Quiz 11

(1) $$\frac{A}{x+3}+\frac{B}{x-4}+\frac{C}{(x-4)^2}+\frac{D}{(x-4)^3}+\frac{E}{(x-4)^4}+\frac{Fx+G}{x^2+9}$$

(2) $$\frac{-x+8}{x(x^2+4)}=\frac{A}{x}+\frac{Bx+C}{x^2+4}\Rightarrow$$

$$-x+8 = A(x^2+4)+(Bx+C)x$$

$$= \underbrace{(A+B)}_{0}x^2+\underbrace{C}_{-1}x+\underbrace{4A}_{8} \Rightarrow \begin{matrix} A=2, C=-1, \\ B=-2 \end{matrix}$$

So $$\int \frac{-x+8}{x(x^2+4)}dx=\int \frac{2}{x}+\frac{-2x-1}{x^2+4}dx=$$

$$2\ln|x|-\ln|x^2+4|-\frac{1}{2}\arctan\frac{x}{2}+C$$

$$=\ln\left|\frac{x^2}{x^2+4}\right|-\frac{1}{2}\arctan\frac{x}{2}+C$$