4/22/ Calc3

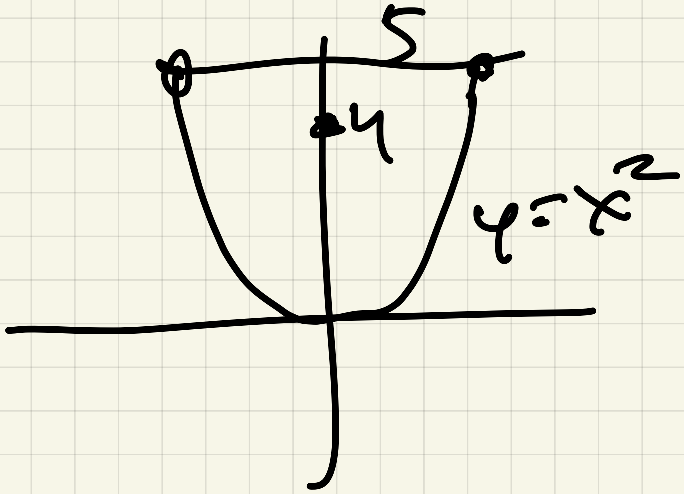
Exam 3

150	
135	9
120	3
105	3

avg 88

med 92

#2



∇

$$f = x(y-4)$$

$\nabla f = (0,0)$ at $\boxed{(0,4)}$ $\boxed{f=0}$

top $f(x,y) = x(5-4) = x$

$f(-\sqrt{5},5) = -\sqrt{5}$

$f(\sqrt{5},5) = \sqrt{5}$

bottom $f(x,x^2) = x(x^2-4) =$

$\to x^3 - 4x$

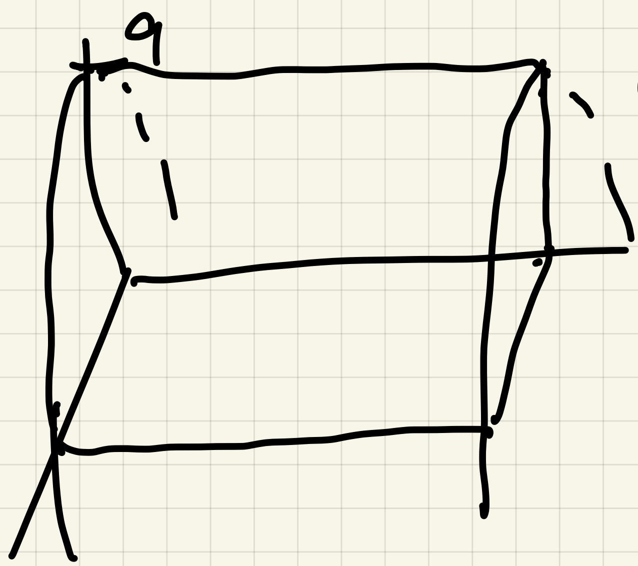
$$f' = 3x^2 - 4 = 0 \Rightarrow x = \pm\frac{2}{\sqrt{3}}$$

$$f\left(\frac{2}{\sqrt{3}}\right) = \frac{8}{3\sqrt{3}} - \frac{4\cdot 2}{\sqrt{3}} = \frac{-16}{3\sqrt{3}} \quad \text{min}$$

$$f\left(-\frac{2}{\sqrt{3}}\right) \qquad +\frac{16}{3\sqrt{3}} \quad \text{max}$$

#5.

$$\int_0^4 \int_0^3 \int_0^{9-x^2=z}$$

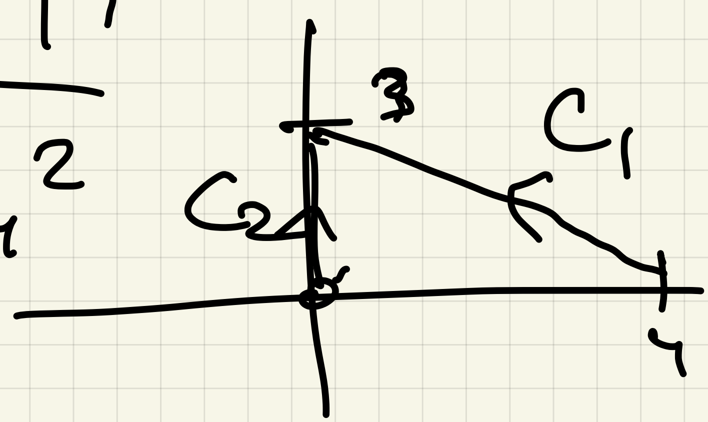


$0 \le z \le$

$0 \le x \le 3$

Quiz 17

$f(x,y) = x + y^2$



y

$=$

1. $C_1 : r(t) = \langle 4-4t, 3t \rangle \quad 0 \leq t \leq 1$

$r'(t) = \langle -4, 3 \rangle$

$|r'(t)| = 5$

$$\int x+y^2 \, ds = \int_0^1 \left(4-4t+\underbrace{(3t)^2}_{9t^2}\right) \cdot 5 \, dt$$

$$5\left(4t - 2t^2 + 3t^3\right)\Big|_0^1 =$$

$$5(4-2+3) = 25$$

2. $r(t) = \langle 0, t \rangle \quad 0 \leq t \leq 3$

$r'(t) = \langle 0, 1 \rangle$

$|r'| = 1$

$$\int_0^3 0 + t^2 \cdot 1 \, dt = \frac{t^3}{3}\Big|_0^3 = 9$$

Last time:

(M, N, P)

FTLI: If $F = \nabla f$ is conservative

and $C: r(t) : a \le t \le b$

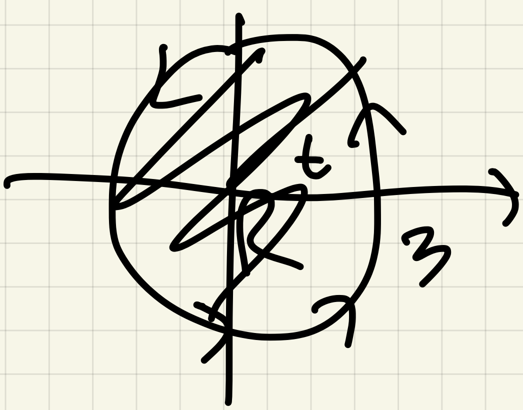
Then $\int_C F \cdot dr = \int_C M\,dx + N\,dy + P\,dz$

$$= f(r(b)) - f(r(a))$$

$(= 0$ if C closed$)$

§15.4 What if C closed, F not conservative

Ex1 $\int_C \underbrace{(x+3y)}_{M}\,dx + \underbrace{2x}_{N}\,dy$



C: $r(t) = \langle \overset{x}{3\cos t}, \overset{y}{3\sin t} \rangle$

$0 \le t \le 2\pi$

$$\int_C (x+3y)\,dx + \underline{2x\,dy} =$$

$$\int_0^{2\pi} (\underbrace{3\cos t}_{x} + \underbrace{9\sin t}_{3y})(-3\sin t) + \underline{6\cos t(3\cos t)}\,dt$$

$$\int_0^{2\pi} \underbrace{-9\sin t\cos t}_{u=\sin t} - 27\underbrace{\sin^2 t}_{\frac{1-\cos 2t}{2}} + 18\underbrace{\cos^2 t}_{\frac{1+\cos 2t}{2}}\,dt$$

$$-\frac{9}{2}\sin^2 t - \frac{27}{2}\left(t - \cancel{\tfrac{1}{2}\sin 2t}\right) + \frac{18}{2}\left(t + \cancel{\tfrac{1}{2}\sin 2t}\right)\Big|_0^{2\pi}$$

$$= 0 - \frac{27}{\cancel{2}}(\cancel{2}\pi) + \frac{18}{\cancel{2}}(\cancel{2}\pi)$$

$$= -27\pi + 18\pi = -9\pi.$$

Easy way$

Green's Theorem: Let C be a counterclockwise simple closed curve bounding region R in $\mathbb{R}^2$. Then

$$\int_C M\,dx + N\,dy = \iint_R (N_x - M_y)\,dA$$

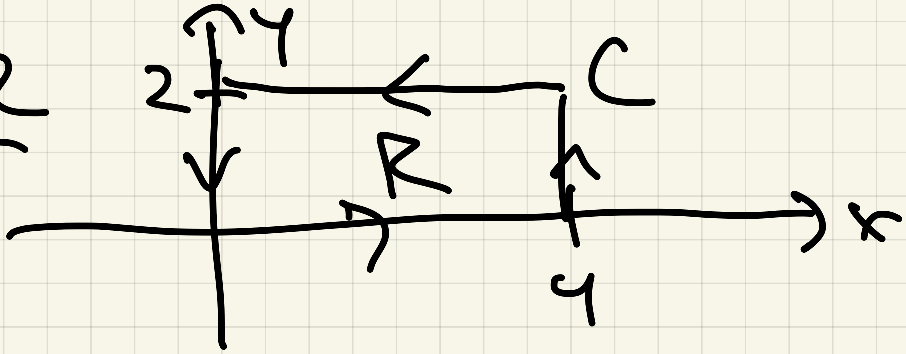
Ex: $\displaystyle\int \underbrace{(x+3y)}_{M}\,dx + \underbrace{2x}_{N}\,dy$

$$\overset{\text{Green's}}{=} \iint_R (\underset{N_x}{2} - \underset{M_y}{3})\,dA =$$

$$-\iint_R 1\,dA = -\text{Area of } 3$$

$$= -9\pi.$$

Ex2



$$\int_C \underbrace{(x+y^3)}_{M}\,dx + \underbrace{(\sin y - e^x)}_{N}\,dy$$

$$\| $$

$$\iint_R (N_x - M_y)\,dA = \iint_R -e^x - 3y^2\,dA$$

$$\int_0^2 \int_0^4 -e^x - 3y^2\,dx\,dy$$

$$-e^x - 3xy^2 \Big|_{x=0}^{x=4} =$$

$$\int_0^2 -e^4 - 12y^2 - (-1)$$

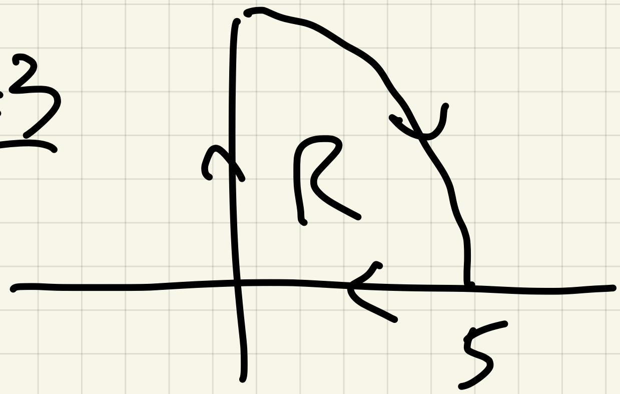
$$\int_0^2 1 - e^4 - 12y^2\,dy =$$

$$(1-e^4)y - 4y^3 \Big|_0^2$$

$$= 2(1-e^4) - 32$$

$$= -2e^4 - 30$$

Ex3



$\frac{1}{4}$ circle radius 5

clockwise

$$\int_C \underset{M}{(x+y^2)}\, dx - \underset{N}{y}\, dy =$$

$$- \underbrace{\int_{C'} (x+y^2)\, dx - \overset{N}{y}\, dy}$$

$C' = C$ but counter clockwise

$\parallel$

$$- \iint_R 0 - 2y \, dA =$$

$$\iint_R 2y\,dA = \int_0^5 \int_0^{\sqrt{25-x^2}} 2y\,dy\,dx$$

$=$ polars

$$\int_0^{\pi/2}\int_0^5 2r\sin\theta\, r\,dr\,d\theta$$

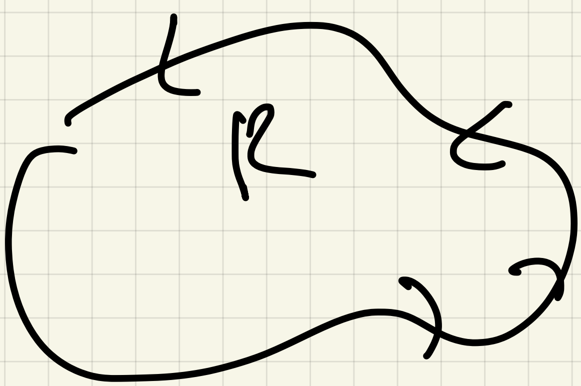
$$\int_0^{\pi/2} \sin\theta \underbrace{\int_0^5 2r^2\,dr}_{\frac{2}{3}r^3\Big|_0^5} \,d\theta$$

$$\int_0^{\pi/2} \frac{250}{3}\sin\theta\,d\theta$$

$$-\frac{250}{3}\cos\theta\Big|_0^{\pi/2} = 0-\left(-\frac{250}{3}\right) = \frac{250}{3}.$$

Rmk

$$\text{Area of } R = \iint_R 1\,dA = \frac{1}{2}\oint \overset{M}{-y}\,dx + \overset{N}{x}\,dy$$



This is the basis for a physical device used by surveyors to find area of a lot.

§ 15.5 **Surface integrals**

Recall: ① $\int_C f\,ds$ f scalar (line integral)

② $\int_C F\cdot dr$ $\quad$ F vector field

($\int_C F\cdot T\, ds$ $\quad$ flux)

If S is a surface, there are two analogous integrals:

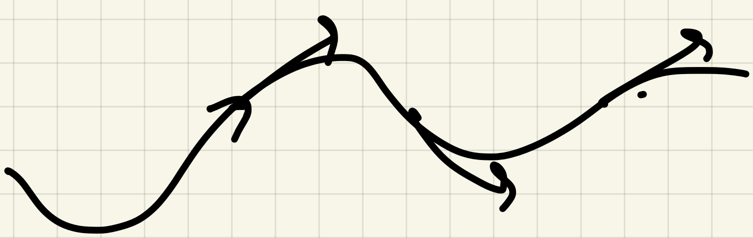
scalar $\qquad\qquad\qquad\qquad$ Flux $\quad$ vector field

① $\iint_S f\, dS$ $\qquad$ ② $\iint_S F\cdot n\, dS$

$\qquad\qquad\qquad\qquad\qquad\qquad$ unit normal

$f=1$ $\quad$ area

$f=$ density $\Rightarrow$ mass



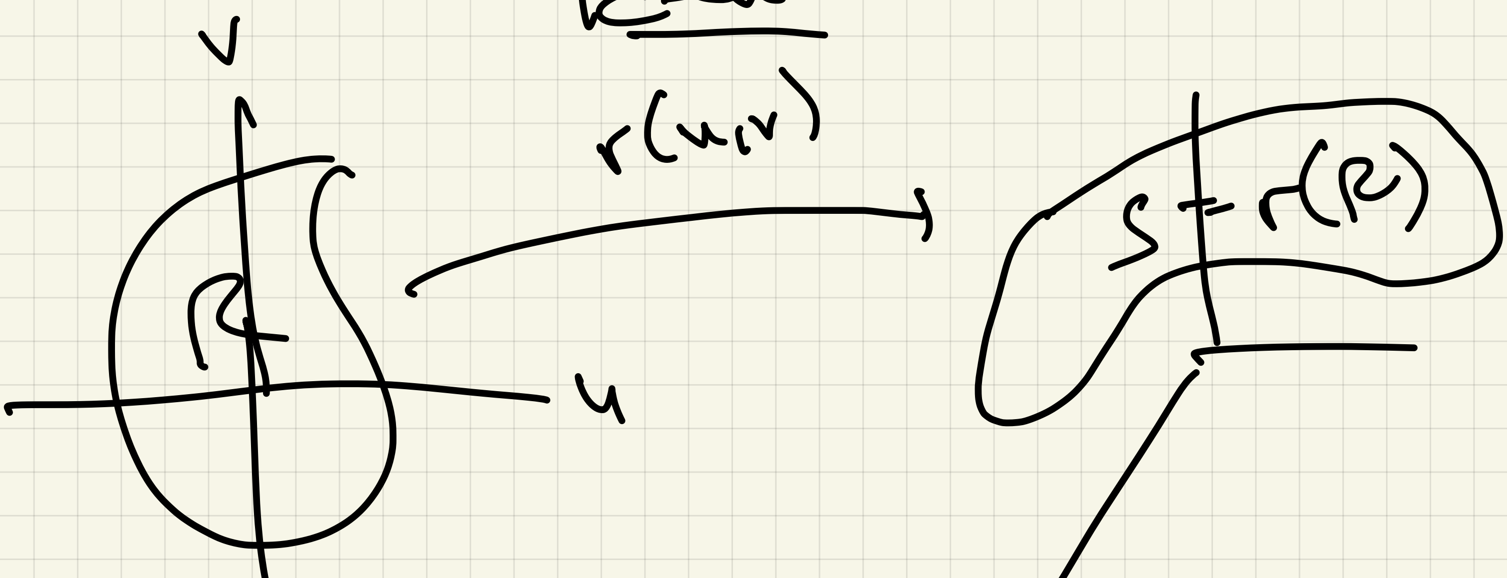
Parametric surfaces in $\mathbb{R}^3$

A parametric surface in $\mathbb{R}^3$ is given by a vector valued function

$$r(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$$

and a region R in u-v plane

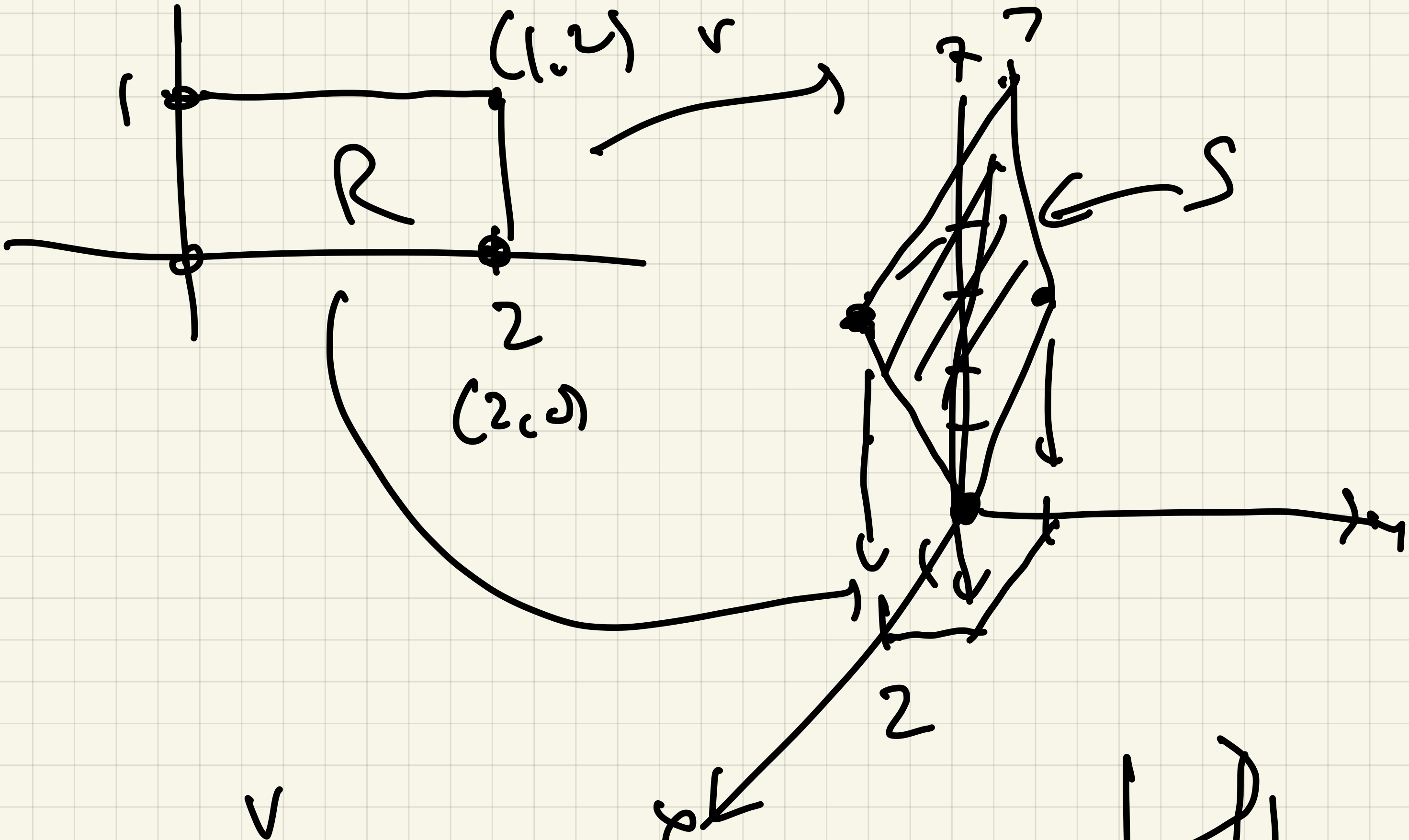
Idea:



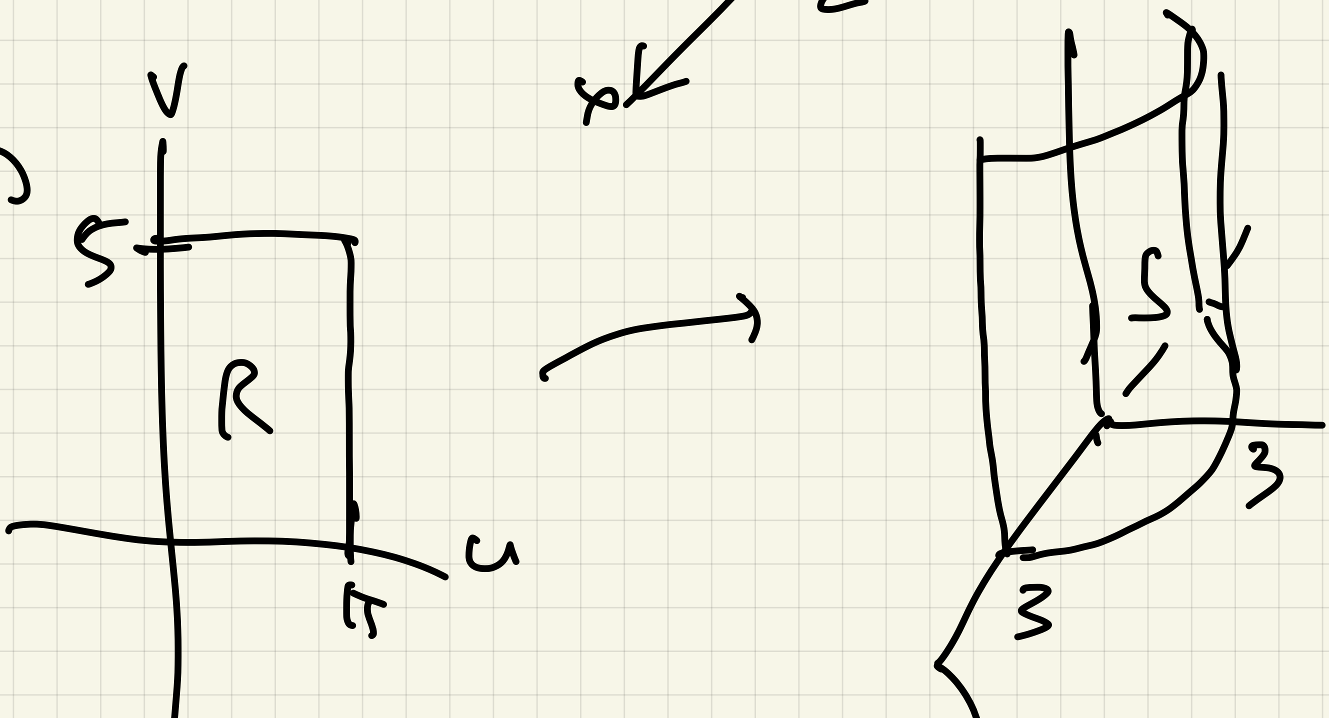
Ex1

(a) $$\bar{r}(u,v) = \langle \underbrace{u}_{x}, \underbrace{v}_{y}, \underbrace{2u+3v}_{z} \rangle$$

$$z = 2x+3y$$



(b)

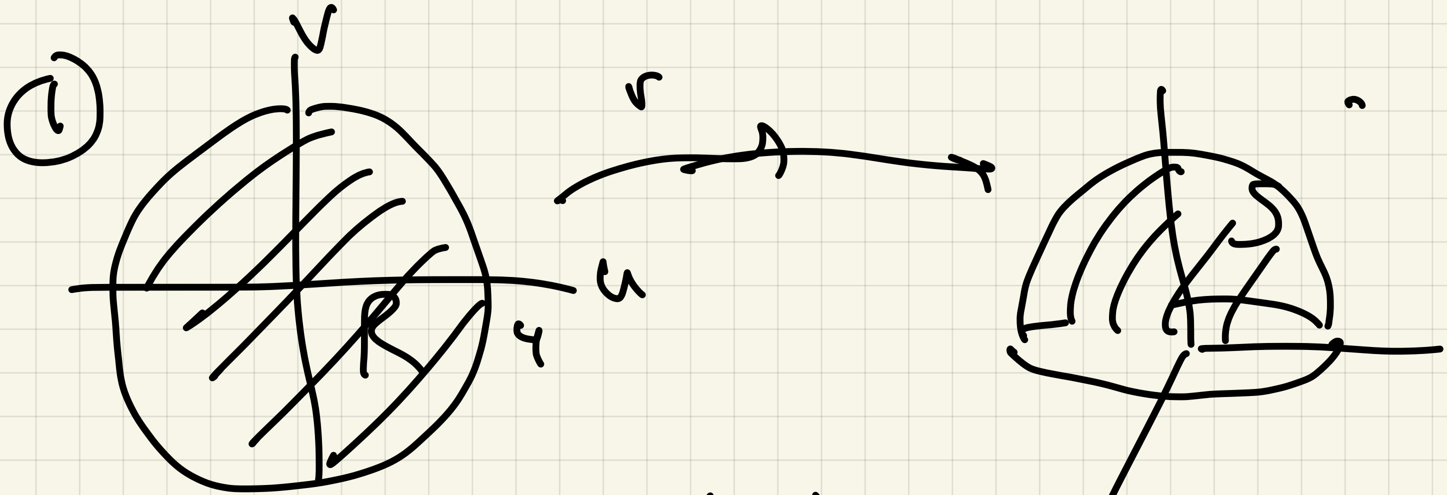


$$r(u,v) = (3\cos u,\ 3\sin u,\ v)$$
$$\quad\ \ x \qquad\quad y \qquad z$$

$$x^2 + z^2 = 9$$

(c) $r(u,v) = \langle u, v, \sqrt{16 - u^2 - v^2} \rangle$

①



cylindrical

② $r(u,v) = \left(u\cos v, u\sin v, \sqrt{16-u^2}\right)$