Quiz 16

(1) $\sum_{k=1}^{\infty} \frac{4k}{k^2+1}$

$f(x) = \frac{4x}{x^2+1}$ cont ✓ pos ✓

$f'(x) = \frac{4-4x^2}{(x^2+1)^2} \le 0$ for $x \ge 1$, so decreasing ✓

So Int test applies!

$\int_1^{\infty} \frac{4x}{x^2+1}\,dx =$

$\lim_{b\to\infty} \int_1^b \frac{4x}{x^2+1}\,dx = \lim_{b\to\infty} 2\ln(x^2+1)\Big|_1^b =$

$\lim_{b\to\infty} 2\ln(b^2+1) - 2\ln 2$ diverges to ∞.

So $\sum_{k=1}^{\infty} \frac{4k}{k^2+1}$ diverges by integral test.

(2) $\sum_{k=1}^{\infty} \frac{1}{k^4}$ p-series, $p = 4 > 1 \Rightarrow$ converges

(3) $\lim_{n\to\infty} \frac{3n^2}{8n^2+1} \overset{LH}{=} \lim_{n\to\infty} \frac{6n}{16n} = \frac{3}{8} \neq 0$,

$\therefore \sum \frac{3k^2}{8k^2+1}$ diverges by nth term.

(4) Geometric, $r = \frac{1}{3} < 1 \Rightarrow$ converges.