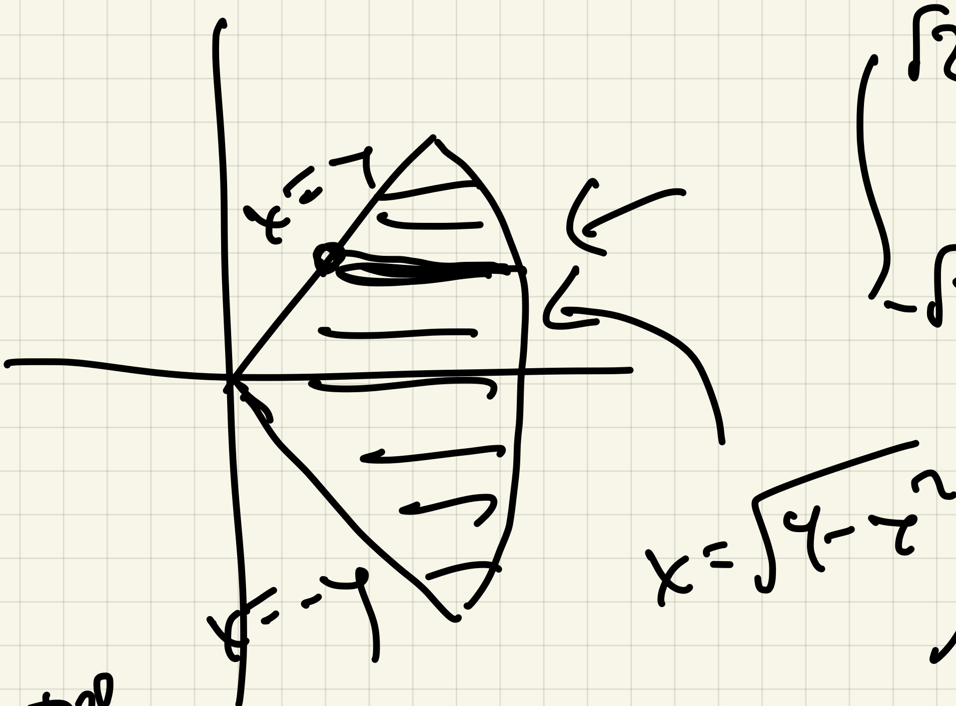
left



$$\Rightarrow \int_0^{\sqrt{2}}\int_{-x}^{x} x\,dy\,dx$$

$$+ \int_{\sqrt{2}}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x\,dy\,dx$$

Also



$$\int_{-\sqrt{2}}^{\sqrt{2}}\int_{|y|}^{\sqrt{4-y^2}} x\,dx\,dy$$

$$\|$$

top

$$\int_{0}^{\sqrt{2}}\int_{y}^{\sqrt{4-y^2}} x\,dx\,dy + \int_{-\sqrt{2}}^{0}\int_{-y}^{\sqrt{4-y^2}} x\,dx\,dy$$

2. $$\int_{-\pi/4}^{\pi/4}\int_{0}^{2} \overbrace{r\cos\theta}^{x}\; r\,dr\,d\theta$$

conversion

$$\frac{r^3}{3}\cos\theta\Big|_0^2 =$$

$$\int_{-\pi/4}^{\pi/4} \frac{8}{3}\cos\theta \, d\theta = \frac{8}{3}\sin\theta \Big|_{-\pi/4}^{\pi/4} =$$

$$\frac{8}{3}\frac{1}{\sqrt{2}} - \frac{8}{3}\left(-\frac{1}{\sqrt{2}}\right) = \frac{8}{3}\frac{2}{\sqrt{2}} =$$

$$\frac{16}{3\sqrt{2}} = \frac{8\sqrt{2}}{3}$$

<u>Last time:</u> triple integrals

$$\iiint_B f(x,y,z)\, dV$$

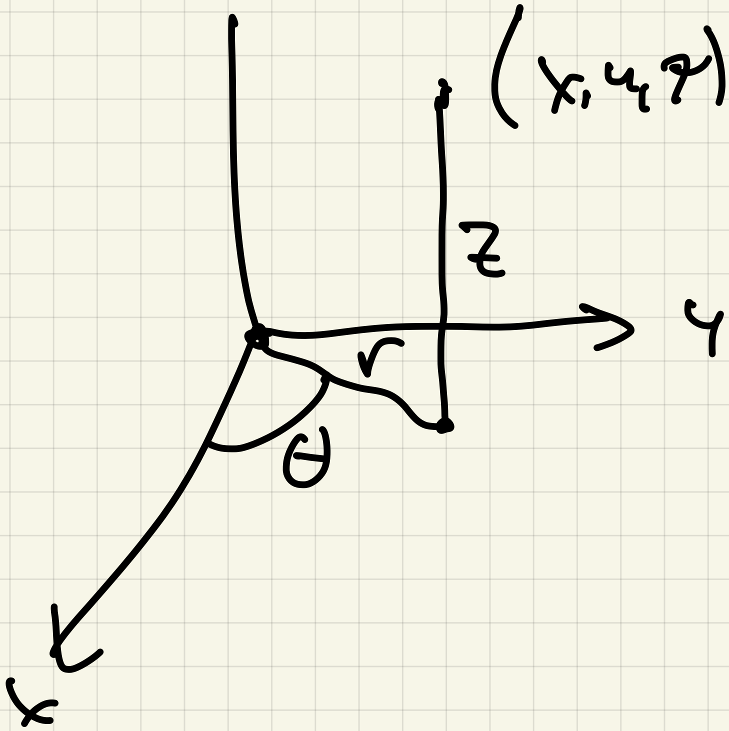
3D solid $\longrightarrow$ B

end points for B

C   <u>x, y, z coordinates</u> $\Rightarrow$

<u>Cylindrical coordinates:</u>

$(x, y, z) \sim (r, \theta, z)$



$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

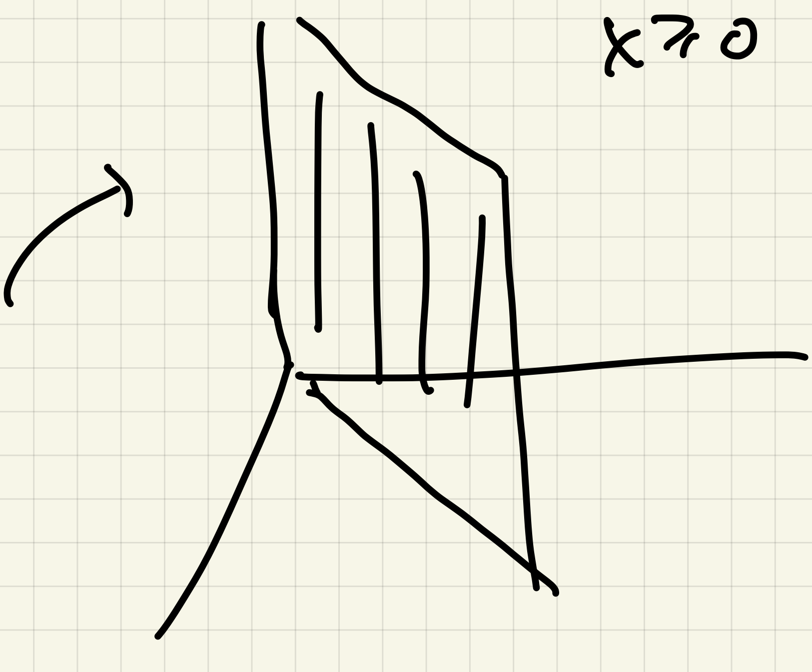
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\frac{y}{x}$$

Ex (a) $\theta = \pi/4$

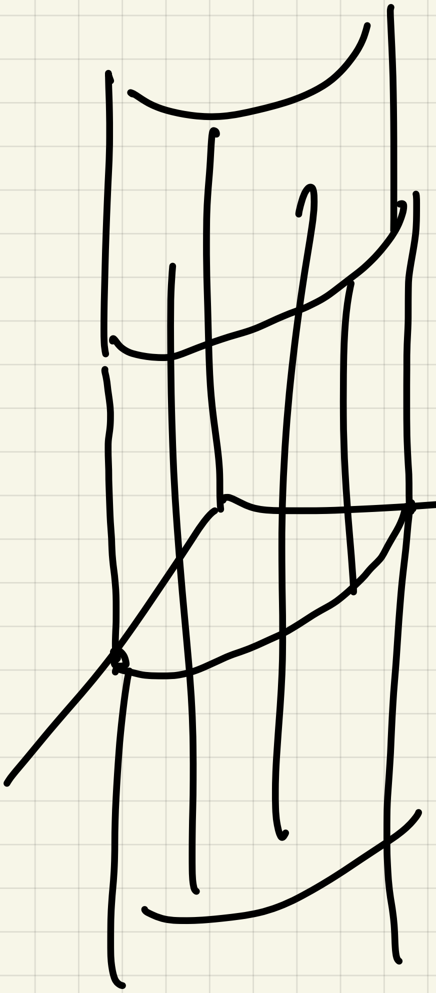
plane $x = y$

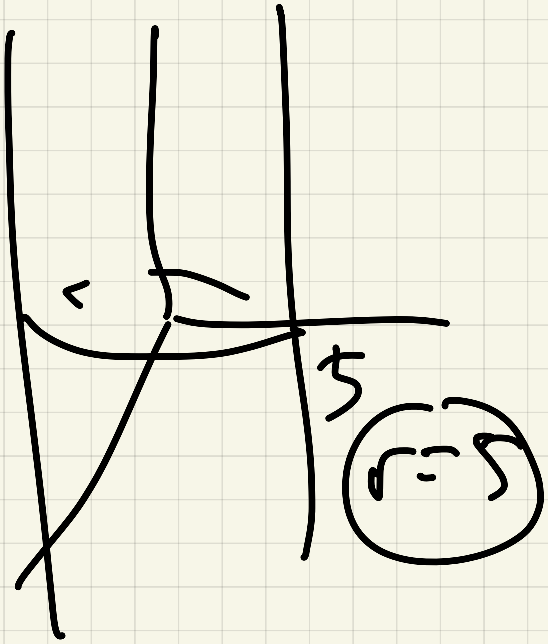
$x \geq 0$ $(r \geq 0)$



(c) $\boxed{r=5}$

$\boxed{0 \le \theta \le \pi/2}$





$x^2+y^2=25,$

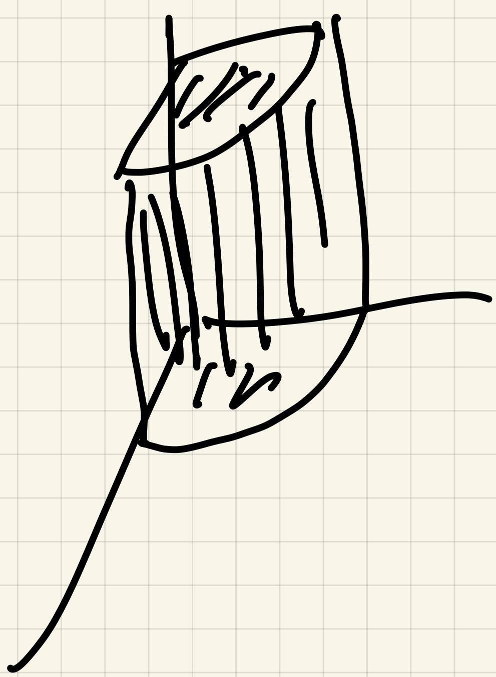
$x \ge 0$

$y \ge 0$

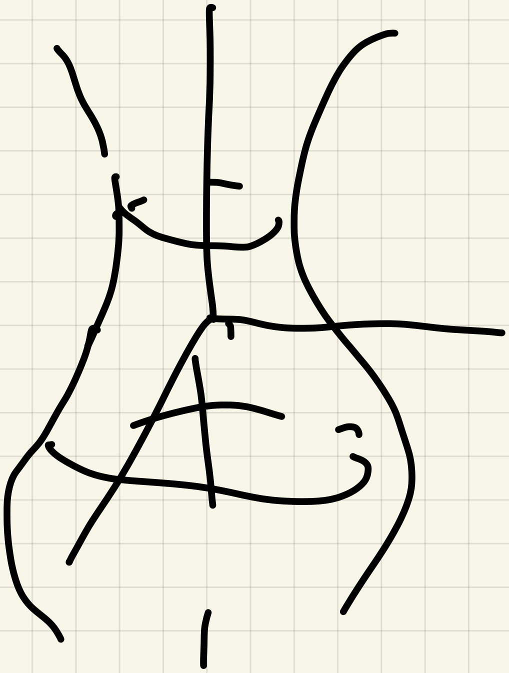
surface

(d) $0 \le r \le 5$

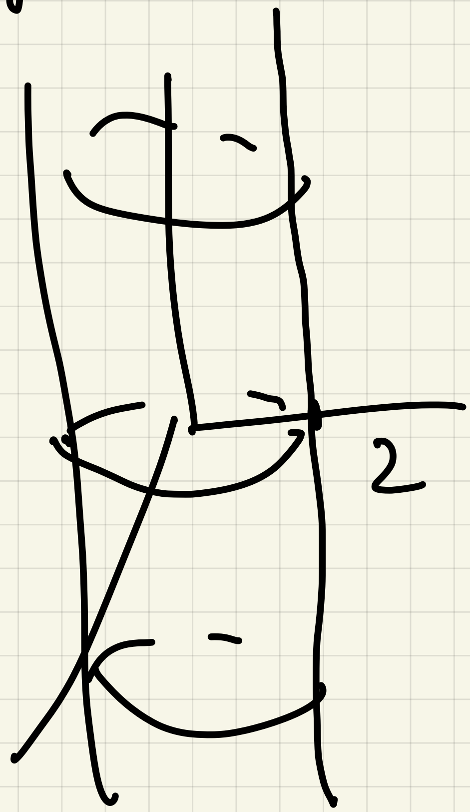
$0 \le \theta \le \pi/2$



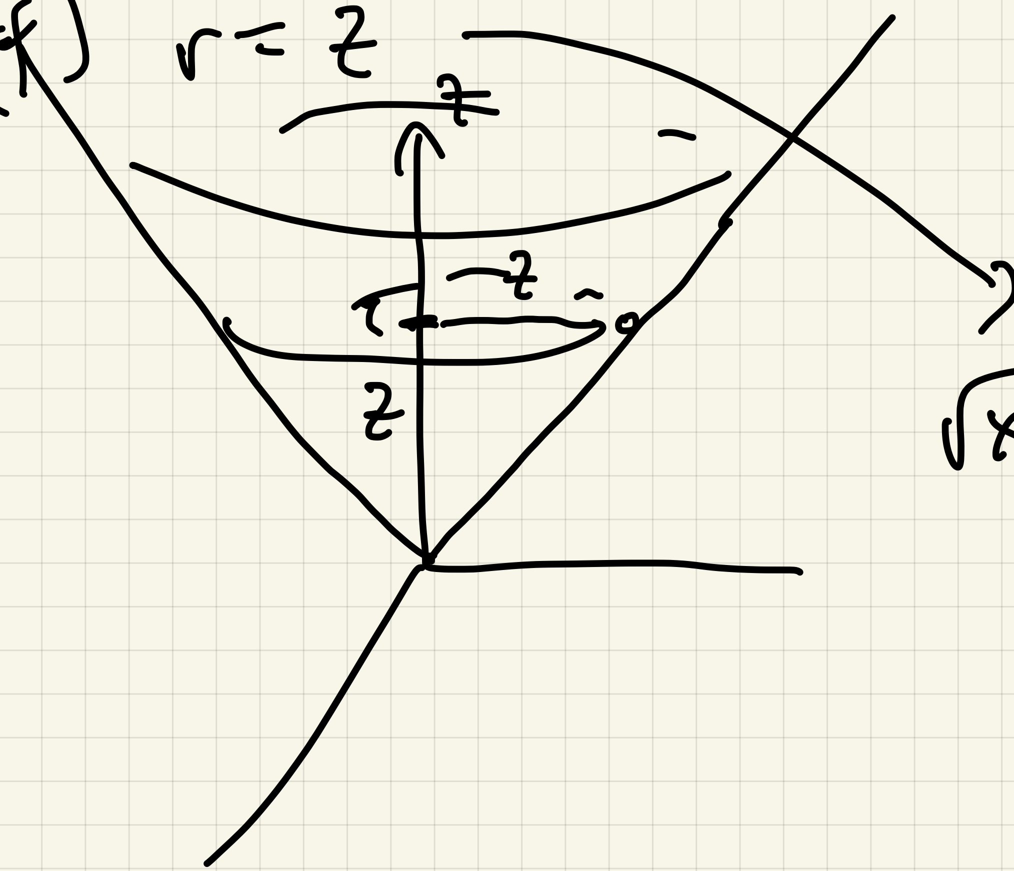
(e) Surfaces of revolution:



$r = 2$



(f) $r = z$

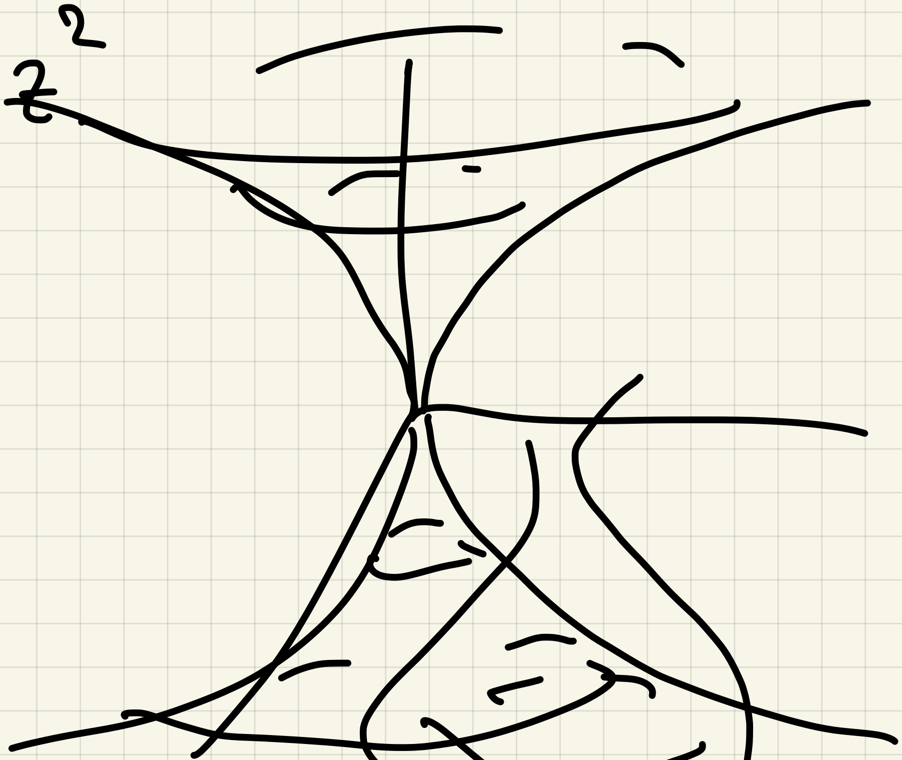


$\sqrt{x^2+y^2} = z$

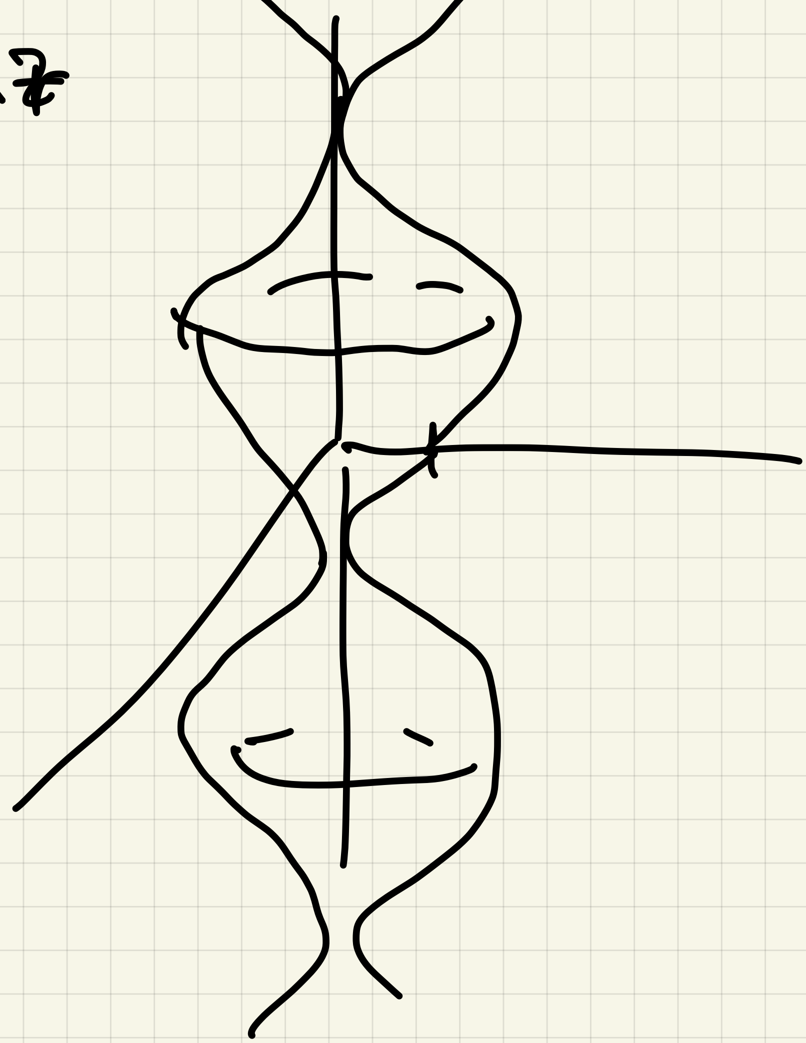
(g) $r = z^2$

$\Downarrow$

$x^2+y^2 = z^4$



(h) $r = 1 + \sin z$



<u>Integrals:</u>

$$\iiint_B f(x,y,z)\,dV =$$

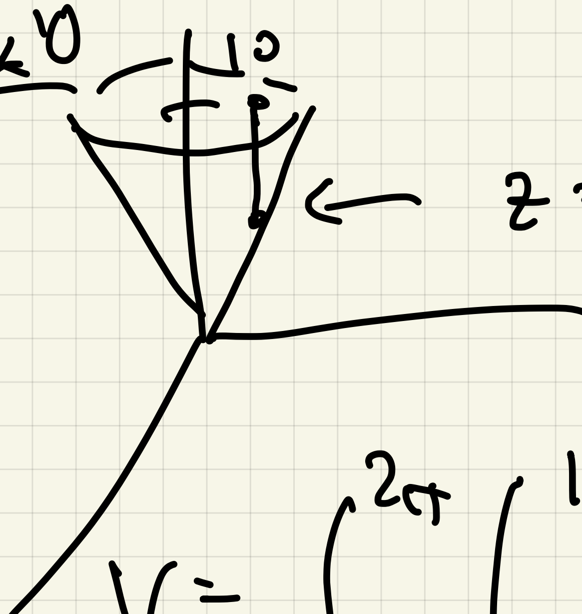
conversion

$$\parallel$$

$$\iiint_B f(r\cos\theta, r\sin\theta, z)\; r \; \underline{dz\,dr\,d\theta}$$

six orders

Ex 0



$$z = 2\underbrace{\sqrt{x^2+y^2}}_{r} = \underline{z = 2r}$$

$$r = \frac{z}{2}$$

$$V = \int_0^{2\pi}\int_0^{10}\int_0^{z/2} r\,dr\,dz\,d\theta$$

$\theta \quad z \quad r$

$$\parallel$$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{5}\int_{z=2r}^{10} \underline{r\,dz}\,dr\,d\theta$$

$$rz\Big|_{z=2r}^{z=10} = 10r - 2r^2$$

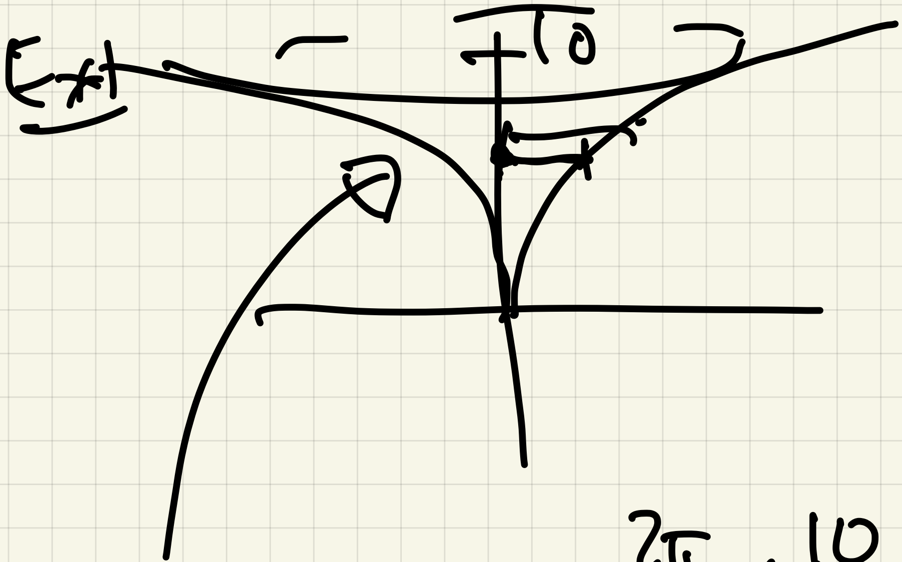
$$\int_0^5 10r - 2r^2\,dr =$$

$$5r^2 - \frac{2}{3}r^3\Big|_0^5 =$$

$$125 - \frac{2}{3}125 = \frac{125}{3}$$

$$\int_0^{2\pi} \frac{125}{3}\,d\theta = \frac{250\pi}{3}\ \checkmark$$

Ex1



$x^2+y^2=z^4$

$r^2=z^4$

$r=z^2$

$$\text{Volume} = \underbrace{\int_0^{2\pi}}_{\theta}\underbrace{\int_0^{10}}_{z}\underbrace{\int_0^{z^2} r\,dr}_{r}\,dz\,d\theta$$

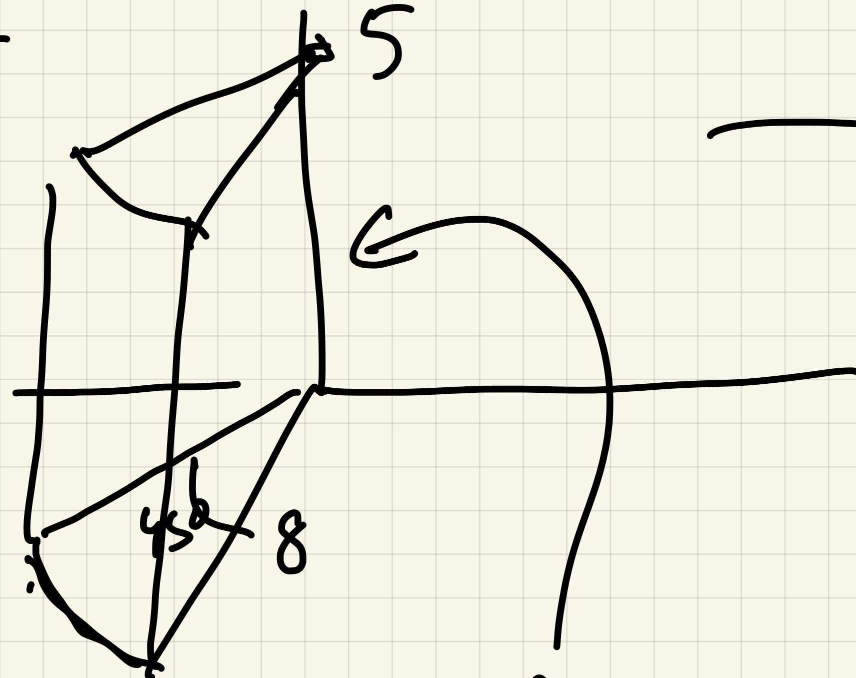
$$\left.\frac{r^2}{2}\right|_0^{z^2} = \frac{z^4}{2}$$

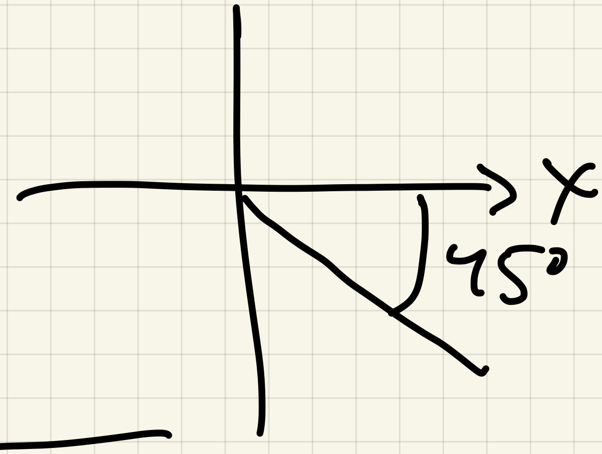
$$\int_0^{10}\frac{z^4}{2}\,dz = \left.\frac{z^5}{10}\right|_0^{10} =$$

$$\frac{100000}{10} = \int_0^{2\pi} 10000\,d\theta =$$

$$20000\pi$$

Ex 2





Volume =

$$\int_{-\pi/4}^{0}\int_{0}^{8}\int_{0}^{5} r\,dz\,dr\,d\theta$$

θ r z

$$rz\Big|_0^5$$

$$\int_0^8 5r \qquad \frac{5}{2}r^2\Big|_0^8 =$$

$$\frac{5\cdot 64}{2} = 160$$

$$\int_{-\pi/4}^{0} 160\, d\theta = 160\theta \Big|_{-\pi/4}^{0} =$$

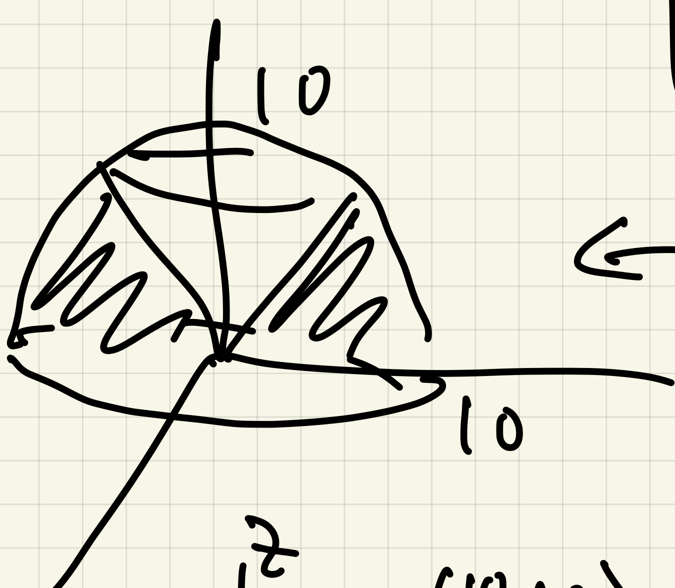
$$160\left(0 - -\frac{\pi}{4}\right) = 40\pi.$$

<u>Ex 3</u> Find Volume solid

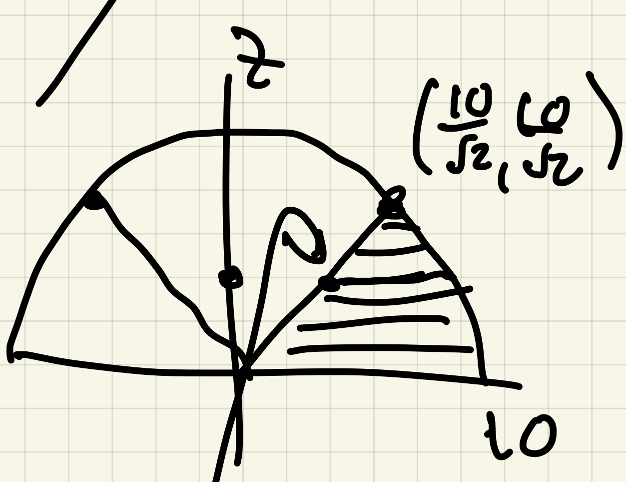
inside $\boxed{x^2+y^2+z^2 = 100}$

outside cone $\boxed{z = \sqrt{x^2+y^2}}$

$\boxed{z \geq 0}$



← difficult using x/y/z



cross section

$$z = \sqrt{x^2+y^2} \rightarrow r = z$$

$$\underbrace{x^2+y^2}+z^2 = 100$$

$$r^2 + z^2 = 100 \Rightarrow r = \sqrt{100-z^2}$$

$$\underline{\text{Volume}} = \underset{\theta}{\int_0^{2\pi}} \underset{z}{\int_0^{10/\sqrt{2}}} \boxed{\underset{r}{\int_z^{\sqrt{100-z^2}}} r \, dr \, dz \, d\theta}$$

$$\frac{1}{2} r^2 \Big|_z^{\sqrt{100-z^2}} = \underbrace{\frac{1}{2}(100 - z^2) - \frac{1}{2} z^2}$$

$$= 50 - z^2$$

$$\int_0^{10/\sqrt{2}} 50 - z^2 \, dz =$$

$$50z - z^3/3 \Big|_0^{10/\sqrt{2}} =$$

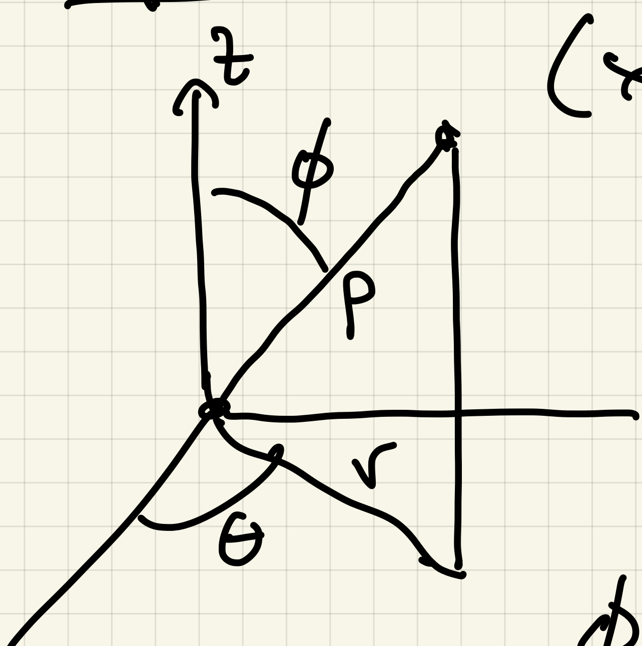
$$\frac{500}{\sqrt{2}} - \frac{\cancel{1000}}{3\cdot \cancel{2}\sqrt{2}} =$$

$$\frac{500}{\sqrt{2}} - \frac{500}{3\cdot\sqrt{2}} =$$

$$\underbrace{\left(1-\frac{1}{3}\right)}_{\frac{2}{3}}\left(\frac{500}{\sqrt{2}}\right) = \frac{1000}{3\sqrt{2}} =$$

$$\int_0^{2\pi} \frac{500\sqrt{2}}{3} = \frac{1000\sqrt{2}\,\pi}{3}$$

Spherical coordinates:

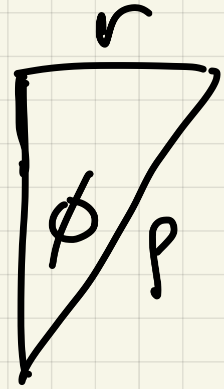


θ = polar angle

ρ = distance to $(0,0,0)$

ϕ = angle from positive z-axis

$r = \rho \sin\phi$



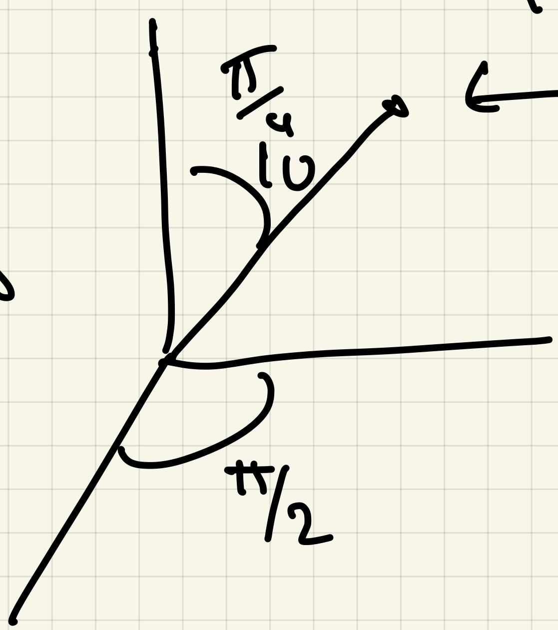
$$\boxed{\begin{aligned} z &= \rho\cos\phi \\ x &= r\cos\theta = \rho\sin\phi\cos\theta \\ y &= r\sin\theta = \rho\sin\phi\sin\theta \end{aligned}}$$

$\rho = \sqrt{x^2+y^2+z^2}$

$$\boxed{\begin{aligned} 0 \le \phi \le \pi \\ 0 \le \theta \le 2\pi \\ 0 \le \rho \end{aligned}}$$

Ex 4 (a) $\rho = 10,\ \theta = \pi/2,\ \phi = \pi/4$

(a)

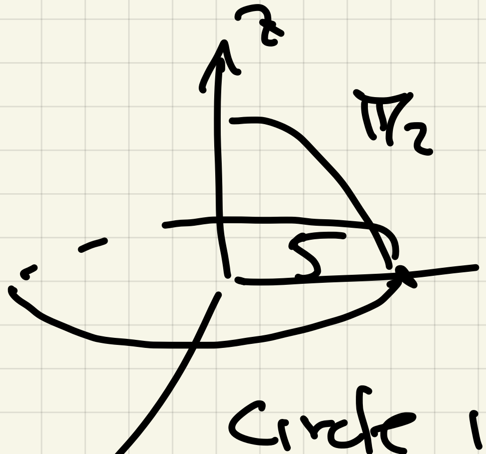


$\leftarrow \left(0, \frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}}\right)$

x y z

(b) $\rho = 5$

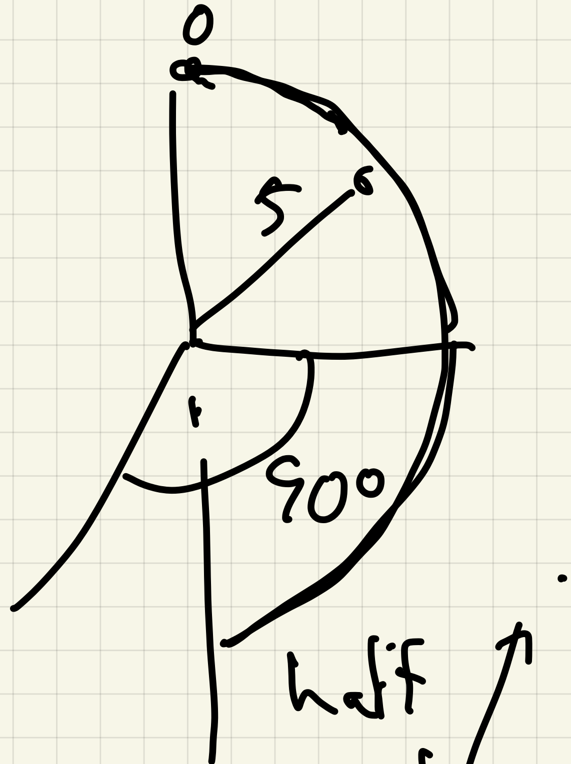
$\phi = \pi/2$



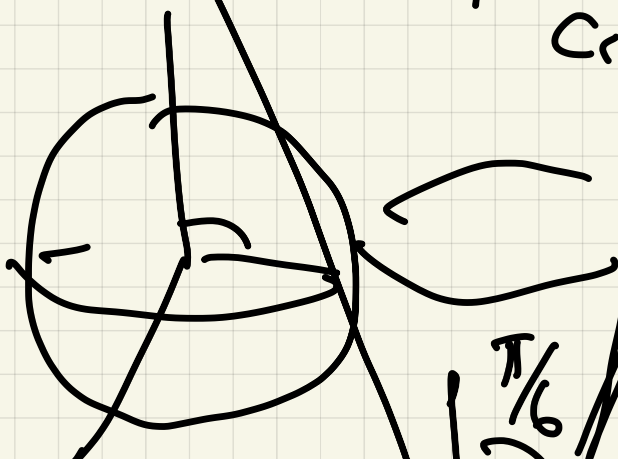
circle in xy plane

(c) $\rho = 5$

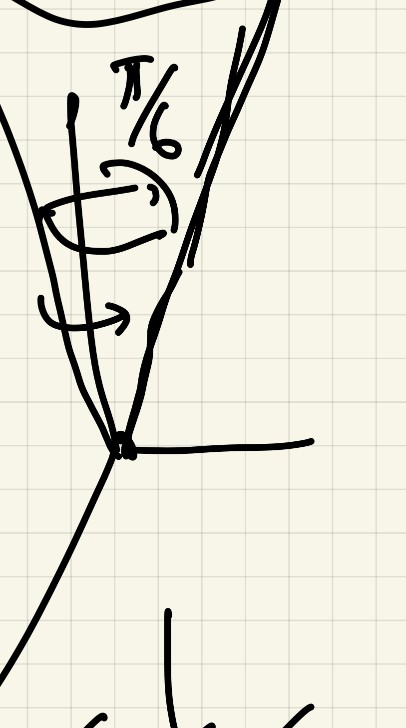
$\theta = \pi/2$



(d) $\rho = 5$

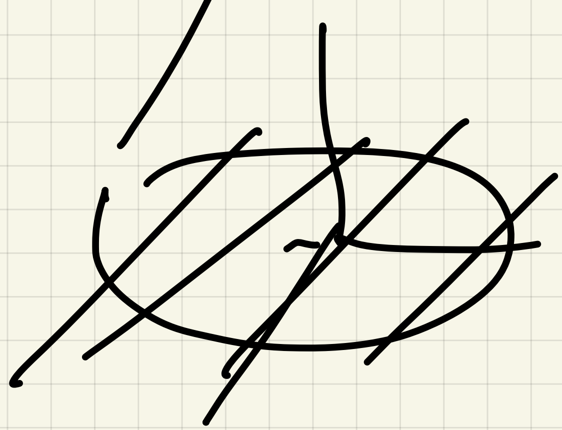


(e) $\phi = \pi/6$



(f) $\phi = \pi/2$

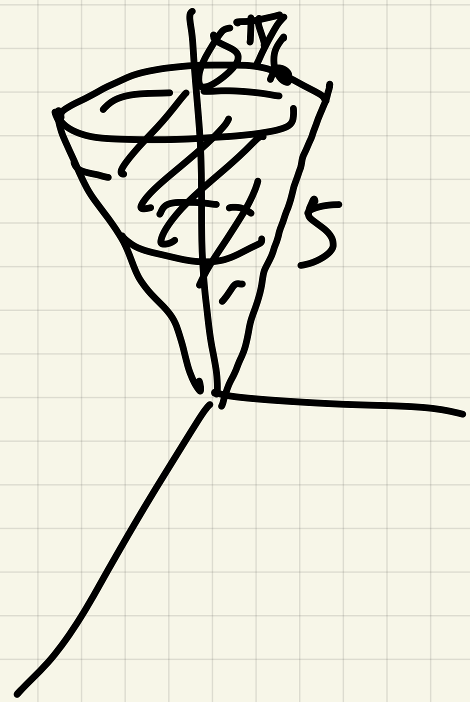
xy plane



(g) $0 \le \rho \le 5$

$0 \le \phi \le \pi/6$

$0 \le \theta \le 2\pi$



Integration!

$$\iiint_B f(x,y,z)\, dV$$

$\Vert$ spherical

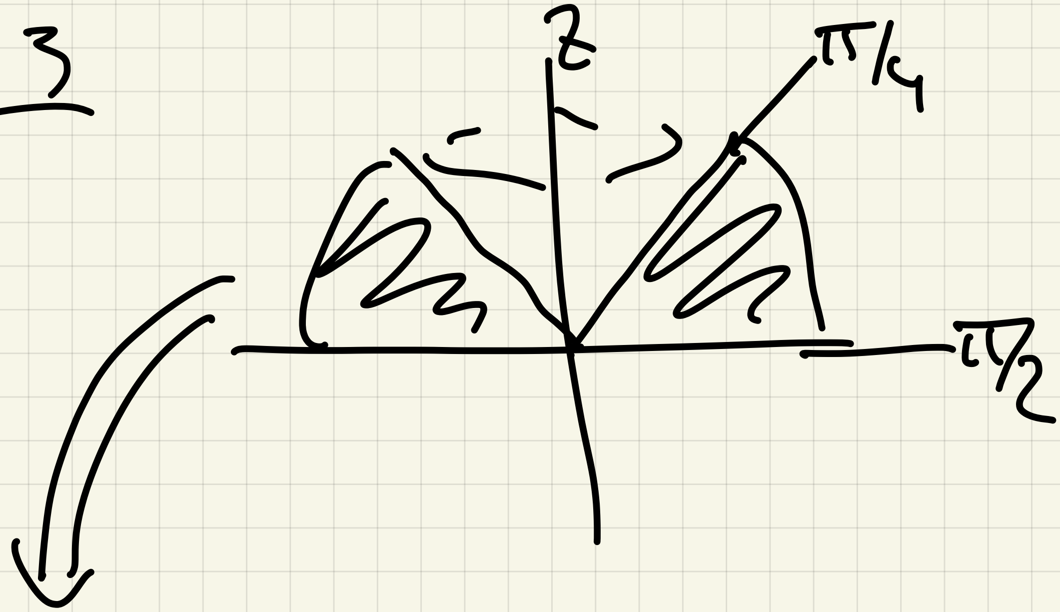
$$\iiint f(\overset{x}{\rho\sin\phi\cos\theta},\ \overset{y}{\rho\sin\phi\sin\theta},\ \overset{z}{\rho\cos\phi})$$

$$\boxed{\rho^2\sin\phi}\ d\rho\, d\phi\, d\theta$$

$\Uparrow$

conversion factor

Ex 3



$$\text{Volume} = \iiint_B 1\, dV =$$

$$\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{10} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\rho^3/3 \Big|_0^{10} = \int_{\pi/4}^{\pi/2} \frac{1000}{3} \sin\phi \, d\phi =$$

$$-\frac{1000}{3}\cos\phi \Big|_{\pi/4}^{\pi/2} = \left(0 - -\frac{1000}{3}\frac{1}{\sqrt{2}}\right)$$

$$\int_0^{2\pi} \frac{1000}{3\sqrt{2}}\, d\theta = \frac{2000\pi}{3\sqrt{2}} \checkmark$$