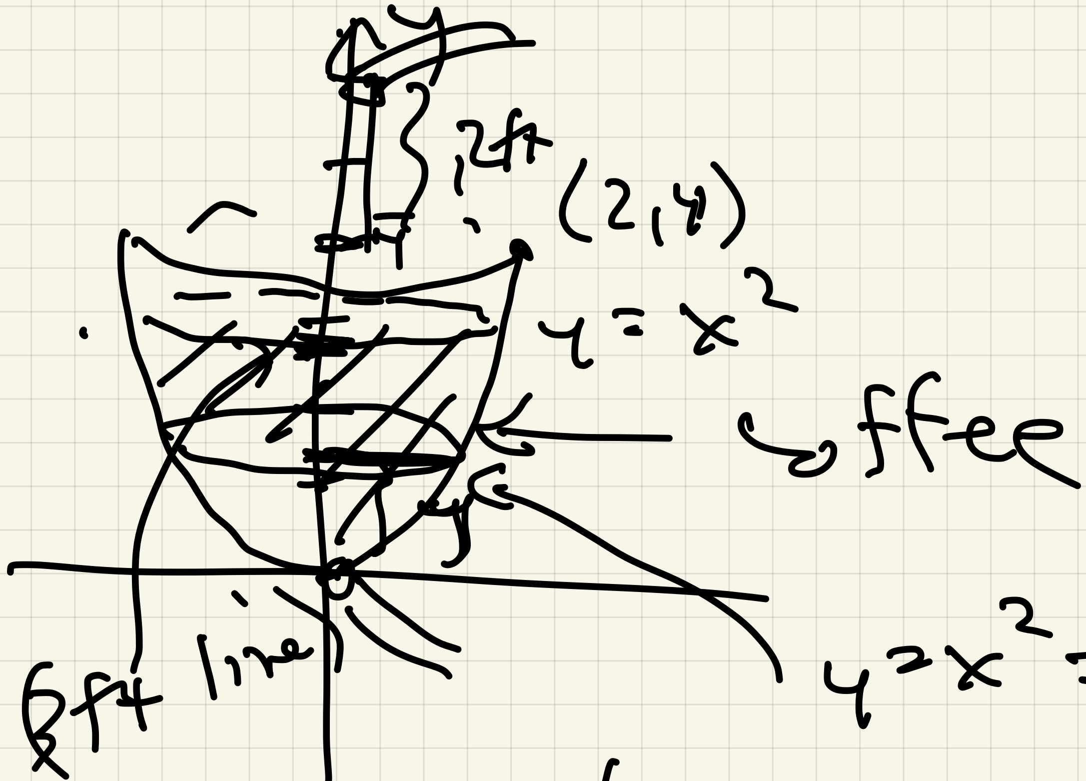
9/24 Calc2

Quiz 6

avg 72
med 78



$y = x^2 \Rightarrow \overset{r}{x} = \sqrt{y}$

cross section area:

$$\pi r^2 = \pi(\sqrt{y}) = \pi y$$

density 60

dist 6 - y

$$W = \int_0^3 60\pi y\underbrace{(6-y)}_{6y-y^2}\,dy =$$

$$60\pi\left(3y^2 - \frac{y^3}{3}\right)\Big|_0^3 = 60\pi(18) =$$

1080π ft-lbs

(b) $$W = \int_0^3 60\pi y \, \underset{\underset{\text{dist}}{\uparrow}}{y} \, dy$$

Last time **Integration by parts**

$$\boxed{\int u \underset{\underset{\frac{dv}{dx}}{\uparrow}}{dv} = uv - \int v \underset{\underset{\frac{du}{dx}}{\uparrow}}{du}}$$

Ex 0 $$\int \underbrace{x}_{u} \underbrace{e^x dx}_{dv} =$$

$$du = 1dx \quad v = e^x$$

$$xe^x - \int e^x dx$$

$$= \underline{xe^x - e^x} + C$$

Note: if we chosen $u = e^x$, $dv = x\,dx$

$du = e^x dx \qquad v = \frac{1}{2}x^2$

$$uv - \int v\,du = \frac{1}{2}x^2 e^x - \int \frac{1}{2}x^2 e^x dx$$

Guiddlines: choose u/dv

① dv is easy to integrate

② du is "simpler" than u

Ex 1 $\int \underbrace{x^2}_{u}\underbrace{e^x dx}_{dv} =$

$du = 2x\,dx \qquad v = e^x$

$$\underset{u}{x^2}\,\underset{v}{e^x} - \int e^x \cdot 2x\,dx$$

$$\|$$

$$x^2 e^x - \int 2\underbrace{x e^x dx} =$$

$$x^2e^x - 2\underbrace{\int xe^x\,dx}$$

$$x^2e^x - 2(xe^x - e^x) + C$$

$$x^2e^x - 2xe^x + 2e^x + C$$

Ex 2 $\int \underbrace{x}_{u}\,\underbrace{\sin(7x)\,dx}_{dv}$

$$du = dx \qquad v = -\frac{1}{7}\cos(7x)$$

$$-\frac{1}{7}x\cos(7x) - \int -\underbrace{\frac{1}{7}\cos(7x)}\,dx$$

$$-\frac{1}{7}x\cos(7x) + \frac{1}{49}\sin(7x) + C$$

Ex 3 $\int \underbrace{\ln x}_{u}\,\underbrace{dx}_{dv}$ $\qquad \int 1 = x + C$

(($du = \frac{1}{x}dx, \; v = x$

$$x\ln x - \int dx$$

$$= \underline{x\ln x - x + C}$$

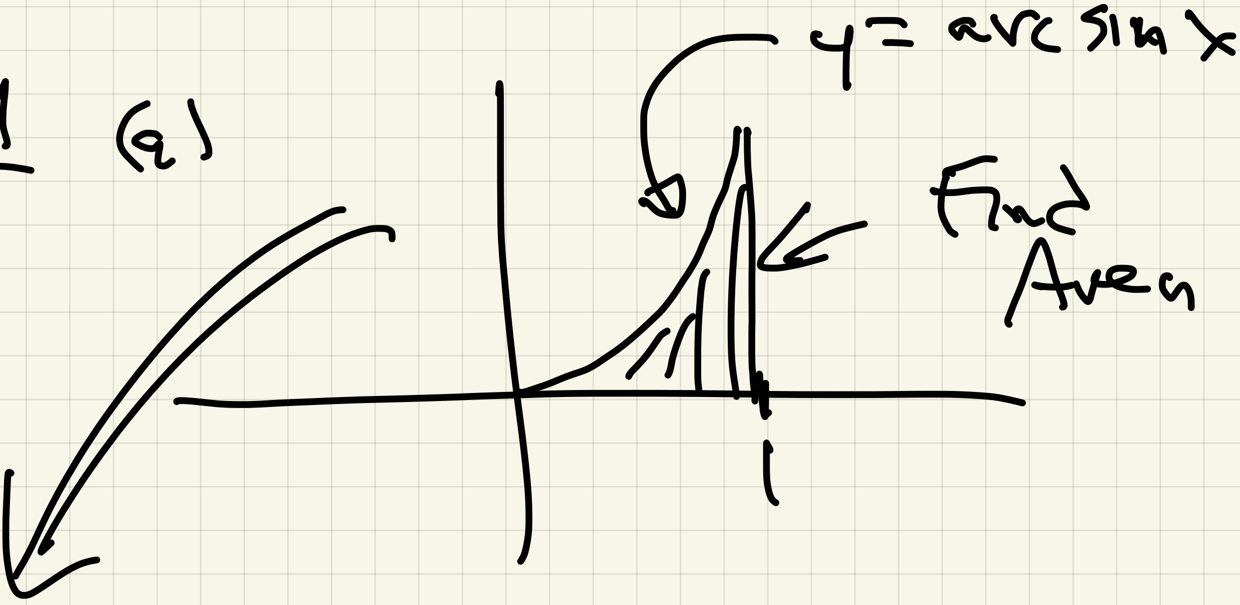
<u>Also</u>: $u = \ln x \Rightarrow x = e^u$

$dx = e^u du$

$$\int \underbrace{\ln x \, dx} = \int u e^u du$$

use Ex 0, go back to x

<u>Ex 4</u> (a)



$$A = \int_0^1 \underbrace{\arcsin x}_{u} \, \underbrace{dx}_{dv}$$

$$du = \frac{1}{\sqrt{1-x^2}} dx \qquad v = x$$

$$x\arcsin x - \underbrace{\int \frac{x}{\sqrt{1-x^2}} dx}$$

$$u = 1-x^2$$

$$dn = -2x\,dx$$

$$-\frac{1}{2} du = x\,dx$$

$$\int \frac{-\frac{1}{2} du}{\sqrt{u}} = -\frac{1}{2} \cdot 2 u^{1/2}$$

$$\uparrow \; u^{-\frac{1}{2}} \qquad \qquad ''$$

$$-\sqrt{u}$$

$$-\sqrt{1-x^2}$$

$$x\arcsin x + \sqrt{1-x^2} + C$$

so $$A = \int_0^1 \arcsin x \, dx =$$

$$x \arcsin x + \sqrt{1-x^2} \Big|_0^1 =$$

$$\left(\frac{\pi}{2} + 0\right) - (0 + 1) =$$

$$\pi/2 - 1$$

(b) V = volume of revolution about y-axis

$$V = \int_0^1 \underbrace{2\pi x}_{u} \, \underbrace{\arcsin x \, dx}_{dv}$$

Similar, but harder

Ex 5

$$\int \underbrace{\cos(7 \ln x)}_{u} \, \underbrace{dx}_{dv}$$

$$\| \; du = -\sin(7\ln x)\left(\frac{7}{x}\right)dx \qquad v = x$$

$$x\cos(7\ln x) + 7\int \underbrace{\sin(7\ln x)}_{u}\underbrace{dx}_{dv}$$

$$du = \cos(7\ln x)\left(\frac{7}{x}\right) \qquad v = x$$

$$x\cos(7\ln x) + \underset{=}{7}\left(x\,\underline{\sin(7\ln x)} - \underset{=}{7}\int \cos(7\ln x)\,dx\right)$$

move to left

Summary :

$$1\int \cos(7\ln x)\,dx =$$

$$x\cos(7\ln x) + 7x\sin(7\ln x) - 49\int \cos(7\ln x)\,dx$$

$$50\,\underline{\int \cos(7\ln x)\,dx} = \boxed{}$$

$$\int \cos(7\ln x)\,dx = \frac{x\cos(7\ln x) + 7x\sin(7\ln x)}{50} + C$$

50

Rmk: $u = \ln x \Rightarrow x = e^u$

$$dx = e^u \, du$$

$\int \cos(7 \ln x)\, dx$ becomes

$$\nearrow \int \cos(7u) \cdot e^u \, du$$

integrate by parts twice

Ex 6 $\int_0^1 \underbrace{\ln(x^2+4)}_{u} \underbrace{dx}_{dv}$

$du = \dfrac{2x}{x^2+4}\, dx \qquad v = x$

$$= x \ln(x^2+4) - \underbrace{\int \frac{2x^2}{x^2+4}\, dx}_{??}$$

$$\int \frac{2x^2}{x^2+4}\, dx = \int \frac{2x^2+8-8}{x^2+4}\, dx =$$

$$\int \frac{2x^2+8}{x^2+4} - \int \frac{8}{x^2+4}$$

$$\int 2 - \frac{8}{x^2+4} \, dx =$$

$$2x - 8 \overbrace{\frac{1}{2} \arctan \frac{x}{2}}^{\text{table}} + C$$

FINAL :

$$x \ln(x^2+4) - 2x + 4 \arctan \frac{x}{2} \Big|_0^1$$

$$\ln 5 - 2 + 4 \arctan \frac{1}{2}$$