**Sequence and Series Review** **April 2026**
**Calculus II** **Prof. Nollet**

1. **Sequences:** You should be able to (1) determine if sequences converge, (2) determine whether they are monotone and (3) determine whether they are bounded. Try the following ten examples.

   (a) $a_n = \frac{1}{2^n}$ (b) $a_n = \frac{n^2}{n+5}$ (c) $a_n = \sin \frac{n\pi}{2}$ (d) $a_n = \frac{3n^2}{(n+1)^2}$
   (e) $a_n = \cos \frac{1}{n}$ (f) $a_n = n^5 e^{-n}$ (g) $a_n = (1 - \frac{1}{n})^n$ (h) $a_n = \frac{(2n+2)!}{n^2 \cdot (2n)!}$
   (i) $a_n = \frac{5n + 2000 \sin n}{n}$ (j) $a_n = \frac{\ln n}{n}$

2. **Infinite series:** What are the **partial sums** of an infinite series $\sum_{k=1}^{\infty} a_k$? What does convergence mean? How does convergence of the *series* $\sum a_n$ relate to convergence of the *sequence* $\{a_n\}$? How does this play out for geometric series and telescoping series?

3. **Series tests:** There are many tests for absolute/conditional convergence of an infinite series $\sum a_k$. Here's a rough guide on how to apply them efficiently:

   (a) Try the $n$th term test. If $\lim a_k \neq 0$, then series diverges, DONE!

   (b) Check for convergence of $\sum |a_k|$ with the following tests:
   *Note: if $\sum |a_k|$ converges, then so does $\sum a_k$.*

      i. Geometric and $p$-series are easy to recognize.

      ii. RATIO/ROOT tests (good with factorials and exponentials)

      iii. DCT/LCT (good with powers of $k$, often compare to $p$-series)

      iv. The integral test (works less often, but sometimes)

   When applying series tests, please (a) state which test you are using, (b) execute the test properly (for example, state the value of $p$ if using $p$-series, state the value of $r$ if using geometric series, carefully compute limits if using the LCT or RATIO/ROOT tests, make sure inequality is useful for DCT) and (c) state the conclusion you reach about convergence or divergence.

(c) If $\sum |a_k|$ diverges, use AST to check for convergence of $\sum a_k$.

4. **General reminders:**

   (a) Don't confuse sequences and series: convergence of the **sequence** $a_k$ does **NOT** imply convergence of **series** $\sum a_k$ (series diverges if $\lim a_k \neq 0$).

   (b) The nth term test is a test for divergence, **NEVER** convergence.

   (c) The A.S.T. is a test for convergence, **NEVER** for divergence.

   (d) $\sum_{n=1}^{\infty} a_n$ converges $\iff$ $\sum_{n=N}^{\infty} a_n$ converges for some $N$.

5. Some practice.

   (a) Find the partial sums $S_n$ for $\sum_{k=1}^{\infty} \frac{6}{k(k+2)}$ and determine convergence.

   (b) Does $\sum_{k=2}^{\infty} 3\frac{2^k}{5^{k+1}}$ converge? If so, to what? How about $\sum_{k=2}^{\infty} 3\frac{(-2)^{3k}}{5^{k+1}}$?

   (c) Use the integral test to determine if $\sum_{k=1}^{\infty} \frac{1}{k \ln k}$ converges.

   (d) Determine absolute/conditional convergence or divergence of each:

      i. $\sum_{k=1}^{\infty} \frac{(-1)^k 3}{3^k}$

      ii. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$

      iii. $\sum_{k=1}^{\infty} \frac{(-1)^k k!}{5^k}$

      iv. $\sum_{k=1}^{\infty} \frac{(-1)^k \sqrt{k^7+4}}{1+k^5}$

      v. $\sum_{k=6}^{\infty} \frac{(-1)^k \sqrt{3}}{k-4}$

      vi. $\sum_{k=1}^{\infty} \frac{(-1)^k (k+8)}{2^k}$

vii. $\sum_{k=1}^{\infty} \frac{(-1)^k 3}{1+k^2}$

viii. $\sum_{k=3}^{\infty} \frac{(-1)^k \sqrt{k}}{k-2}$

ix. $\sum_{k=1}^{\infty} \frac{(-1)^k (k-1)}{5k}$

x. $\sum_{k=1}^{\infty} (\frac{k-1}{5k})^k$

xi. $\sum_{k=1}^{\infty} \frac{\ln k}{k}$