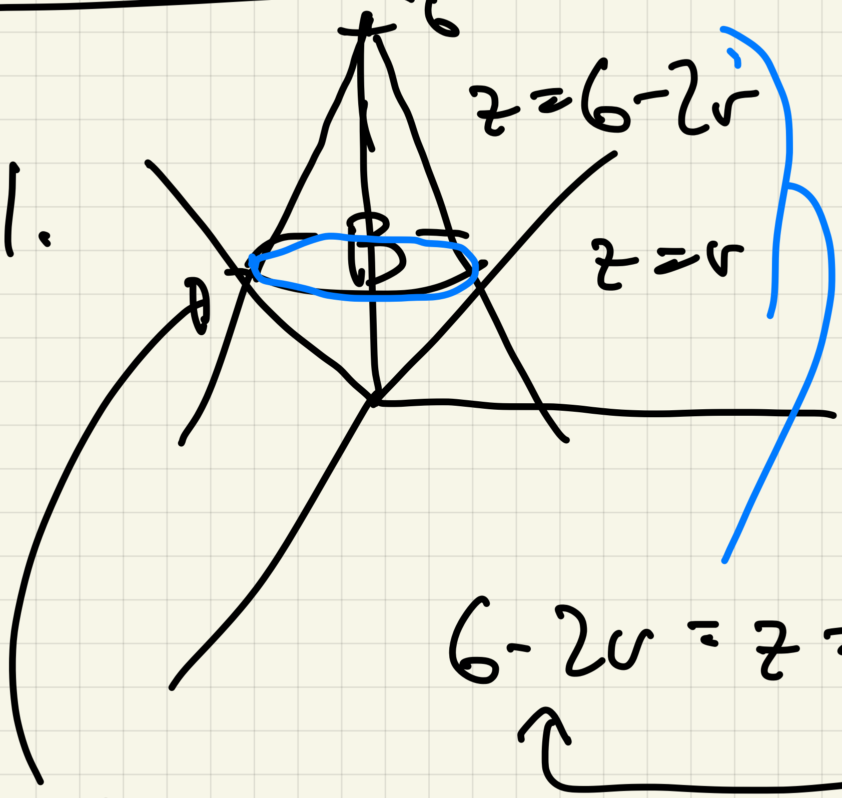
4/11/ Calc3

Quiz

avg 85
med 90

1.



$z = 6 - 2r$

$z = r$

$$\iiint_B y \, dV$$

$$6 - 2r = z = r$$

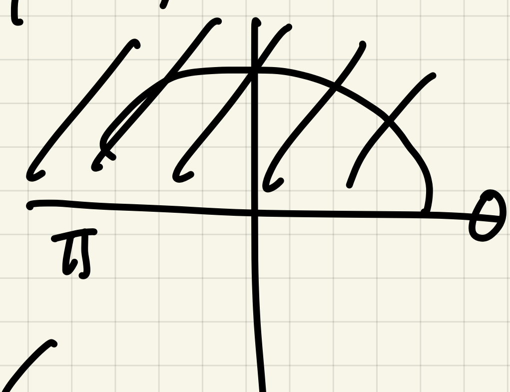
$$6 = 3r \to r = 2$$

$r = 2$
$z = 2$

$$\int_0^{2\pi} \int_0^{2} \int_r^{6-2r} r \sin\theta \; r \; dz \, dr \, d\theta$$

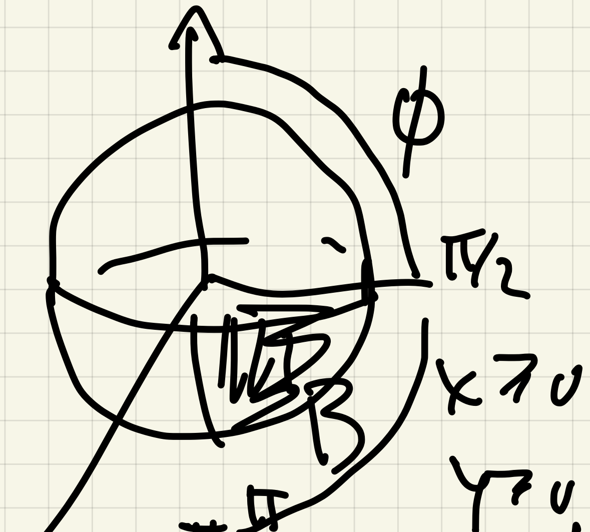
$y = r\sin\theta$

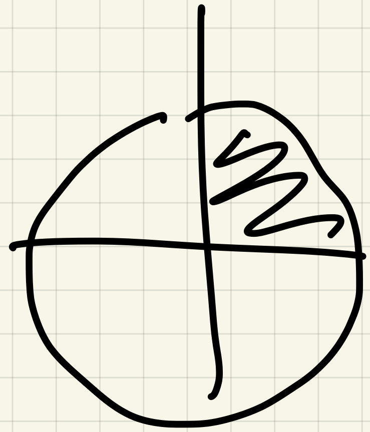
$(y \geq 0)$



$$\to \int_0^{\pi} \int_0^{2} \int_r^{6-2r} r^2 \sin\theta \ldots$$

2.





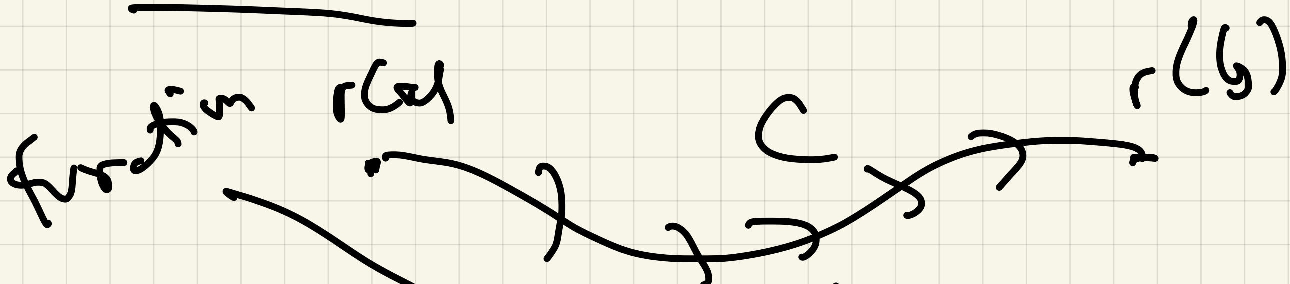
$$\underset{\theta}{\int_0^{\pi/2}} \underset{\phi}{\int_{\pi/2}^{\pi}} \int_0^4 \rho\sin\phi\sin\theta\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$r = \rho\sin\phi$

$y = \underline{\underline{r}}\sin\theta$

$a \leq t \leq b$

<u>Last time</u>: Curve: $r(t)$



<u>§15.1</u> $\int_C f\,ds = \int_a^b f(r(t))\cdot|r'(t)|\,dt$

§15.2

$$\int_C F\cdot dr = \int_a^b F(r(t))\cdot r'(t)\,dt$$

Vector field

Flow integral

work integral

$$F(x,y,z) = (M,N,P)$$

unit tangent

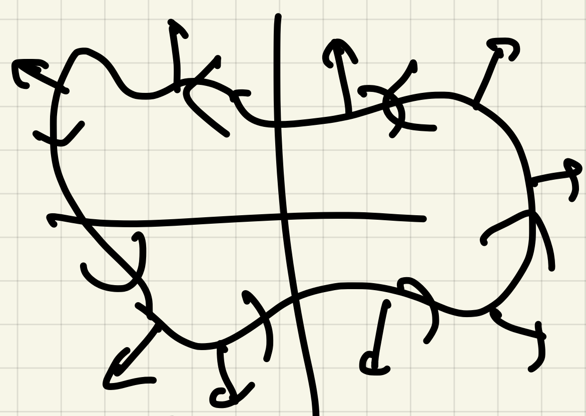
$$\int M\,dx + N\,dy + P\,dz = \int_C F\cdot T\,ds$$

In 2 variables: $F(x,y) = (M,N)$

$$\int_C F\cdot n\,ds = \text{Flux}$$

simple closed curves

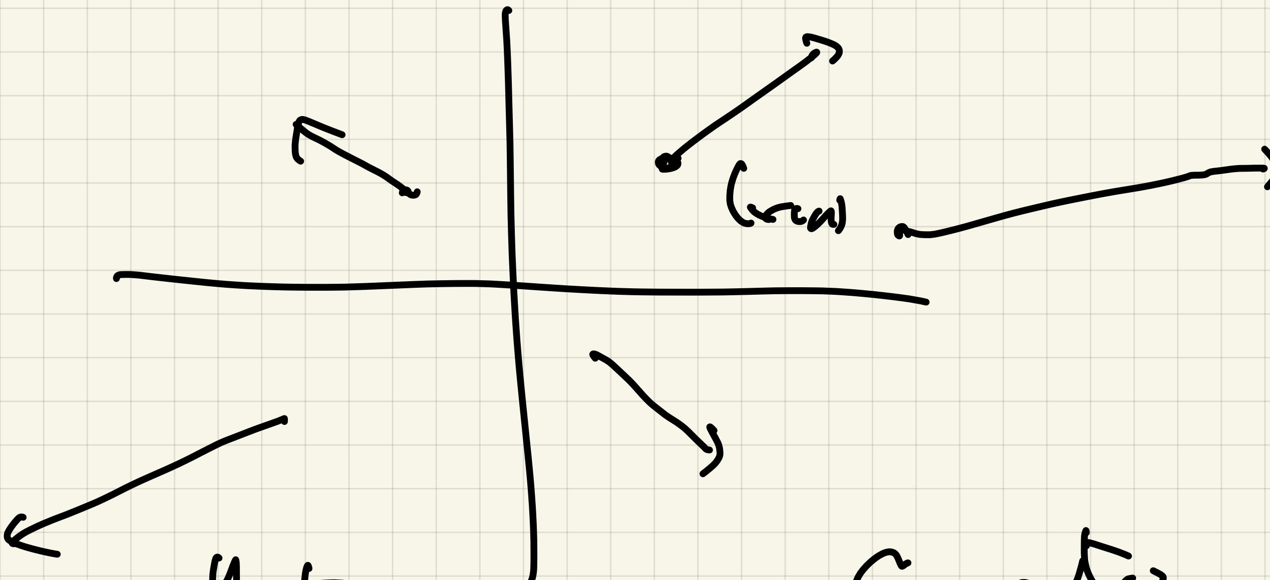
unit outward normal

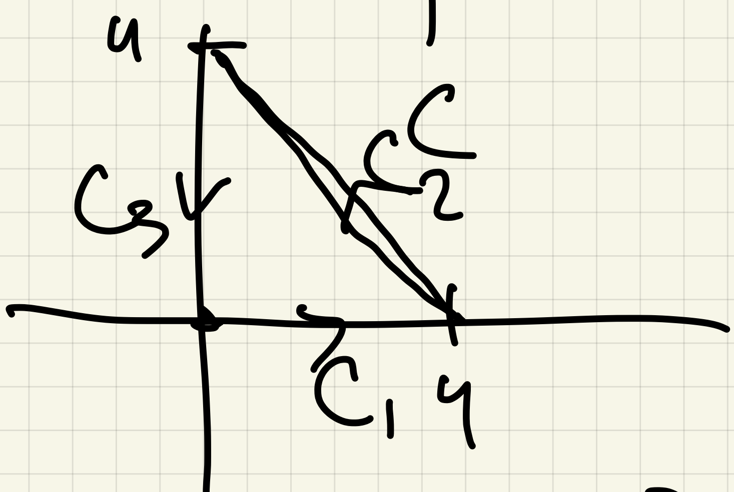


$$\int -N\,dx + M\,dy = \text{Flux}$$

$$\int M\,dx + N\,dy = \text{flow}$$

Ex] $F(x,y) = \langle M, N \rangle = \langle x, y \rangle$





Compute (a) (b) flux & flow integrals

(a) Split into 3 integrals

$$\int_C x\,dx + y\,dy = \int_{C_1} + \int_{C_2} + \int_{C_3}$$

$C_1: \quad r(t) = (\overset{x}{4t}, \overset{y}{0})$

$0 \le t \le 1$

$(t, 0)$

$0 \le t \le 4$

$(4\sin t, 0)$

$0 \le t \le \pi/2$

$$\int_0^1 (4t) \cdot \underset{\frac{dx}{dt}}{4} + 0 \cdot 0$$

$$= \int_0^1 16t = 8t^2 \Big|_0^1 = 8$$

$C_2: \quad r(t) = (4-4t, 4t)$

$0 \le t \le 1$

$$\int_0^1 x\,dx + y\,dy = \int_0^1 \underline{(4-4t)(-4)} + \frac{(4t)(4)}{dt}$$

$$= \int_0^1 \left(x \frac{dx}{dt} + y \frac{dy}{dt} \right) dt$$

$$\int_0^1 -16 + 16t + 16t =$$

$$\int_0^1 -16 + 32t \, dt = -16t + 16t^2 \Big|_0^1$$

$$= 0$$

C_3 $\quad r(t) = (\underline{0}, 4-4t) \quad 0 \le t \le 1$

$$\int x\,dx + y\,dy = \int_0^1 0 - 0 + (4-4t)(-4)\,dt$$

$$= \int_0^1 -16 + 16t \, dt =$$

$$-16t + 8t^2 \Big|_0^1 = -8$$

$$\int_C = \int_{C_1} + \int_{C_2} + \int_{C_3} =$$

$$8 \quad 0 \quad -8 = 0$$

(circulation = 0)

(b) <u>Flux</u>: $\int -y\,dx + x\,dy$

C_1: $\underset{x}{(4t}, \underset{y}{0)}$ $0 \le t \le 1$

$$\int_0^1 0 \cdot \; + (4t) \cdot 0 = 0$$

C_2

$r = (\underset{x}{4-4t}, \underset{y}{4t})$

$$\int -y\,dx + x\,dy$$

$$= \int_0^1 (-4t)(-4) + (4-4t)4$$

$$= \int_0^1 16t + 16 - \cancel{16t}\;dt = \int_0^1 16 = 16$$

C_3 $r = (0, 4-4t)$ $0 \le t \le 1$

$$\int_0^1 -y\underset{0}{dx} + \underset{0}{x}\,dy = \int_0^1 0 + 0 = 0$$

$$\underset{\text{Flux}}{\overset{S_7}{}} = \int_C F \cdot n \, ds = 16$$

§15.3

Definition

A vector field F is conservative

if $F = \nabla f$

f is called the potential function for F.

Ex) If F is conservative, find a potential function for F.

(a) $F(x,y) = \langle \underset{f_x}{2x}, \underset{f_y}{2y} \rangle$

$$f = x^2 + y^2$$

$$\nabla f = \langle 2x, 2y \rangle$$

(b) $F(x,y) = \langle 2y, 2x \rangle$

$$f = 2xy$$

(c) $F = \langle 2x, 1 \rangle$

$$f = x^2 + y$$

(d) $F = \langle 1, 2x \rangle$ $\quad f = ??$

$x \quad 2xy$

There's no potential function!

If $F = \langle f_x, f_y \rangle$

$$(f_x)_y = (f_y)_x$$

$$\frac{\partial}{\partial y} 1 \neq \frac{\partial}{\partial x}(2x)$$

$0 \neq 2$

Test $F = (M, N)$ is conservative

$\Updownarrow$

$$\partial M/\partial y = \partial N/\partial x$$

Test: $F = (M, N, P)$ is conservative

$\quad f_x \; f_y \; f_z$

$\Updownarrow$

$$\partial M/\partial y = \partial N/\partial x, \quad \partial M/\partial z = \partial P/\partial x$$

$$\partial N/\partial z = \partial P/\partial y$$