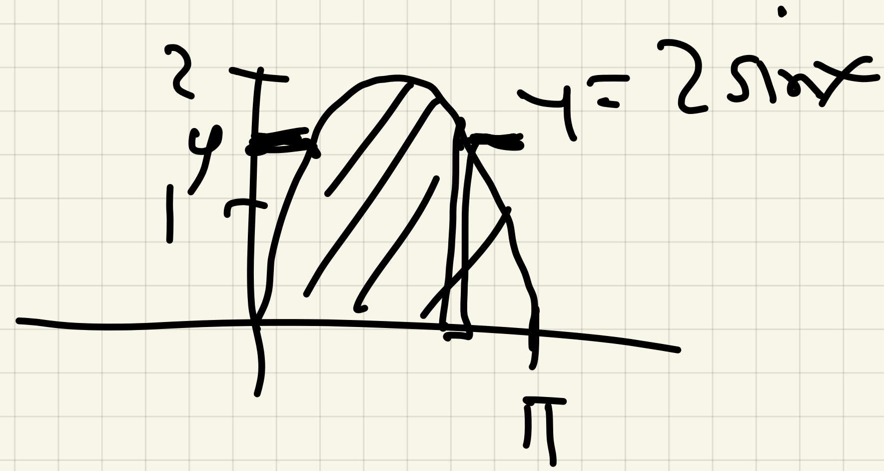
9/17 | Calc2 Quiz 4



$\boxed{1}$ $$\int_0^{\pi} 2\pi\, x\, 2\sin x \, dx$$

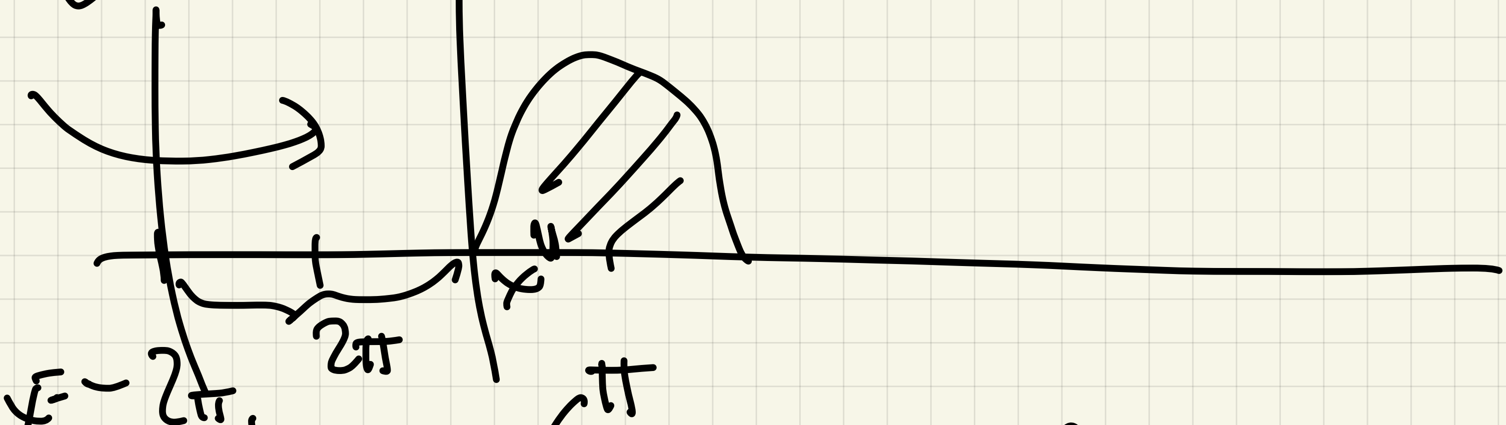
Also: $y = 2\sin x \Rightarrow$

$x = \arcsin \frac{y}{2}$

Washers: $$\int_0^{2} \pi \left(\left(\pi - \arcsin\frac{y}{2}\right)^2 - \left(\arcsin\frac{y}{2}\right)^2 \right) dy$$

$\boxed{2}$ $$\int_0^{\pi} \pi \, (2\sin x)^2 \, dx$$

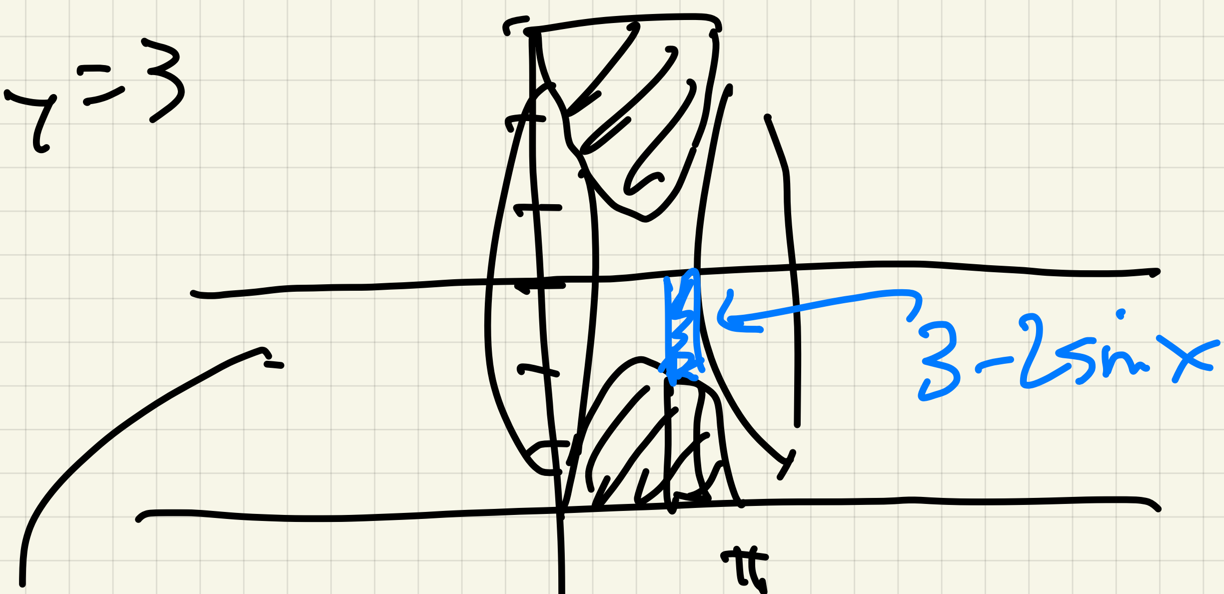
$\boxed{3}$



shells

$$\int_0^{\pi} 2\pi(x+2\pi)\,2\sin x\,dx$$

$\boxed{4}$ $y=3$



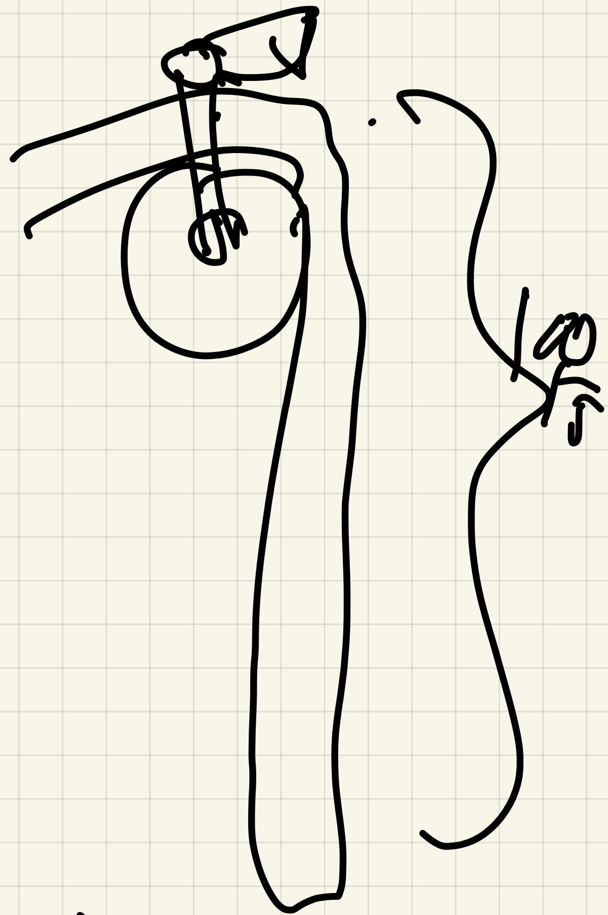
$$V=\int_0^{\pi}\pi\left(3^2-(3-2\sin x)^2\right)dx$$

Last time: Work problems

$$W = F \cdot d$$

$$W = \int_a^b F(x)\,dx \quad F = \text{variable force}$$

Chain problems



Ex 1

If x is distance from top,
Then

5 lbs/ft

$$W = \int_0^{100} x \cdot 5\,dx = 25000 \text{ ft-lbs}$$

$5\Delta x$

(b) First work to crank

the chain half way up.

$$W_{\substack{top \\ half}} = \int_0^{50} 5x\,dx = 6250$$

$$+$$

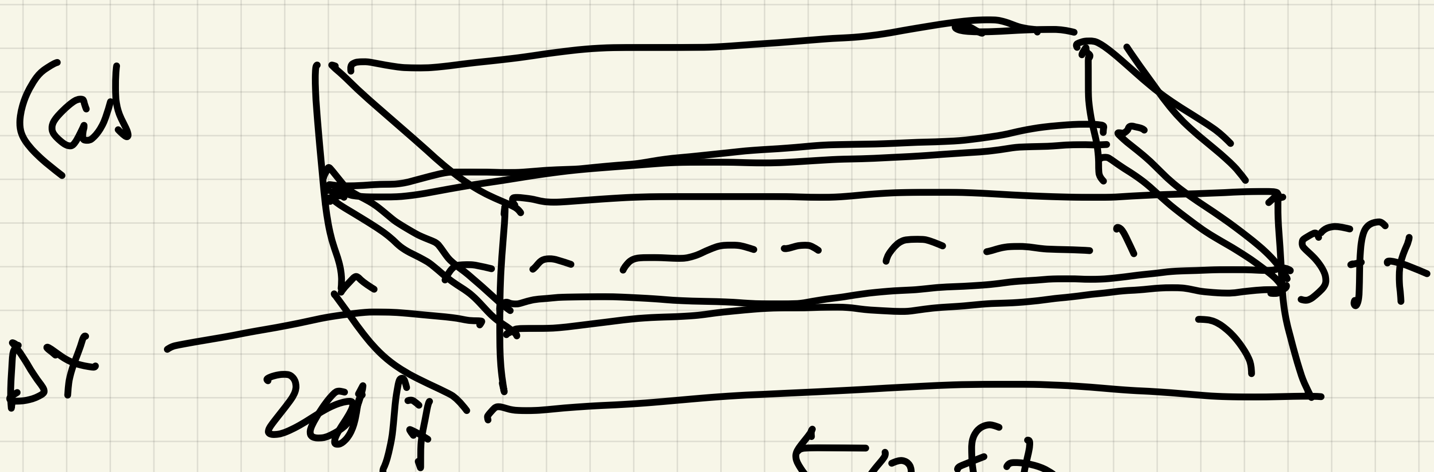
$$W_{\substack{bottom \\ half}} = \int_{50}^{100} 50 \cdot 5\,dx = 12500$$

$$18750$$

ft-lbs

~~Ex~~ Tank problems

(a)



filled with water,

weighs $62.4\ \text{lbs}/\text{ft}^3$

Find Work done to pump all water over the top of pool.

Idea:

Work done to move a layer of water at depth x_i of thickness Δx_i is

slab water weighs:

$$\underbrace{\underbrace{(50 \times 20 \times \Delta x_i)}_{\text{volume}} \cdot \underbrace{62.4}_{\text{density}}}_{\text{weight}}$$

so work done to move it top of pool is

$$\underbrace{(1000\,\Delta x_i)(62.4)}_{\text{wt}} \times \underbrace{x_i}_{\text{dist}}$$

Subdividing into n slabs of water

$$W \approx \sum_{i=1}^{n} (1000\,\Delta x_i)(62.4)(x_i)$$

exact work

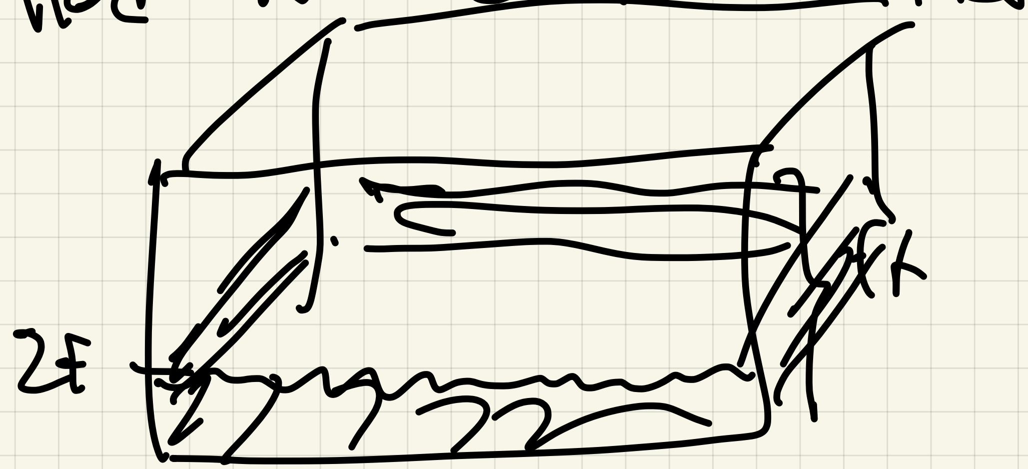
$$\int_0^5 62400\, x \, dx$$

$$\frac{x^2}{2}$$

$$W = 62400 \times \frac{25}{2} = 780{,}000$$

ft-lbs

(b) What if pool is only filled to 2-ft line

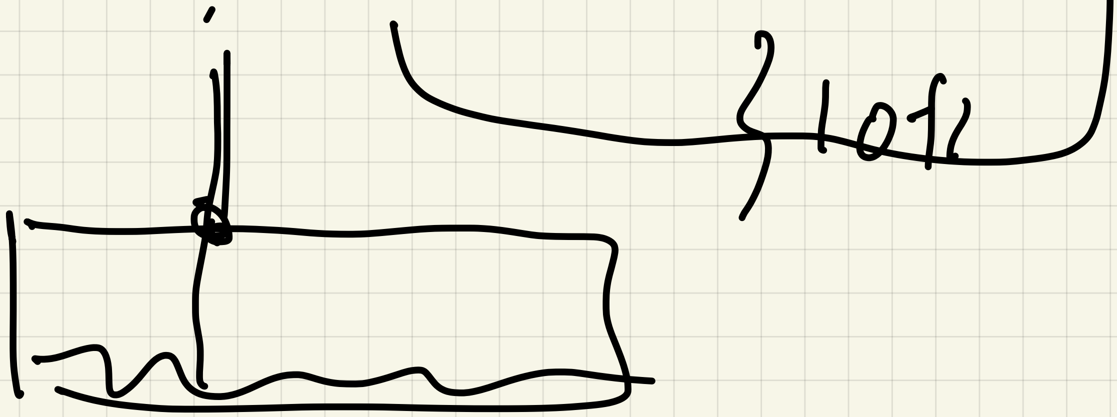


$$\int_3^5 62400\, x\, dx =$$

<u>Note</u>: $x = 3$ ft measured from <u>top</u>,

∴ corresponds to 2 ft line

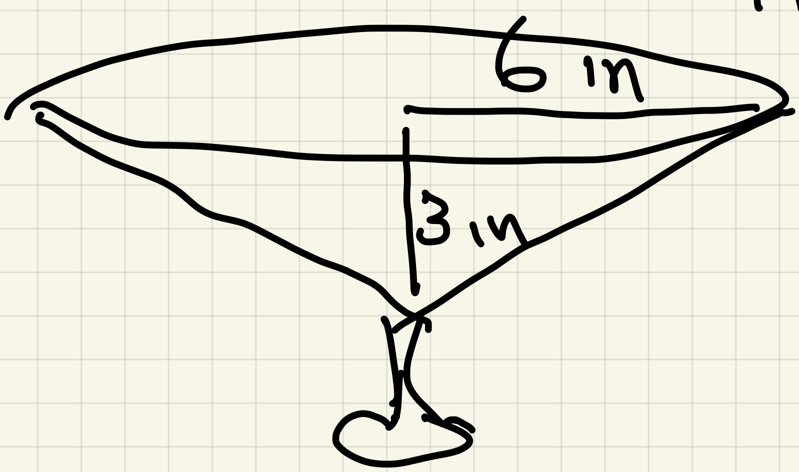
(c) What if we pump water to 10 ft over top



dist becomes $x+10$

$$\int_3^5 62400(x+10)\,dx$$

Ex2 Cone shaped glass filled with drink

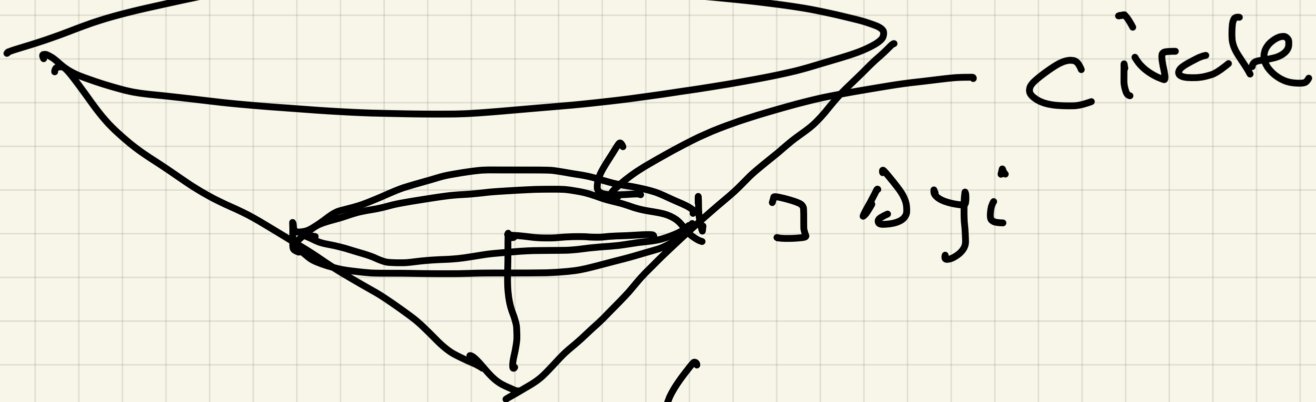


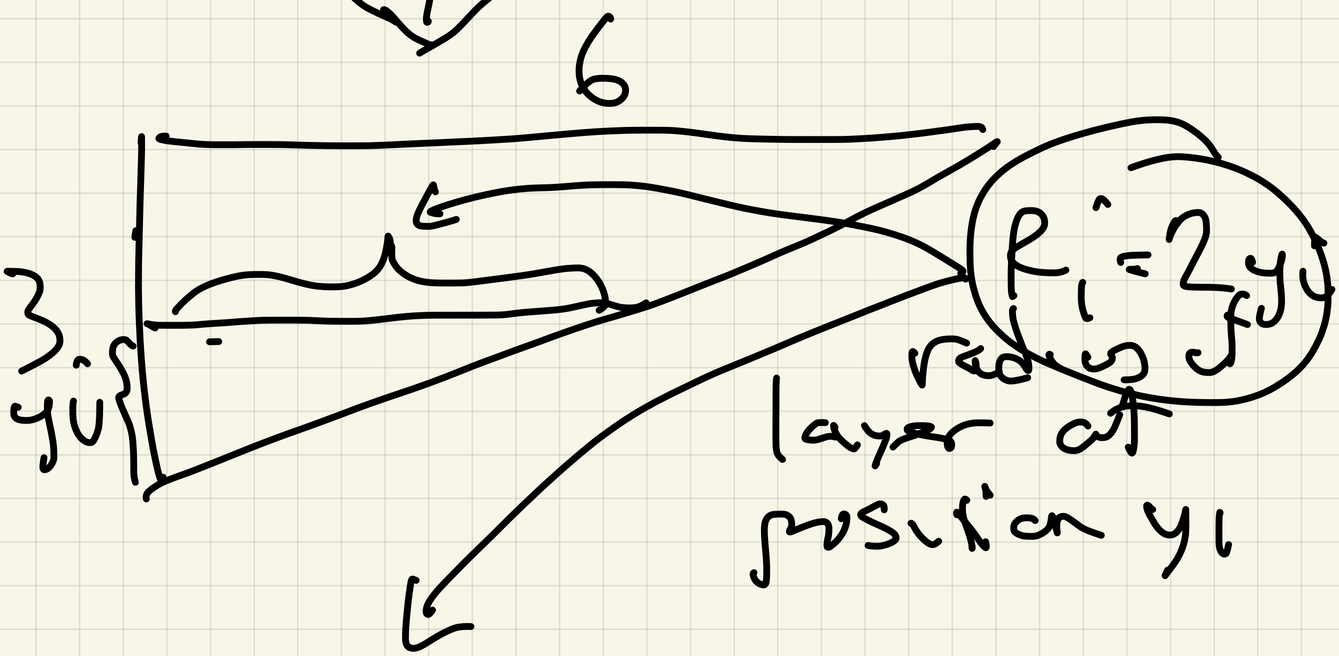
$.02\ lb/in^3$

Find work to empty martini glass:

y_i = vertical location of

a layer of drink,
measured from bottom





cross section area

$$\pi (R_i)^2 = \pi \cdot (2y_i)^2$$

∴ volume of layer

$$\pi (2y_i)^2 \Delta y_i$$

$\therefore$ weight of layer

$$\pi (2y_i)^2 \Delta y_i \cdot (.02)$$

dist to move layer is

$$\text{dist} = 3 - y_i$$

$\therefore$

$$W \approx \sum_{i=1}^{n} \underbrace{\pi (2y_i)^2 \Delta y_i (.02)(3-y_i)}$$

so

$\therefore$ Exact work

$$W = \int_0^3 \pi 4y^2 (3-y)(.02)\,dy$$

$$= \frac{27\pi}{50} \text{ in-lbs}$$

<u>Summary of method</u>:

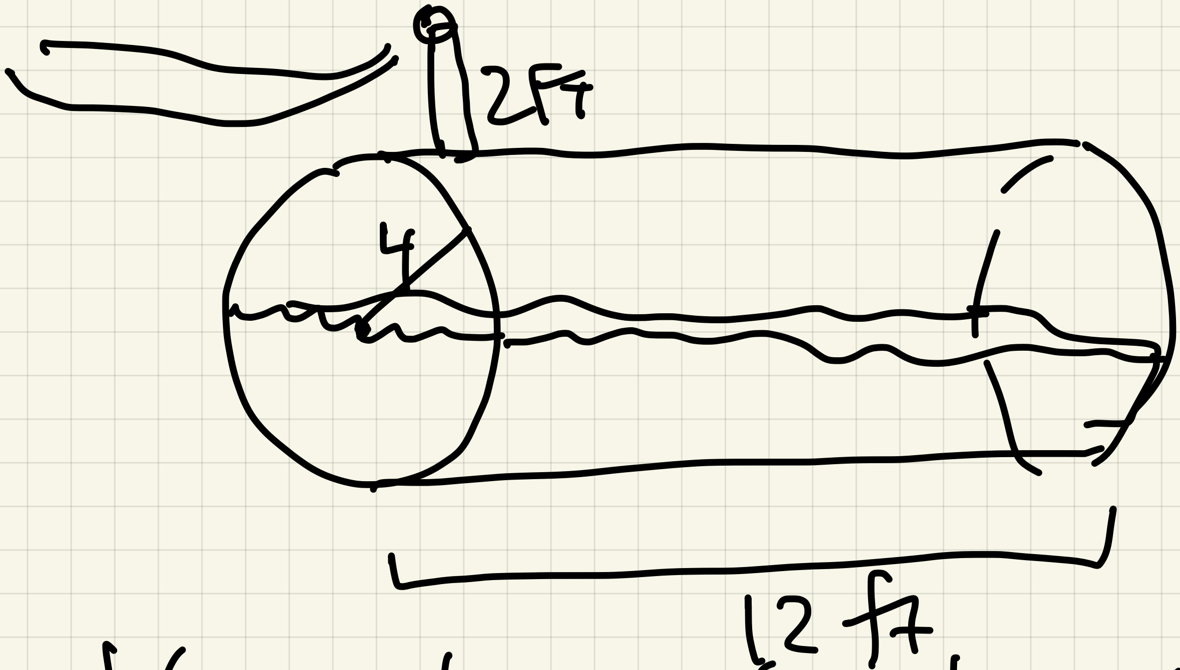
1. sketch
2. Set vertical variable y
3. Endpoints correspond to liquid
4. $A(y)$ cross section area
5. $d(y)$ = distance moved

$$\Downarrow$$

$$W = \int_a^b \rho \cdot A(y) \cdot d(y)\,dy$$

ρ: density

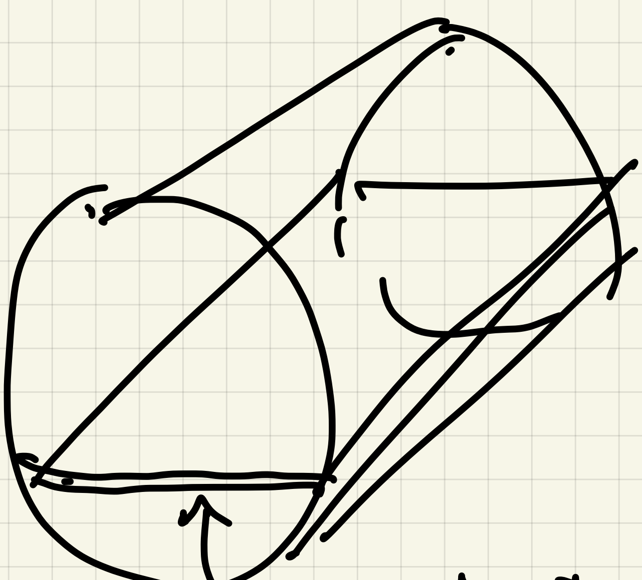
Ex3 A cylindrical gas tank lies on its side as shown:



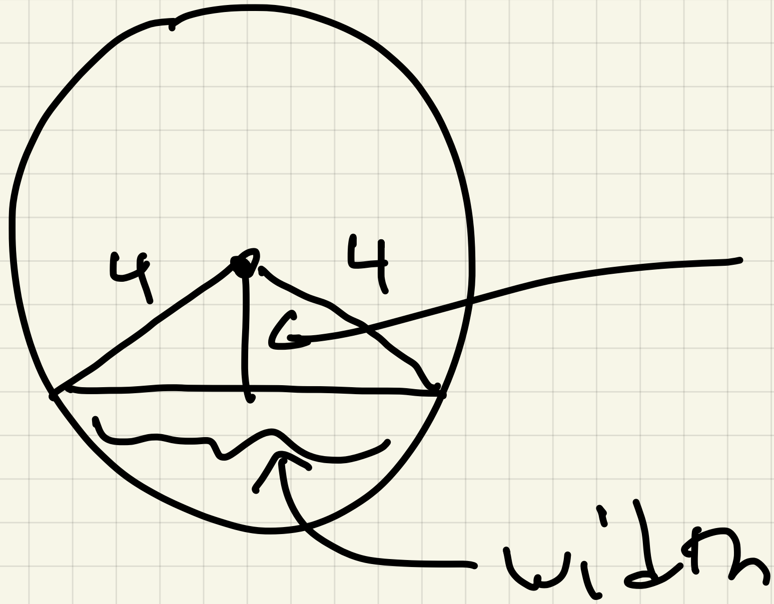
If tank is filled with oil weighing 50 lbs/ft^3

What is work done to pump out oil to 2 feet over top of tank?

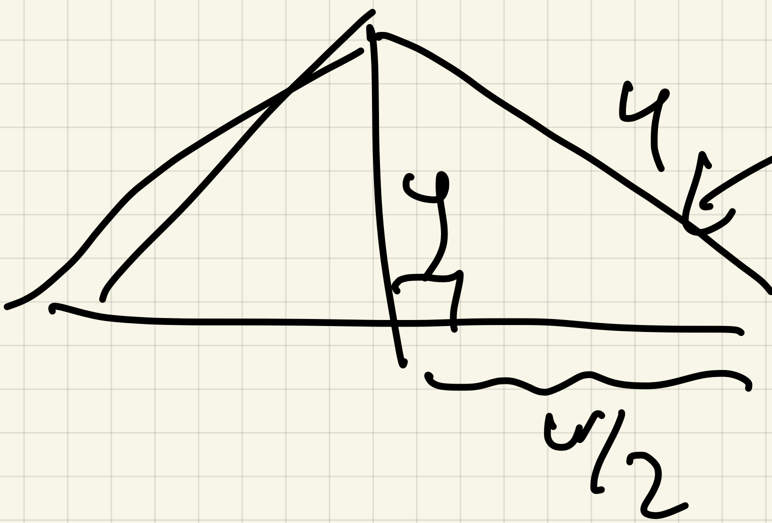
Ans:



what is width?



$y =$ ~~posit~~ height from center



$$4^2 = y^2 + \left(\frac{w}{2}\right)^2$$

$$\frac{w}{2} = \sqrt{16 - y^2}$$

$$w = 2\sqrt{16 - y^2}$$

$$\therefore \text{Area} = 12w = 12 \cdot 2\sqrt{\ }$$

$$= 24\sqrt{16 - y^2}$$

dist

$$W = \int_{-4}^{4} (50)\, 2y\sqrt{16-y^2}\,(y+6)\, dy$$