Quiz 19

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n^2 9^n}(x-4)^n$$

1. (a) $\lim_{n\to\infty} \left| \frac{\frac{(x-4)^{n+1}}{(n+1)^2 9^{n+1}}}{\frac{(x-4)^n}{n^2 9^n}} \right| = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^2 \frac{|x-4|}{9} =$ ($\frac{n}{n+1} \to 1$)

$$-1 < \frac{x-4}{9} < 1 \Rightarrow -9 < x-4 < 9 \Rightarrow -5 < x < 13$$

$x = -5$ $\sum \frac{(-9)^n}{n^2 9^n} = \sum \frac{(-1)^n}{n^2}$ conv by A.S.T.

$x = 13$ $\sum \frac{9^n}{n^2 9^n} = \sum \frac{1}{n^2}$ conv p-series

$\therefore$ I.O.C. $= [-5, 13]$

(b) $R = 9$

(c) $f'(x) = \sum_{n=1}^{\infty} \frac{n(x-4)^{n-1}}{n^2 \cdot 9^n} =$

$$\sum_{n=1}^{\infty} \frac{(x-4)^{n-1}}{n \cdot 9^n} = \sum_{m=0}^{\infty} \frac{(x-4)^m}{(m+1)9^{m+1}}$$