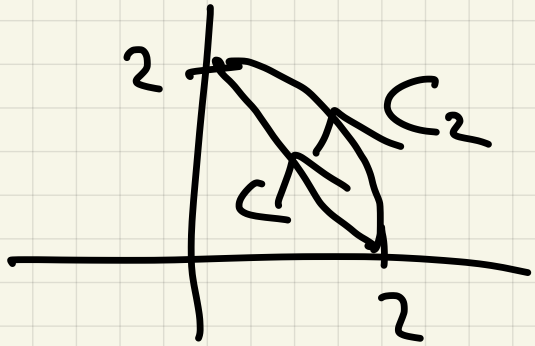
4/24/Calc3   Quiz 18

$r(t) = \langle \overset{x}{2-t}, \overset{y}{t} \rangle$

$0 \le t \le 2$



1. $\int_{C_1} F \cdot dr$ $F(x,y) = \langle 4y, 6x^2 \rangle$

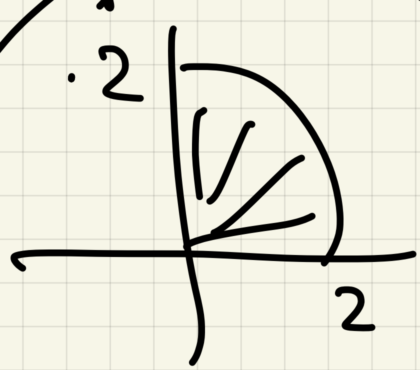
$$\int_0^2 4y \frac{dx}{dt} + 6x^2 \frac{dy}{dt} \, dt$$

$$\int_0^2 4t(-1) + 6(2-t)^2 \cdot 1 \, dt$$

$$= -2t^2 + 2(t-2)^3 \Big|_0^2 = 8$$

2. D: $r(t) = (2\cos t, 2\sin t)$

$0 \le t \le \pi/2$



$x^2 + y^2 = 4$

$$y = \sqrt{4-x^2}$$

$$r(t) = \left(t, \sqrt{4-t^2}\right)$$

$$0 \le t \le 2$$

wrong way!

$$r(t) = \left(2-t, \sqrt{4-(2-t)^2}\right)$$

$$0 \le t \le 2$$

Also: $x = \sqrt{4-y^2}$

$$r(t) = \left(\sqrt{4-t^2}, t\right) \quad 0 \le t \le 2$$

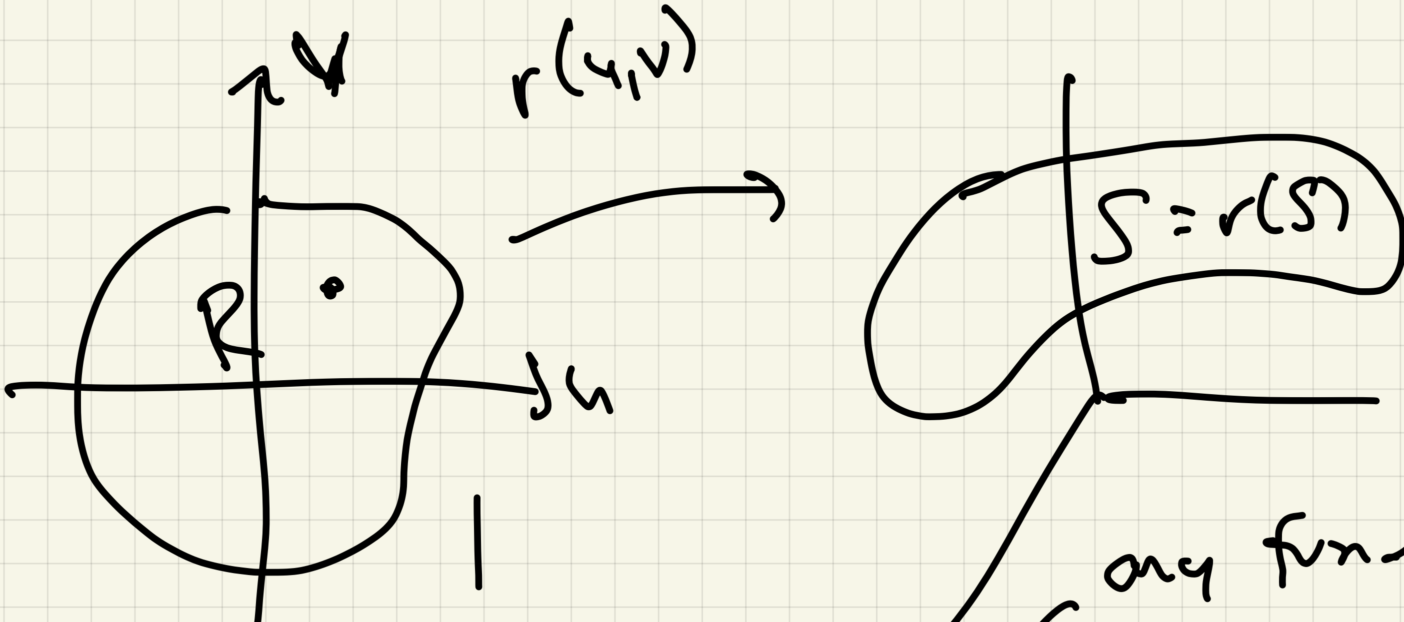
$$r = (\underline{2\cos t}, \underline{2\sin t})$$

$$\int_C \underline{4y}\, dx + 6x^2\, dy$$

$$= \int_0^{\pi/2} 8\sin t(-2\sin t) + 6(2\cos t)^2 \cdot 2\cos t\, dt$$

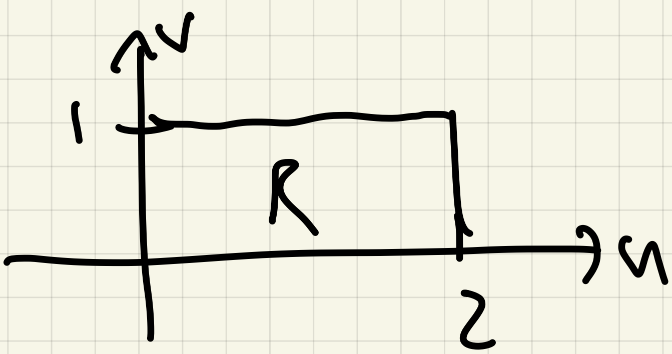
$$= \int_0^{\pi/2} -16\sin^2 t + 48\cos^3 t\, dt$$

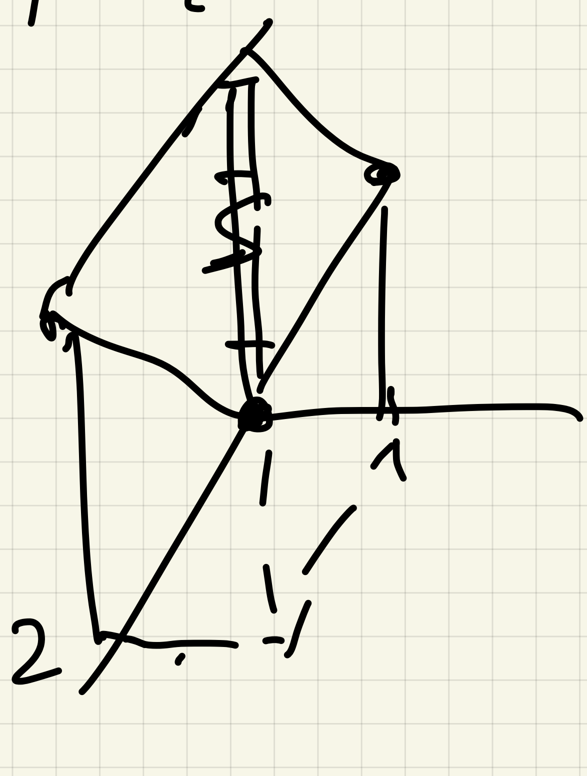
Last time: Parametric surfaces and surface integrals



any function

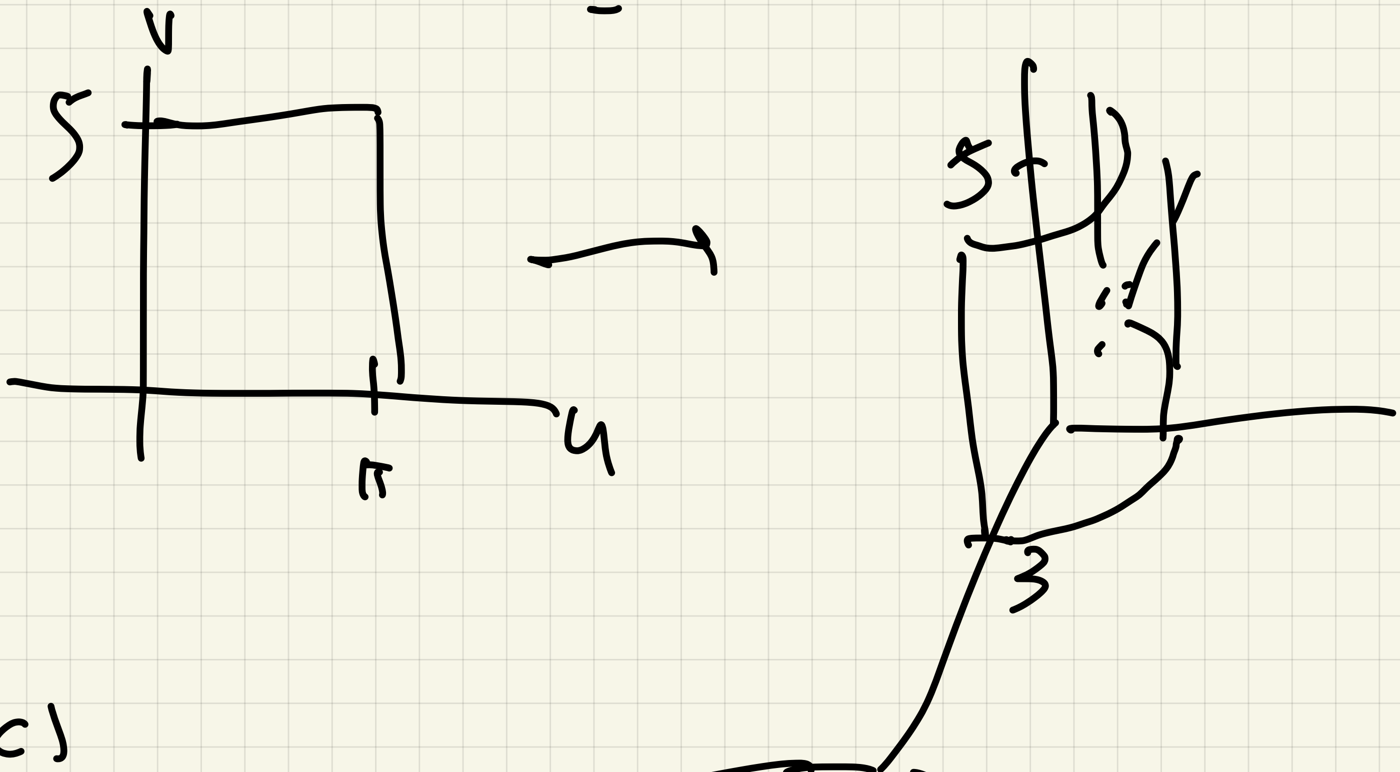
Ex 1 $r(u,v) = (\underset{x}{u}, \underset{y}{v}, \underset{z}{2u+3v})$





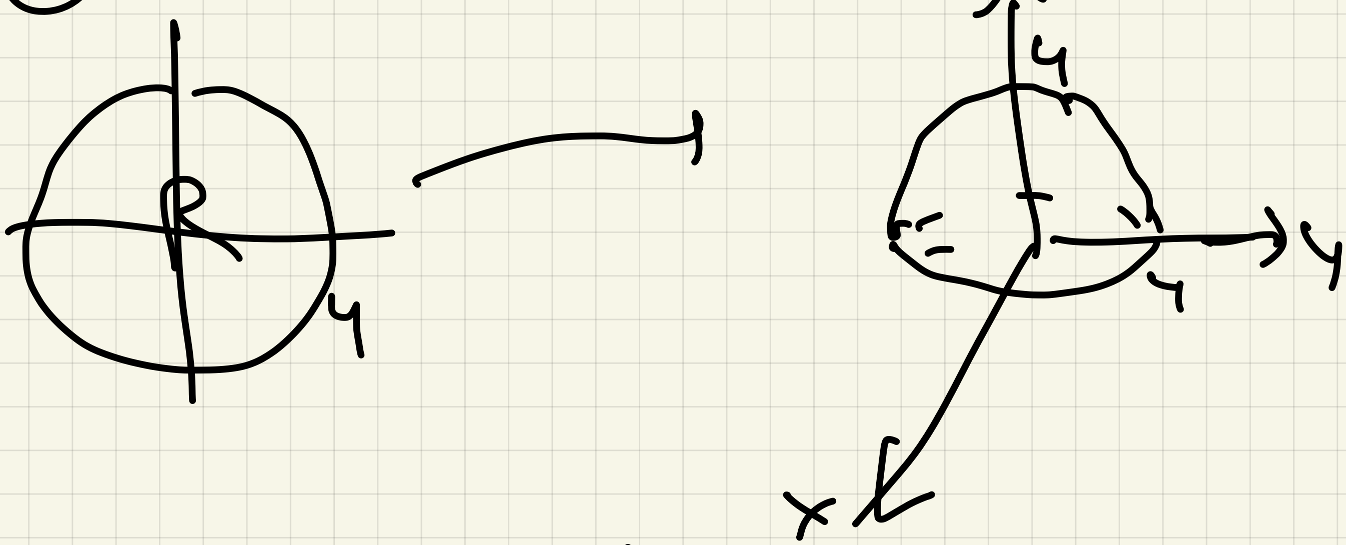
Cal

(b) $r(u,v) = (3\cos v, 3\sin u, v)$



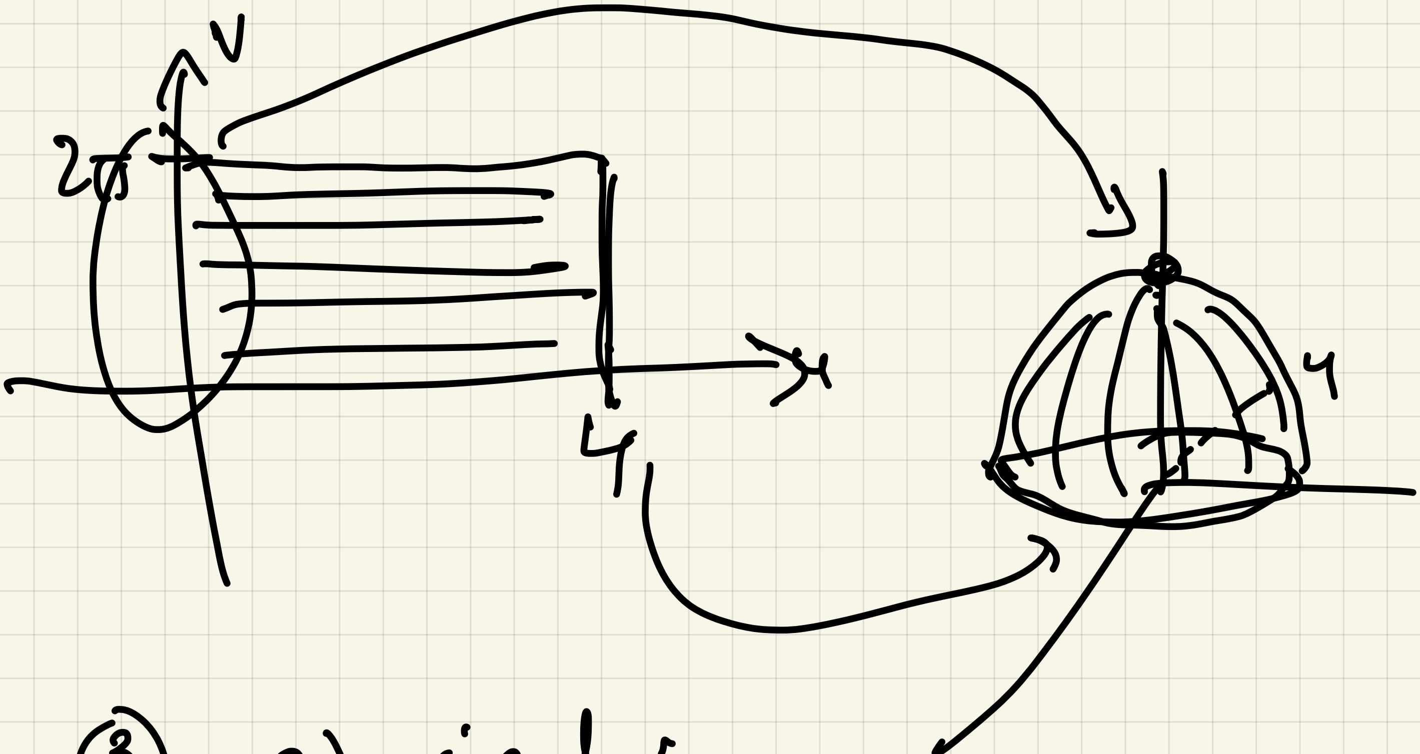
(c)

① $r(u,v,\sqrt{16-u^2-v^2})$



② Cylindrical:

$r(u\cos v, u\sin v, \sqrt{16-u^2})$

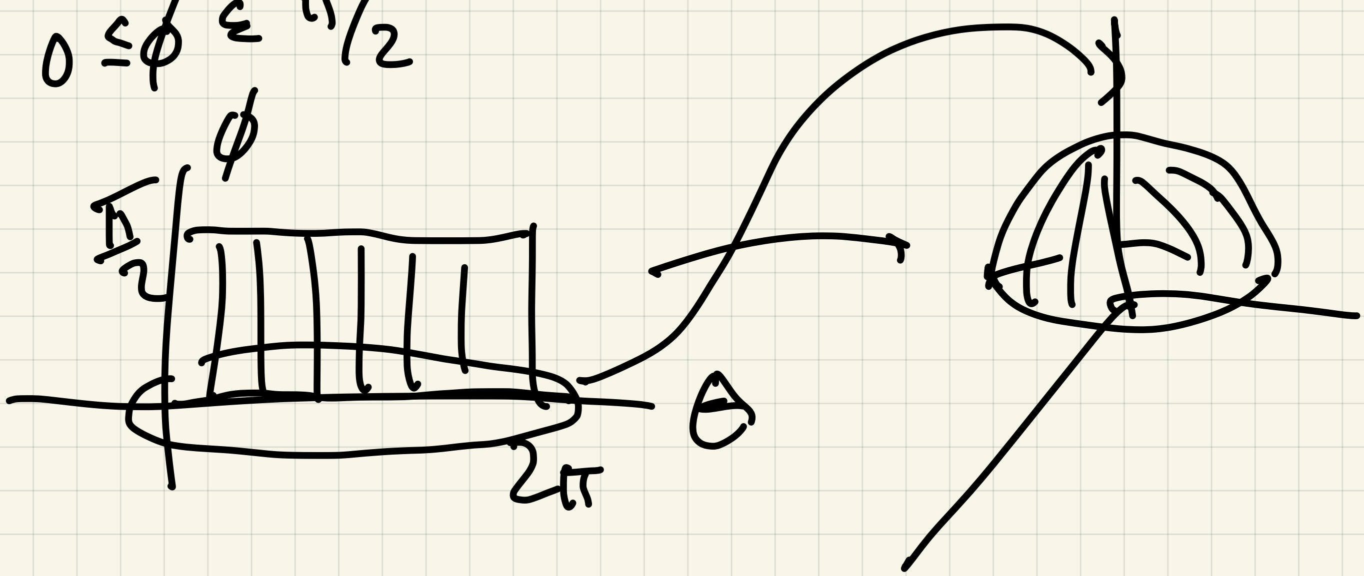


③ spherical :

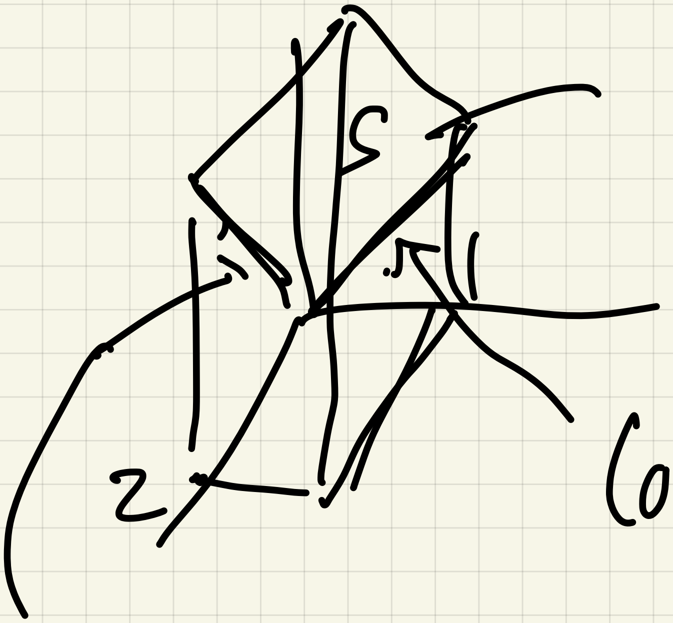
$$r(\theta, \phi) = (\underset{x}{4\sin\phi\cos\theta},\ \underset{y}{4\sin\phi\sin\theta},\ \underset{z}{4\cos\phi})$$

$0 \le t \le 2\pi$

$0 \le \phi \le \pi/2$



Area :



S **Find area:**

$$\text{Area} = |(2,0,4) \times (0,1,3)| = 2\sqrt{14}$$

$(0,1,3)$

$(2,0,4)$

$$r(u,v) = (u, v, 2u+3v)$$

$$\partial r/\partial u = (1,0,2)$$

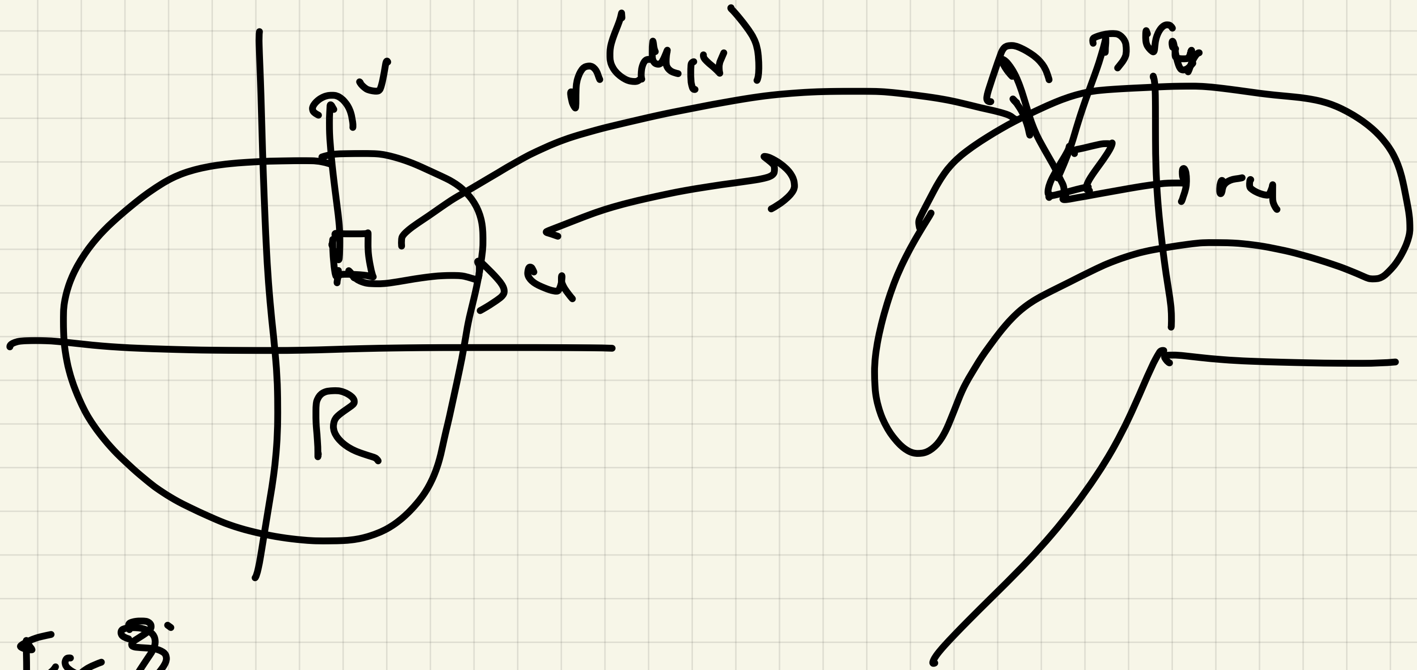
$$\partial r/\partial v = (0,1,3)$$

$$\text{Area} = \left| 2 \cdot \partial r/\partial u \times \partial r/\partial v \right|$$

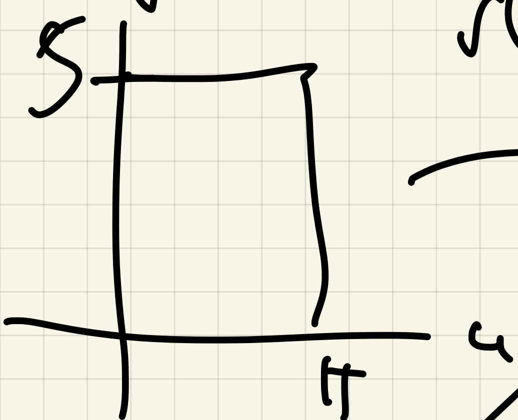
Motivates: Area formula:

$$\boxed{\text{Area } S = \iint_R |r_u \times r_v| \, dA}$$

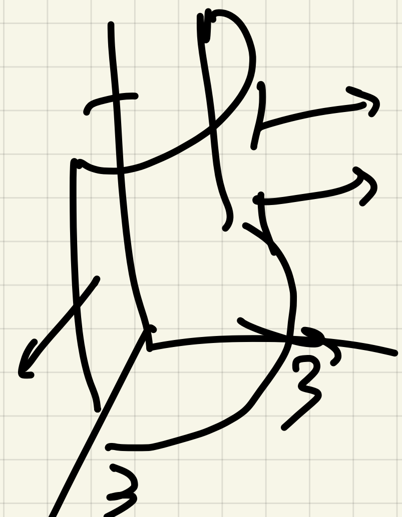
Idea!



Ex 8:



$$r(u,v) = (3\cos u, 3\sin u, v)$$



$$r_u = (-3\sin u, 3\cos u, 0)$$

$$r_v = (0, 0, 1)$$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ & r_u & \\ & r_v & \end{vmatrix} = \langle 3\cos u, 3\sin u, 0 \rangle$$

$$|r_u \times r_v| = 3$$

$$\text{so } SA = \int_0^{\pi} \int_0^{5} 3 \, dv \, du = 15\pi$$

$u \quad v$

<u>Two surfaces:</u>

scalar function

$f(x,y,z)$

① $\iint_S f \, d\sigma = \iint_R \underline{f(r(u,v))} \, |\underbrace{r_u \times r_v}| \, dA$

(<u>Note</u>: $f = 1$, $\iint 1 \, d\sigma =$ Surface area)

② $\iint_S F \cdot \hat{n} \, d\sigma$ <u>Flux</u> <u>integral</u>

F vector field

unit normal

Note: $r_u \times r_v \perp r_u$ and r_v

so $r_u \times r_v \perp$ to surface

$\therefore \bar{n} =$ unit normal $= \dfrac{r_u \times r_v}{|r_u \times r_v|}$

$$\iint_S F \cdot n \, d\sigma = \iint_R F(r(u,v)) \cdot \frac{r_u \times r_v}{\cancel{|r_u \times r_v|}} \cancel{|r_u \times r_v|} \, dA$$

$$= \iint_R F(r(u,v)) \cdot (r_u \times r_v) \, dA$$

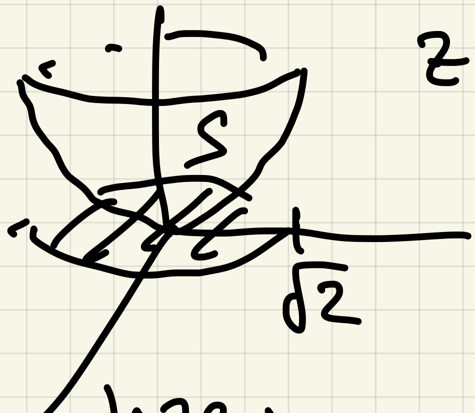
Aside:

$$\int_C f \, ds = \int f(r(t)) |r'(t)| \, dt$$

$$\int_C F \cdot dr = \int F(r(t)) \cdot r'(t) \, dt$$

Let's compute some!

Ex| ;

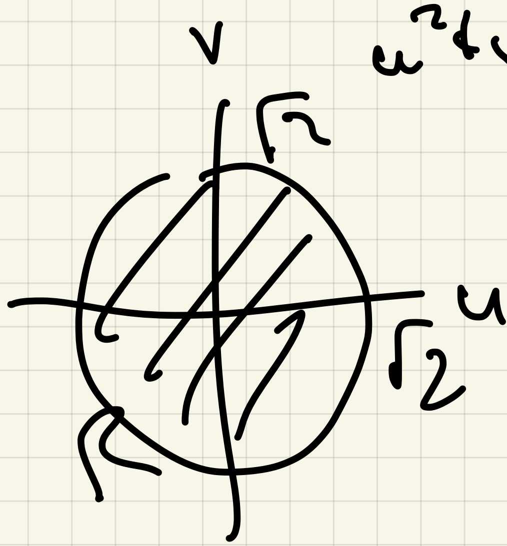


$z = x^2 + y^2$

Parametrize :

$r(u,v) = (u, v, u^2 + v^2)$

$u^2 + v^2 \leq 2$



$r_u \times r_v$

$r_u = (1, 0, 2u)$

$r_v = (0, 1, 2v)$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 1 & 0 & 2u \\ 0 & 1 & 2v \end{vmatrix} = \underline{\langle -2u, -2v, 1 \rangle}$$

(a) Surface area :

$$\iint_R 1 \, |r_u \times r_v| \, dA =$$

$$\iint_R \sqrt{4u^2+4v^2+1}\, dA$$

$$\|$$

<u>polars</u>:

$$\int_0^{2\pi} \underbrace{\int_0^{\sqrt{2}} \sqrt{\underline{4r^2+1}} \cdot r\, dr}\, d\theta$$

$$\frac{1}{8}\frac{2}{3}(4r^2+1)^{3/2}\Big|_0^{\sqrt{2}} =$$

$$\frac{2}{24}\left(9^{3/2} - 1^{3/2}\right) = \int_0^{2\pi} \frac{26}{24}\, d\theta =$$

$$27-1$$

$$26\frac{1}{12}\cdot 2\pi = \frac{13\pi}{3}$$

(b) $\iint_S z\, d\sigma = \iint_R z\, d\sigma$

$$\|$$

$$\iint_R (u^2+v^2)\,|r_u \times r_v|\, dA$$

polar $\qquad \frac{w-1}{4}$

$$\int_0^{2\pi}\int_0^{\sqrt{2}} \underbrace{r^2 \sqrt{4r^2+1}\; r\,dr}_{r}\, d\theta$$

$w = 4r^2+1$

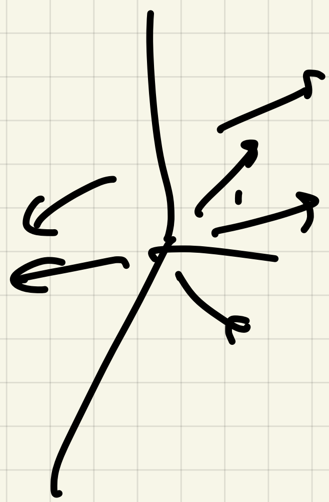
$dw = 8r\,dr$

$$\int_1^9 \frac{w-1}{4}\sqrt{w}\,\frac{1}{8}\,dw =$$ numbers

$$\int_0^{2\pi} \frac{298}{60}$$

(c) Flux integral:

$F(x,y,z) = (x,y,0)$



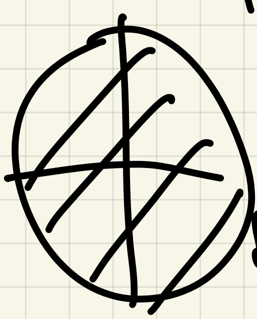
$\bar{n}$ = outward normal

$-(r_u \times r_v)$ outward normal

$$\iint_S F\cdot n\,d\sigma = \iint_R \underline{F(r(u,v))}\cdot(r_u\times r_v)\,dA$$

$$\parallel$$

$$\iint_R (\underline{u},\underline{v},0)\cdot -(-\underline{2u},\underline{2v},1)\,dA$$



$\sqrt{2}$

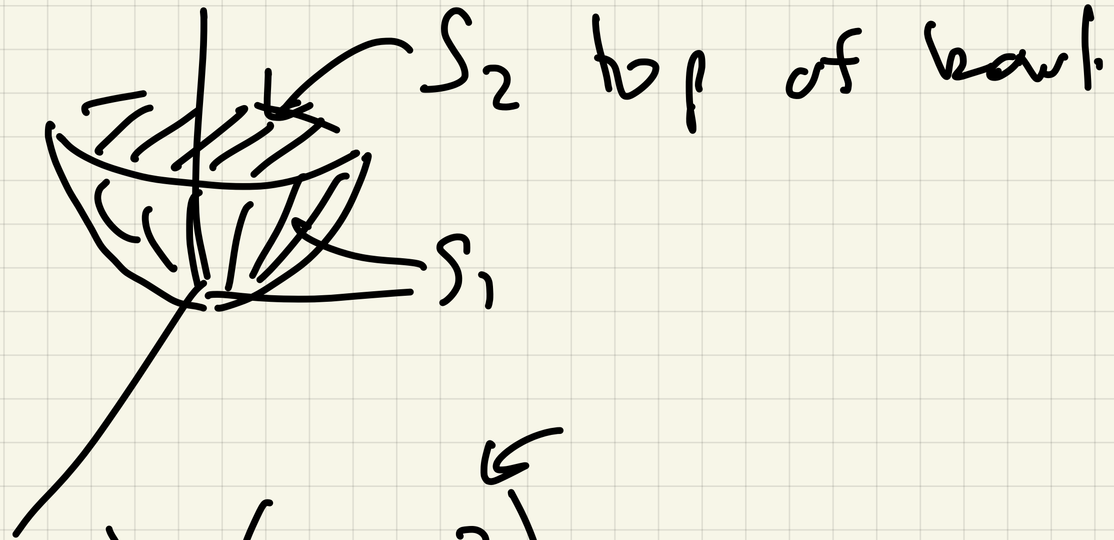
$$\iint_R \underline{2u^2+2v^2}\,dA$$

$$\parallel \quad \text{polars}$$

$$\int_0^{2\pi}\int_0^{\sqrt{2}} 2r^2\cdot r\,dr\,d\theta = 4\pi$$

<u>Ex 2</u> : compute same (a) + (b) (c)

for surface $S_2 =$ lid on bowl



$$r(u,v) = (u, v, 2)$$

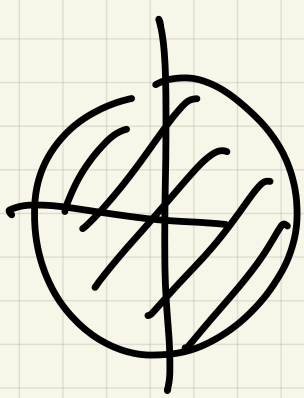
$$r_u = (1, 0, 0)$$

$$r_v = (0, 1, 0)$$

$$r_u \times r_v = (0, 0, 1)$$

$$|r_u \times r_v| = 1$$

<u>SA</u> : (a) $\iint_R 1 \, dA = 2\pi$



S_2 (b) $\iint_R 2 \, dA = 4\pi$

(c) : $F(x, y, z) = (x, y, 0)$

$$\iint_S F \cdot n \, d\sigma = \iint_R \underbrace{(u,v,0)\cdot(0,0,1)}_{0} \, dA$$

$$= 0$$

<u>Theorem</u> (Gauss's Thm)!

If surface S bounds 3-D solid B, $F = \underline{(M,N,P)}$ vector field, then

$$\iint_S F \cdot n \, d\sigma = \iiint_B \operatorname{div} F \, dV$$

where

$$\operatorname{div} F = \partial M/\partial x + \partial N/\partial y + \partial P/\partial z$$