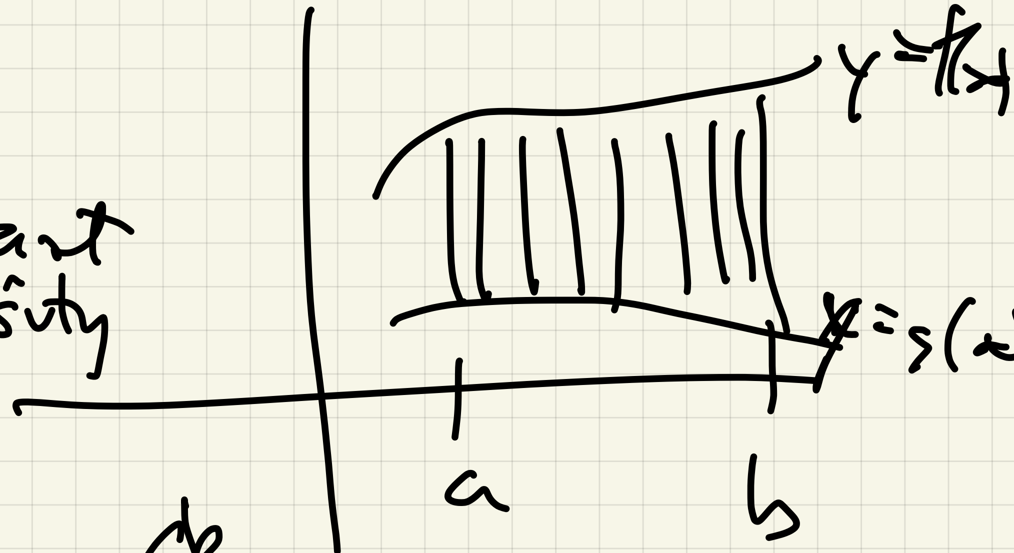
Last time:

δ = constant density

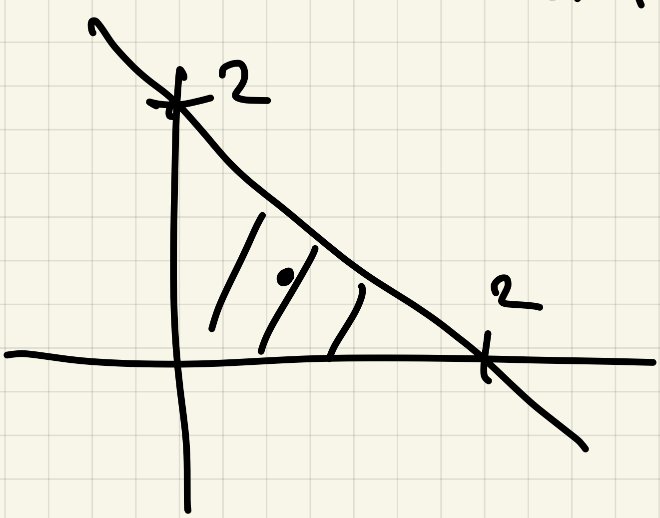


$$\text{Mass} = M = \delta \int_a^b f(x) - g(x)\, dx$$

$$M_y = \delta \int_a^b (f(x) - g(x))\, x\, dx$$

$$M_x = \delta \int_a^b \frac{f(x)^2 - g(x)^2}{2}\, dx$$

$$\text{COM} = (\bar{x}, \bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right)$$



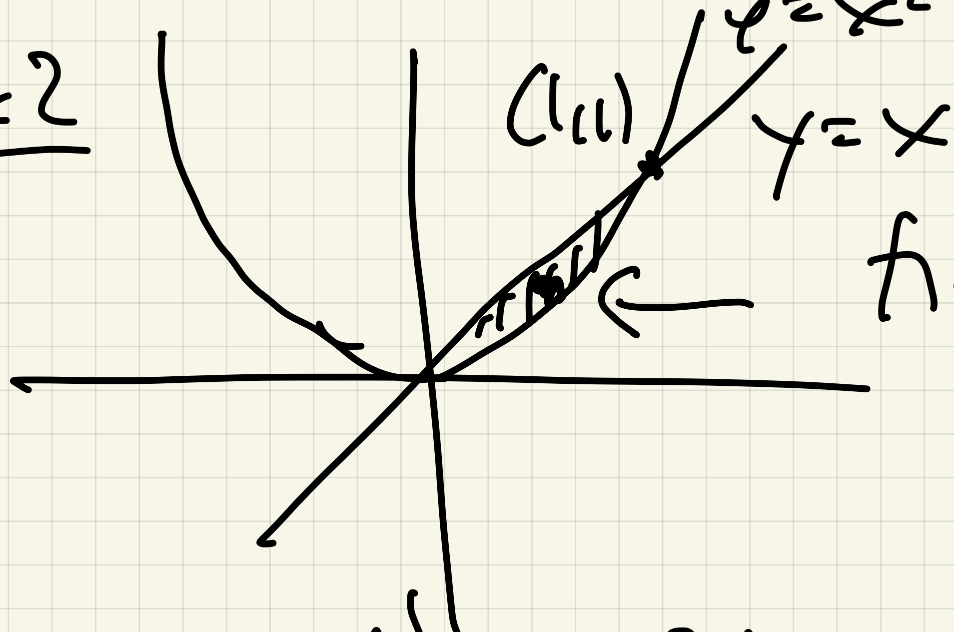
$$M = 2\delta \quad \begin{pmatrix} \text{take} \\ \delta = 1 \end{pmatrix}$$

$$M_y = M_x = 4/3$$

$$\Downarrow$$

$$(\bar{x}, \bar{y}) = (2/3, 2/3)$$

Ex 2



find COM

$$M = A = \int_0^1 x - x^2 \, dx = \frac{x^2}{2} - \frac{x^3}{3} \Big|_0^1 = \frac{1}{6}$$

$$M_y = \int_0^1 (x - x^2)\, x \, dx =$$

$$\int_0^1 x^2 - x^3 \, dx = \frac{x^3}{3} - \frac{x^4}{4} \Big|_0^1 = \frac{1}{12}$$

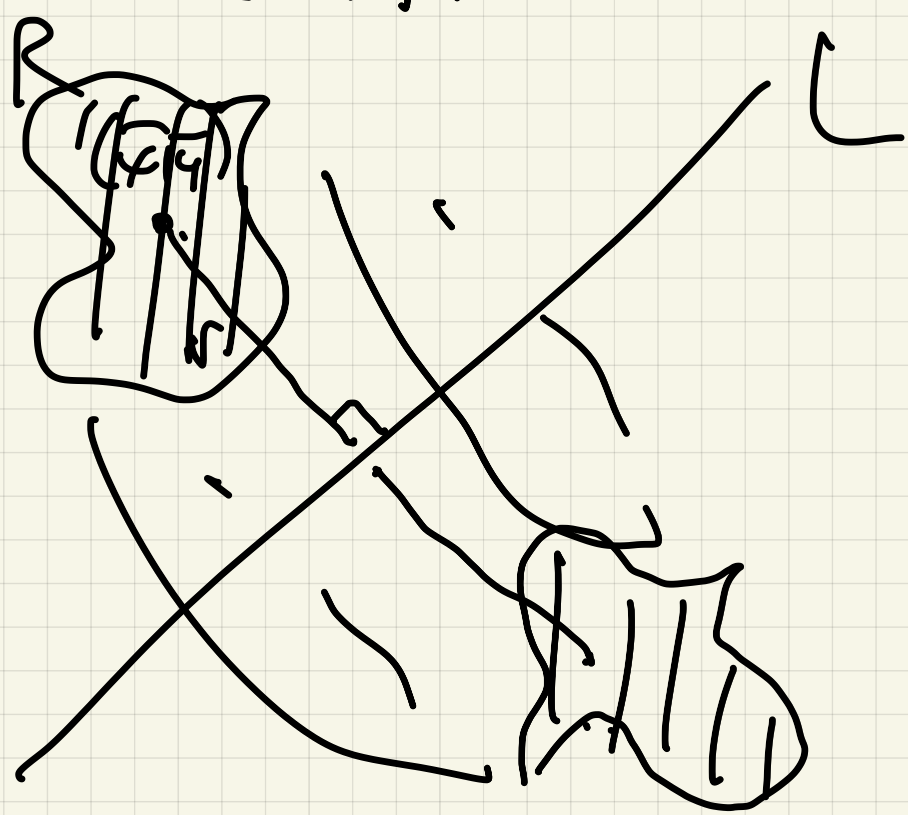
$$M_x = \int_0^1 \frac{x^2 - x^4}{2} \, dx = \frac{1}{2}\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1$$

$$= \frac{1}{2} \, \frac{2}{15} = \frac{1}{15}$$

$$s_U\ (\bar{x}, \bar{y}) = \left(\frac{\frac{1}{12}}{\frac{1}{6}}, \frac{\frac{1}{15}}{1/6} \right) = \left(\frac{1}{2}, \frac{2}{5} \right)$$

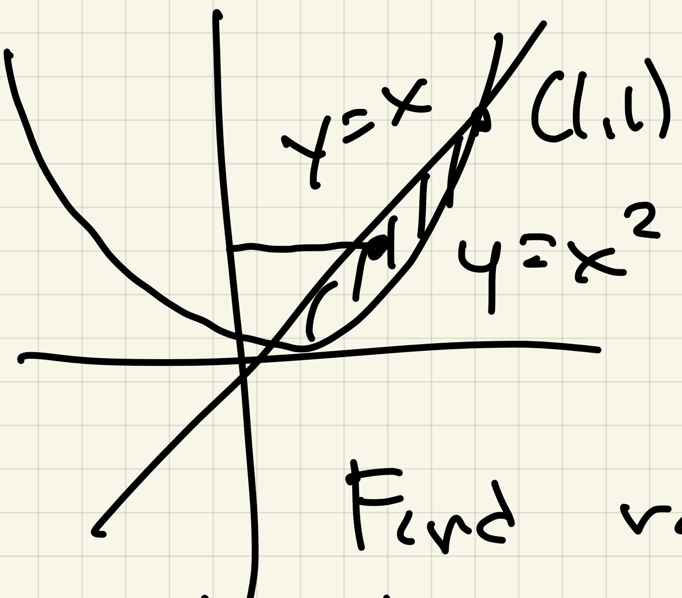
<u>Fun fact from Ancient Greeee!</u>

<u>Pappas</u>: If R is a region in plane with area A and COM $= (\bar{x}, \bar{y})$, L is a line of distance r to $(\bar{x}, \bar{y})$



Then volume of revolution is $V = 2\pi r A$

Ex 3 For region R is Ex2

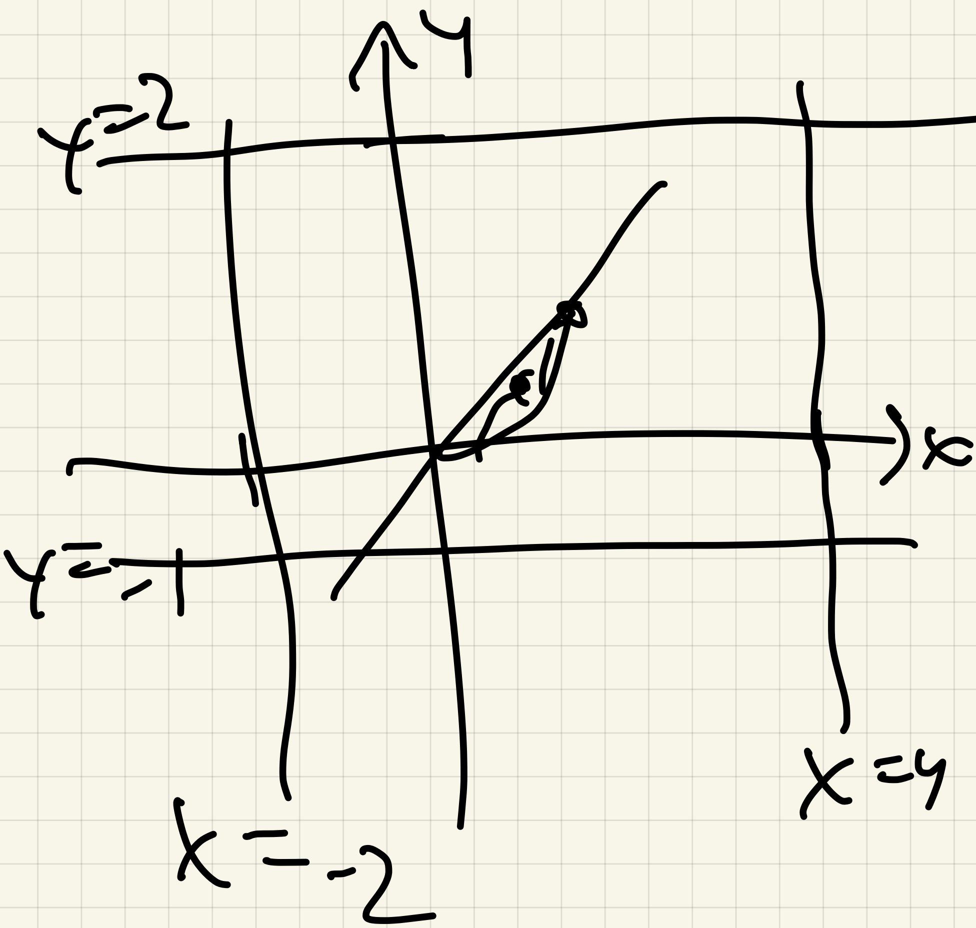


$$\boxed{COM = \left(\frac{1}{2}, \frac{2}{5}\right)}$$

$$\boxed{A = 1/6}$$

Find volume of revolution about

a) y-axis
b) x-axis
c) $x=4$
d) $y=2$
e) $x=-2$
f) $y=-1$



a) $A = 1/6, \quad r = 1/2 = \bar{x},$

$V = 2\pi r A = 2\pi \cdot \frac{1}{2} \cdot \frac{1}{6} = \pi/6$

b) $A = 1/6, \quad r = 2/5$

$V = 2\pi \cdot \frac{2}{5} \cdot \frac{1}{6} = 4\pi/30$

c) $r = 4 - 1/2 = 7/2$

$V = 2\pi \cdot \frac{7}{2} \cdot \frac{1}{6} = \frac{7\pi}{6}$

d) $r = 2 - 2/5 = 8/5 \implies$

$V = 2\pi \frac{8}{5} \cdot \frac{1}{6} = \frac{8\pi}{15}$

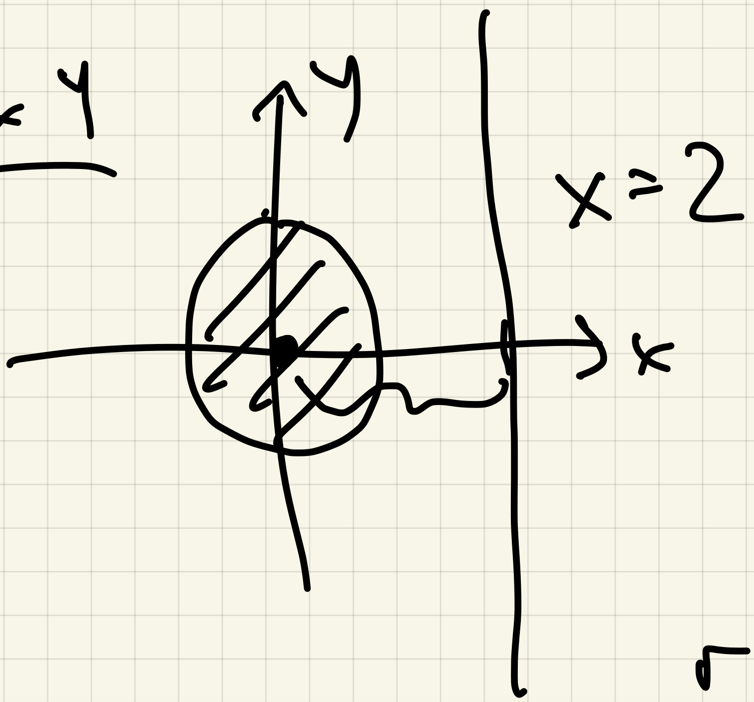
e) $r = \frac{1}{2} + 2 = \frac{5}{2}$

so $V = 2\pi \cdot \frac{5}{2} \cdot \frac{1}{6} = \frac{5\pi}{6}$

f) $r = 1 + 2/5 = 7/5$

$$V = 2\pi \cdot \frac{7}{5} \cdot \frac{1}{6} = \frac{7\pi}{15}$$

Ex 4



COM = (0,0)

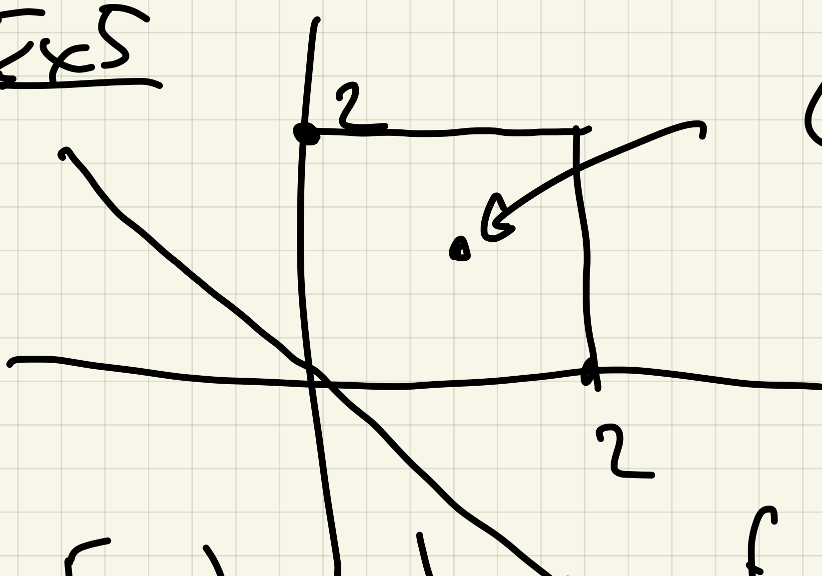
$A = \pi r^2 =$

$= \pi$

$r =$

So $V = 2\pi \cdot 2 \cdot \pi = 4\pi^2$

Ex 5



(1,1) = COM

$A = 4$

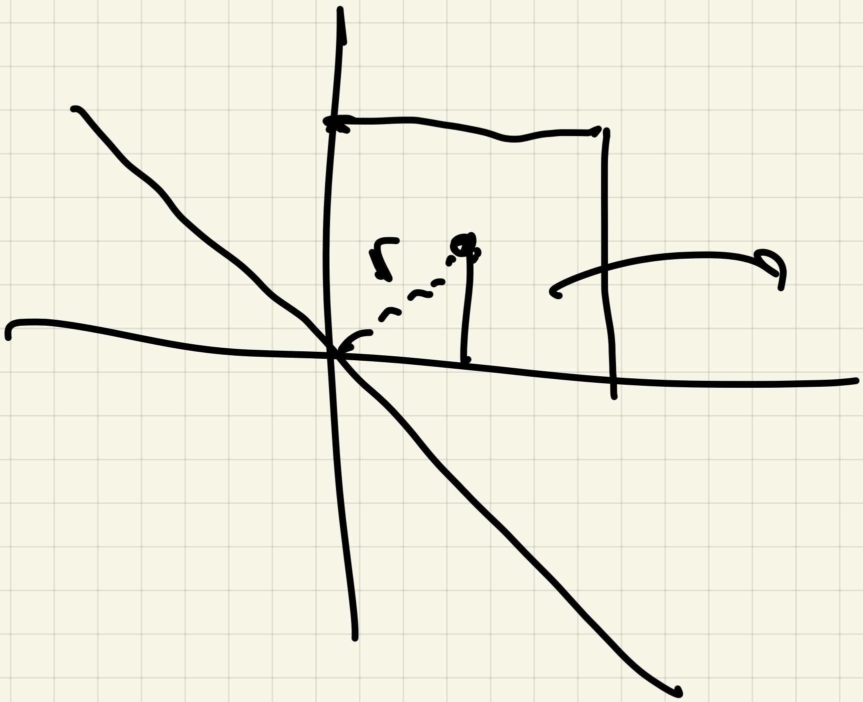
Find volume of revolution

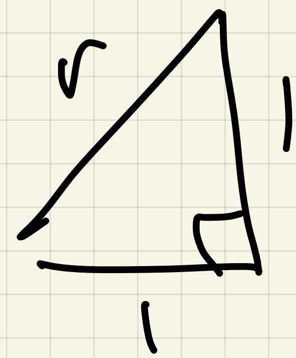
about (1) x-axis

② about line $y=-x$

① $r=1$, $A=4$, $V=2\pi r A$
$= 8\pi$

②





$r=\sqrt{2}$

so $V = 2\pi \cdot \sqrt{2} \cdot 4 = 8\sqrt{2}\pi$

Ch 8 Integration technique

Check table: $\boxed{\text{Table 8.1}}$

Know this except:

#5, 16, 17, 21, 22

$$\int a^x$$

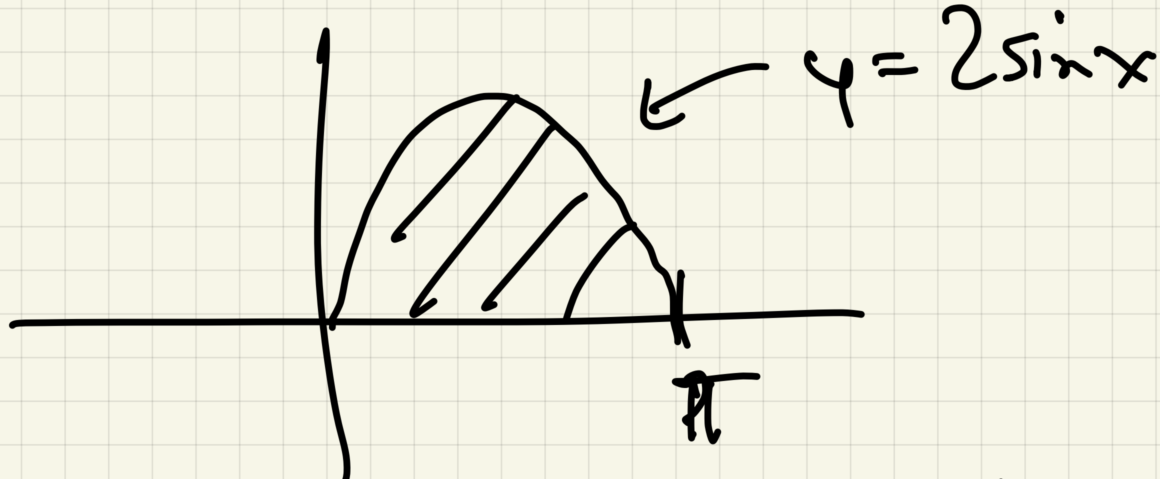
$$\int \tan x\,dx = -\ln|\cos x| + C$$
$$= \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$$

<u>Ex 0</u>



Find the volume ~~of~~ of revolution about

$\boxed{x\text{-axis}}$ & $\boxed{y\text{-axis}}$

$$\int_0^{\pi} \pi(2\sin x)^2\,dx = 4\pi \int_0^{\pi} \sin^2 x\,dx$$

idea

$$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$4\pi \int_0^{\pi} \frac{1-\cos 2x}{2}\,dx =$$

$$2\pi \int_0^{\pi} 1-\cos 2x\,dx =$$

$$2\pi\left(x - \cancel{\frac{\sin 2x}{2}}\right)\Big|_0^{\pi} =$$

0_π

$$= 2\pi^2$$

x-axis ✓

y-axis ; shells ;

$$V = \int_0^{\pi} 2\pi x \, 2\sin x \, dx$$

$$= 2\pi \int_0^{\pi} 2\underline{x\sin x}\, dx$$

x ??

$$\int 2x\sin x \, dx = \frac{x^2}{\cancel{2}}(-\cos x) \; ?!$$

NO

check: that

$$\boxed{-2x\cos x + 2\sin x}$$

this works!

$$-\cancel{2\cos x} + 2x\sin x + \cancel{2\cos x}$$

$$\parallel$$

$$2x\sin x$$

Product Rule:

$$(uv)' = u'v + uv'$$

$\parallel$ APPLY $\int$

$$uv = \int u'v + \int uv' \Rightarrow$$

$$\boxed{\int uv' = uv - \int u'v}$$

$$\int \underbrace{2x}_{u}\,\underbrace{\sin x}_{v'}\,dx = \underbrace{-2x\cos x}_{uv} - \int \underbrace{2(-\cos x)}$$

$$u' = 2 \qquad v = -\cos x$$

$$= -2x\cos x + 2\sin x + C$$

Integration by Parts:

$$\int u\,\underbrace{dv}_{=\frac{dv}{dx}} = uv - \int v\,\underbrace{du}_{=\frac{du}{dx}}$$

Ex) $\int \underbrace{x}_{u}\,\underbrace{e^x\,dx}_{dv} = uv - \int v\,du$

$=$

$du = dx \quad v = e^x$ //

$$xe^x - \int e^x dx$$

$$xe^x - e^x + C$$