9/15/ Calc2

Work

Idea: $W = F \cdot d$

Work = W, force = F, distance = d

$N \cdot m$ = Joule

ft-lbs / in-lbs

Ex] Find the work done in

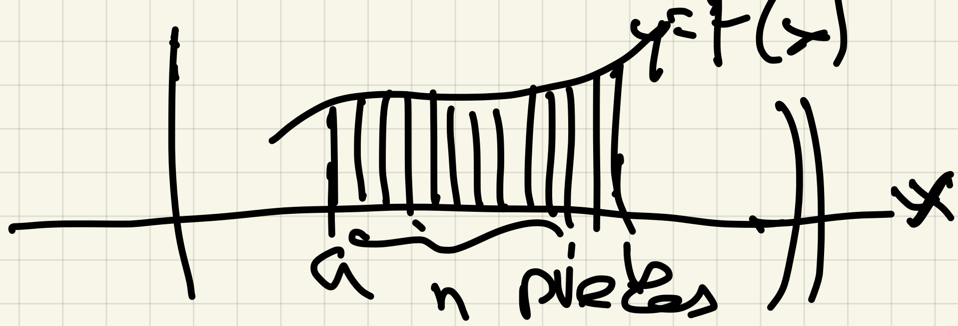
(a) lifting 200 lbs up by 3ft.

$W = 200 \times 3 = 600$ ft-lbs

(b) ~~Work~~ lifting 200 lbs up by 4 miles

Subtle: b/c gravity force is less far away.

Variable force



we move from a to b Force at position

Estimate

$$W \approx \sum_{i=1}^{n} F(x_i)\,\Delta x_i$$

force dist

$$\Big\Downarrow$$

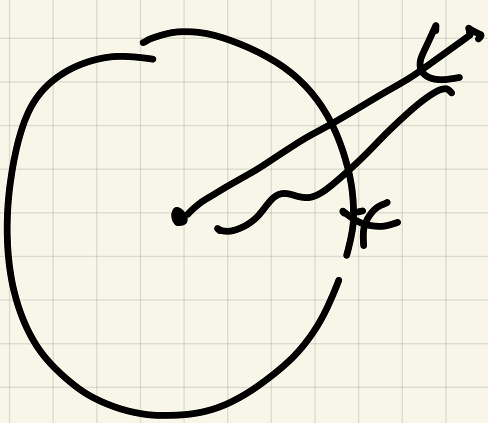
Exact work

$$W = \lim_{n\to\infty} \sum F(x_i)\,\Delta x_i = \int_a^b F(x)\,dx$$

Physics:

gravity $F(x)$ proportional $\frac{1}{x^2}$

x = dist to center of earth



i.e. $F(x) = \dfrac{c}{x^2}$

c constant

Radius of earth is 4000 mil

$$200 \text{ lbs} = \frac{C}{(4000)^2}$$

$$C = 200 \times 4000^2 = 3.2 \times 10^9$$

(b) $$W = \int_{4000}^{4004} \frac{3.2 \times 10^9}{x^2}\, dx =$$

$$(3.2 \times 10^9) \left. -\frac{1}{x} \right|_{4000}^{4004} =$$

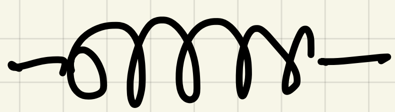
$$799.2808 \text{ mile-lbs}$$

(<u>Compare</u>: using const force $200 \times 4 = 800$)

(c) If we lift weight 4000 miles

$$W = \int_{4000}^{8000} \quad dx = 400{,}000 \text{ mile-lbs}$$

$$(\text{compare } 200 \times 4000 = 800{,}000)$$

<u>Spring Problems</u> 

<u>Hooke's law</u>: If a spring is pulled/pushed x units beyond natural length, the spring reacts with force $\underline{\underline{F = k \cdot x}}$, k = spring const

$$(x = 0,\ F = 0)$$

<u>Ex 1</u> A force of 60 lbs holds a spring 2 ft

beyond its natural length

(a) find spring const

(b) work to stretch spring from rest to 3 ft.

(c) work to compress spring to 1 ft shorter than rest length.

(d) work to stretch from 1 ft shorter than natural length out to 3 ft longer.

(a) $\underset{60 \text{ lbs}}{F} = k \cdot \underset{2 \text{ ft}}{x} \Rightarrow k = 30 \text{ lbs/ft}$

(b) $9 \times 15 =$

$$W = \int_0^3 30x\,dx = 15x^2\Big|_0^3 = 135 \text{ ft-lbs}$$

(c) $$W = \int_0^{-1} 30x\,dx = 15x^2\Big|_0^{-1} = 15 \text{ ft-lbs}$$

(d)
$$W = \int_{-1}^{3} 30x\,dx = 15x^2\Big|_{-1}^{3} =$$
$$120 \text{ ft-lbs}$$

<u>Ex2</u> 150 ft-lbs work is required to stretch a spring 6 inches natural length.

(a) Find spring constant

(b) Work done to stretch to 12 inches

(c) 50 ft-lbs work compresses the spring how far.

Units: Use inches & inch-lbs

$150 \text{ ft-lbs} \times \frac{12 \text{ inch}}{1 \text{ ft}} = 1800 \text{ in-lbs}$

(a) $= 1800 = \int_0^6 kx\,dx$

$= \frac{1}{2}kx^2 \Big|_0^6$

$= \frac{1}{2}k\,36 = 18k$

$k = 100 \text{ lbs/in}$

(b) $\int_0^{12} 100x\,dx = 50x^2 \Big|_0^{12} = 7200$ in-lbs

(c) $\int_0^{l} 100x\,dx = 50$ ft-lbs $= 600$ in-lbs

$l =$ end length

$=$

$$50x^2 \Big|_0^{\ell} =$$

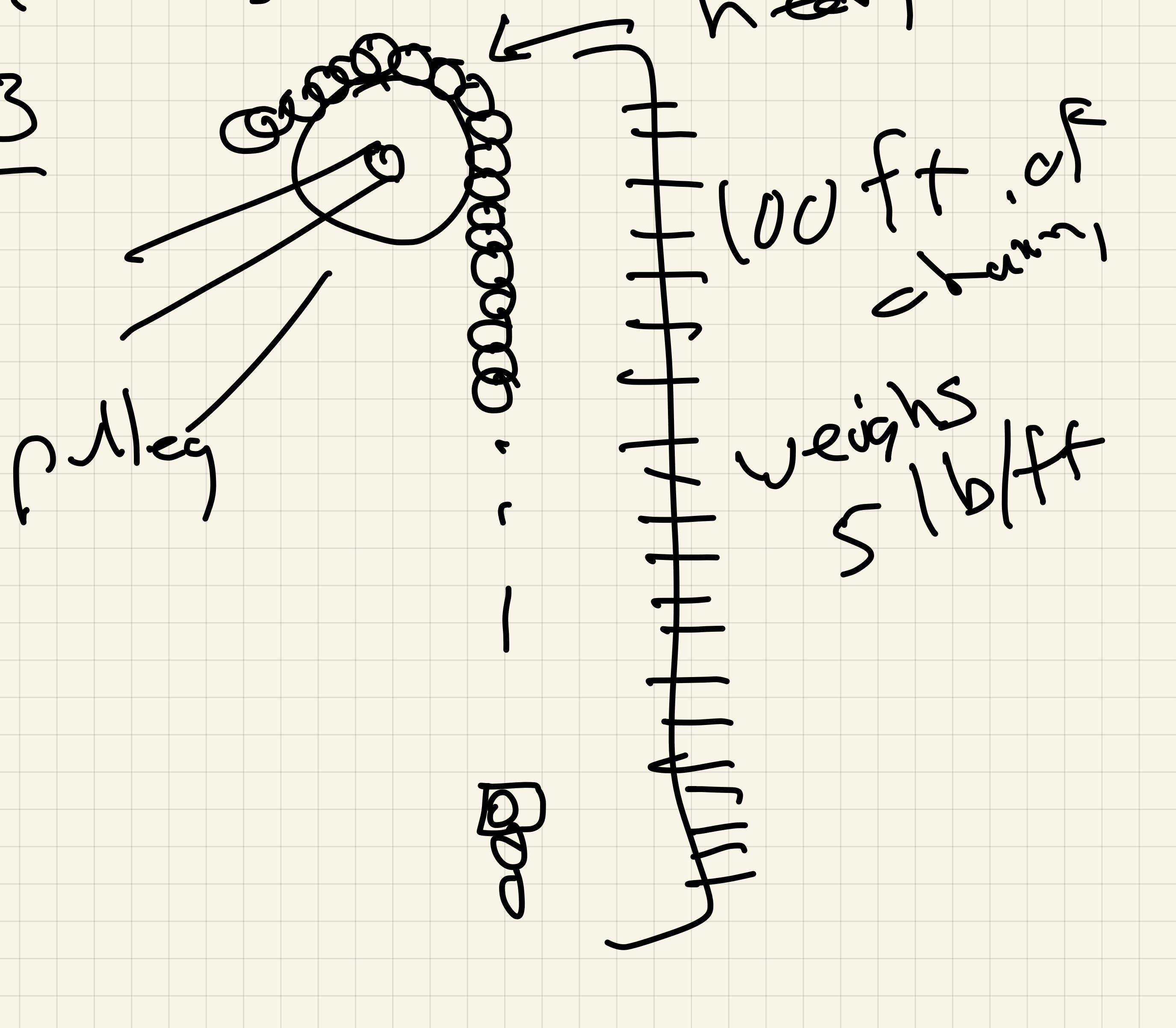
$$50\ell^2 = 600$$

$$\ell^2 = 12$$

$$\ell = \sqrt{12} \text{ inches}$$

Varying distance

Ex3



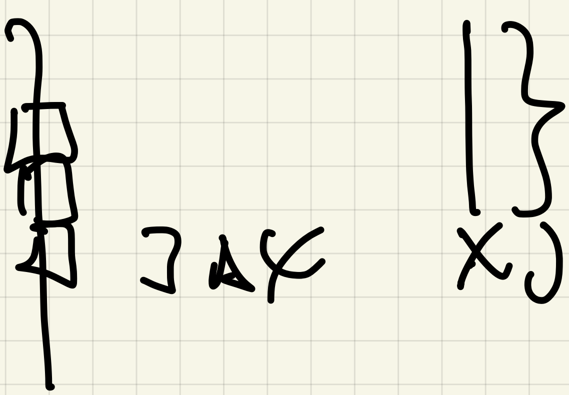
(a) How much work to pull ~~all~~ chain over the top of pulley?

(b) " " " " the chain halfway up?

Idea: ~~consider~~ a

break the 100 ft into n equal segments, of length Δx

consider a segment at position x_i feet from top: the work to move it to top is



$$\underbrace{5\Delta x}_{\text{weight}} \times \underbrace{x_i}_{\text{dist}}$$

Adding up over segments

$$W_i = 5\Delta x \, x_i$$

so

Total work $\approx \sum W_i = \sum_{i=1}^{n} 5 x_i \Delta x$

<u>Exact work</u> $= \int_0^{100} 5x \, dx$

$$= \frac{5x^2}{2}\Big|_0^{100} = \frac{50000}{2} = 25000 \text{ ft·lbs}$$