Quiz 13

① $\int_1^\infty \frac{dx}{x^3}$ converges to $\frac{1}{2}$ $(p=3>1)$

② $\int_1^\infty \frac{dx}{\sqrt{x}}$ diverges $(p=\frac{1}{2}<1)$

③ $\int_1^\infty \frac{1}{x+1} - \frac{2}{x+3} + \frac{1}{x+5}\,dx =$

$$\lim_{b\to\infty} \ln(x+1) - 2\ln|x+3| + \ln|x+5| \Big|_1^b =$$

$$\lim_{b\to\infty} \ln\left|\frac{(x+1)(x+5)}{(x+3)^2}\right|_1^b =$$

$$\lim_{b\to\infty} \ln\left|\frac{x^2+6x+5}{(x^2+6x+9)}\right|_1^b =$$

$$\lim_{b\to\infty} \ln\left|\frac{b^2+6b+5}{b^2+6b+9}\right| - \ln\left|\frac{12}{16}\right| =$$

$$\lim_{b\to\infty} \ln\left|\frac{1+6/b+5/b^2}{1+6/b+9/b^2}\right| - \ln 3/4 = 0 + \ln\frac{4}{3}$$

$$= \ln 4/3.$$